Construction Economics & Finance

Module 1

Engineering economics

Lecture-1

Basic principles:–

<mark>Time value of money:</mark>

The time value of money is important when one is interested either in investing or borrowing the money. If a person invests his money today in bank savings, by next year he will definitely accumulate more money than his investment. This accumulation of money over a specified time period is called as time value of money.

Similarly if a person borrows some money today, by tomorrow he has to pay more money than the original loan. This is also explained by time value of money.

The time value of money is generally expressed by interest amount. The original investment or the borrowed amount (i.e. loan) is known as the **principal**.

The amount of interest indicates the increase between principal amount invested or borrowed and the final amount received or owed.

In case of an investment made in the past, the total amount of interest accumulated till now is given by;

Amount of interest = Total amount to be received – original investment (i.e. principal amount)

Similarly in case of a loan taken in past, the total amount of interest is given by;

Amount of interest = Present amount owed – original loan (i.e. principal amount)

In both the cases there is a net increase over the amount of money that was originally invested or borrowed.

When the interest amount is expressed as the percentage of the original amount per unit time, the resulting parameter is known as the *rate of interest* and is generally designated as '*i*'.

The time period over which the interest rate is expressed is known as the *interest period.* The interest rate is generally expressed per unit year. However in some cases the interest rate may also be expressed per unit month.

Example: 1

A person deposited Rs.1,00,000 in a bank for one year and got Rs.1,10,000 at the end of one year. Find out the total amount of interest and the rate of interest per year on the deposited money.

Solution:

The total amount of interest gained over one year = Rs.1,10,000 - Rs.1,00,000 = Rs.10,000

The rate of interest 'i' per year is given by;

$$i(\%) = \frac{Rs.10,000}{Rs.1,00,000} \times 100 = 10\%$$

Similarly if a person borrowed Rs.1,50,000 for one year and returned back Rs.1,62,000 at the end of one year.

Then the amount of interest paid and the rate of interest are calculated as follows;

The total amount of interest paid = Rs.1,62,000 - Rs.1,50,000 = Rs.12,000

The rate of interest 'i' per year is given by;

$$i(\%) = \frac{Rs.12,000}{Rs.1,50,000} \times 100 = 8\%$$

Simple interest:

The interest is said to simple, when the interest is charged only on the principal amount for the interest period. No interest is charged on the interest amount accrued during the preceding interest periods. In case of simple interest, the total amount of interest accumulated for a given interest period is simply a product of the principal amount, the rate of interest and the number of interest periods. It is given by the following expression.

$$I_T = P \times n \times i$$

Where

I_T = total amount of interest

P = Principal amount

n = number of interest periods

i = rate of interest

Simple interest reflects the effect of time value of money only on the principal amount.

Compound interest:

The interest is said to be compound, when the interest for any interest period is charged on principal amount plus the interest amount accrued in all the previous interest periods. Compound interest takes into account the effect of time value of money on both principal as well as on the accrued interest also.

The following example will explain the difference between the simple and the compound interest.

Example: 2

A person has taken a loan of amount of Rs.10,000 from a bank for a period of 5 years. Estimate the amount of money, the person will repay to the bank at the end of 5 years for the following cases;

a) Considering simple interest rate of 8% per year

b) Considering compound interest rate of 8% per year.

Solution:

a) Considering the simple interest @ 8% per year;

The interest for each year $= Rs.10,000 \times 0.08 = Rs.800$

The interest for each is year is calculated only on the principal amount i.e. Rs.10,000. Thus the interest accumulated at the end of each year is constant i.e. Rs.800.

The year-by-year details about the interest accrued and amount owed at the end of each year are shown in Table1.1.

Table 1.1 Payment using simple interest

End of year (EOY)	Amount of interest (Rs.)	Total amount owed (Rs.)
1	800	10,800
2	800	11,600
3	800	12,400
4	800	13,200
5	800	**14,000***

* The amount repaid to the bank at the end of year 5 (since the person has to repay at EOY 5).

The total amount owed at the end of each year using simple interest is graphically shown in Fig. 1.1.
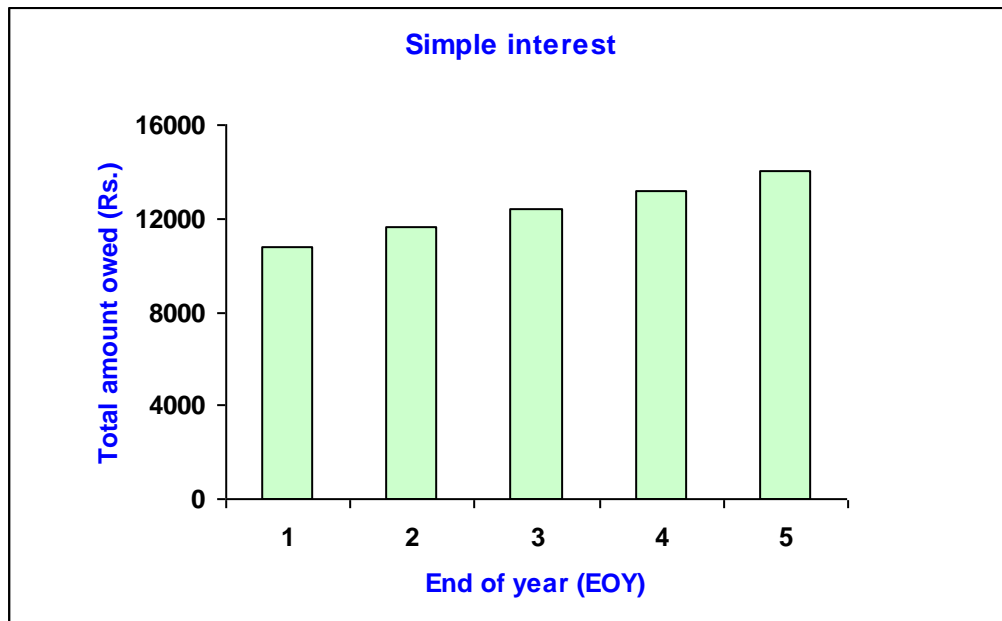

Fig. 1.1 Total amount owed (using simple interest)

b) Considering the compound interest @ 8% per year;

The amount of interest and the total amount owed at the end of each year, considering compound interest are presented in Table 1.2.

Table 1.2 Payment using compound interest

End of year (EOY)	Amount of interest (Rs.)	Total amount owed (Rs.)
1	800.00	10,800.00
2	864.00	11,664.00
3	933.12	12,597.12
4	1007.77	13,604.89
5	1088.39	**14,693.28****

** *The amount repaid to the bank at the end of year 5.*

The year-by-year values of amount of interest and the total amount owed (as shown in Table 1.2) are calculated as follows;

$\text{Amount of interest accumulated at the end of Year } 1 = \text{Rs.}10,000 \times 0.08 = \text{Rs.}800$

$\text{Total amount owed at the end of Year } 1 = \text{Rs.}10,000 + \text{Rs.}800 = \text{Rs.}10,800$

$\text{Amount of interest accumulated at the end of year } 2 = \text{Rs.}10,800 \times 0.08 = \text{Rs.}864$

$\text{Total amount owed at the end of Year } 2 = \text{Rs.}10,800 + \text{Rs.}864 = \text{Rs.}11,664$

Similarly the interest amount and the total amount owed at the end of year 3, year 4 and year 5 can be calculated in the same manner.

From these calculations it is clear that, in case of compound interest the interest for each year is calculated on the principal amount plus the interest amount accumulated till that period.

The total amount owed at the end of each year using compound interest is also graphically shown in Fig. 1.2.
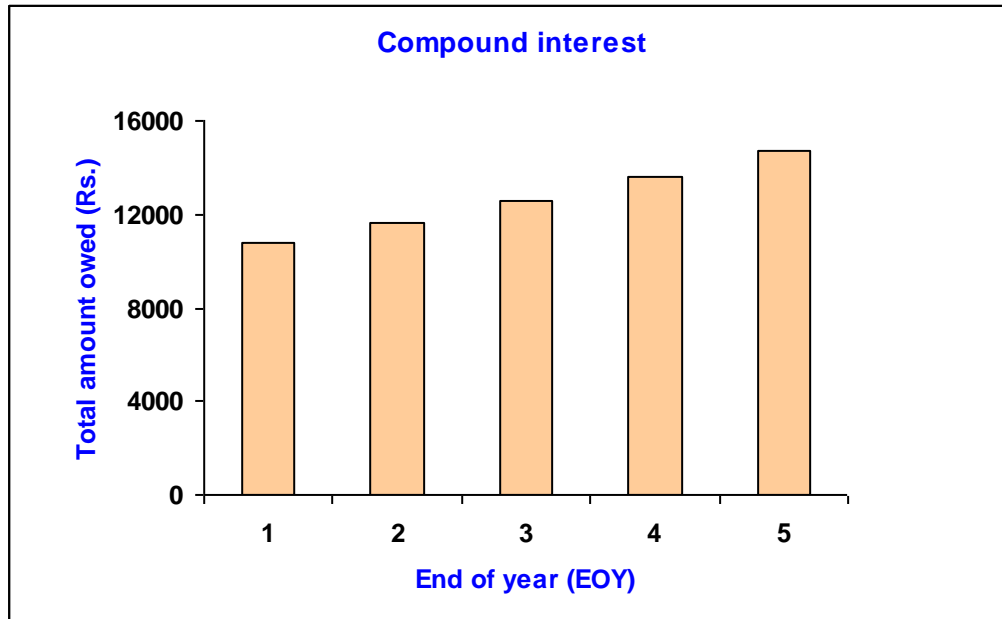
Fig. 1.2 Total amount owed (using compound interest)

The accrued interest at the end of each year considering both simple and compound interest is also shown in Fig. 1.3.

From Fig. 1.3, one can see that in case of simple interest, the amount of interest accumulated each year is constant. However in case of compound interest, the interest amount accumulated at the end of each year is not constant and increases with interest period as evident from Fig. 1.3. Thus considering compound interest, the total amount to be repaid at the end of year 5 is Rs.14,693.28, which is greater as compared to Rs.14,000 that is to be repaid on the basis of simple interest.
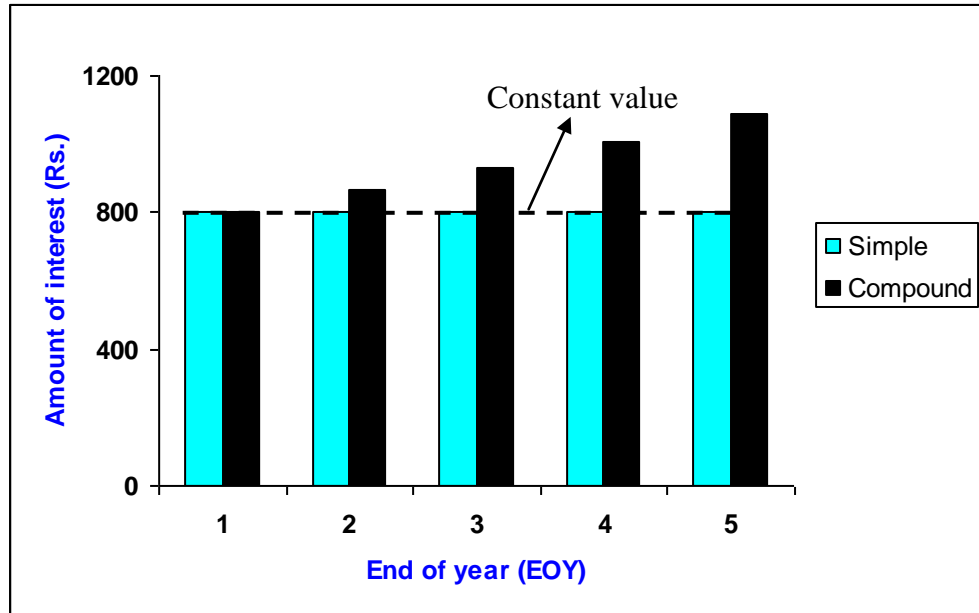
Fig. 1.3 Accumulated interest at the end of each year

Lecture-2

Equivalence:

Equivalence indicates that different amount of money at different time periods are equivalent by considering the time value of money. The following simple example will explain the meaning of equivalence.

Example: 3

What are the equivalent amounts of Rs.10000 (today) at an interest rate of 10% per year for the following cases?

a) 1 year from now (future)

b) 1 year before

Solution:

a) At interest rate of 10% per year, Rs.10000 (now) will be equivalent to Rs.11000 one year from now as shown below;

$$\text{Amount accumulated at the end of one year} = \text{Rs.}(10000 \times 1.10) = \text{Rs.}11000$$

b) Similarly Rs.10000 now was equivalent to Rs.9090.90 one year ago at interest rate of 10% per year.

$$\text{Amount one year before} = \text{Rs.}\left(10000 \middle/ 1.10\right) = \text{Rs.}9090.90$$

Thus due to the effect of time value of money, these amounts Rs.9090.90 (one year before), Rs.10000 (today) and Rs.11000 (one year from now) are equivalent at the interest rate of 10% per year. It is shown in Fig. 1.4.
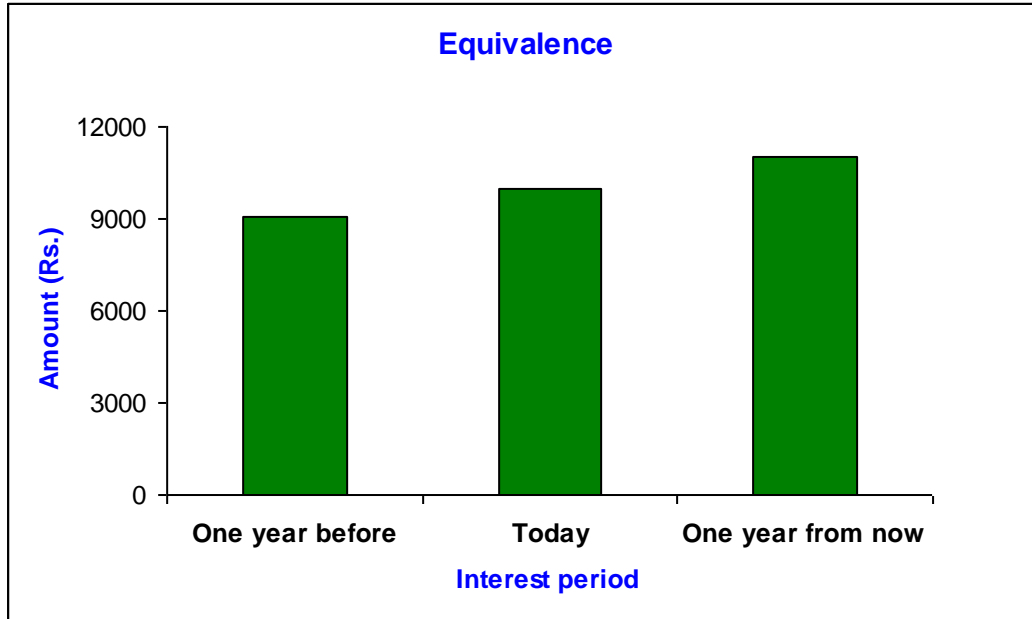
Fig 1.4 Schematic representation of equivalence

The equivalent value of an amount that is borrowed now, at future time period at a given interest rate depends on the type of interest whether simple or compound and the different loan repayment arrangements like payment of accumulated interest annually and principal at the end of the stipulated interest periods or payment of both the principal and interest at the end interest periods or payment of uniform amounts annually that comprises a portion towards the payment of principal amount and remaining for the accumulated interest throughout the interest periods.

Quantifying alternatives for decision making:

Quantifying alternatives for any item is the most important aspect of decision making for selecting the best option. For example, a construction company is planning to purchase a new concrete mixer for preparing concrete at a construction site. Let's say there are two alternatives available for purchasing the mixer; a) an automatic concrete mixer and b) a semi-automatic concrete mixer. Then the task is to find out best alternative that the company will purchase that will yield more profit. For this purpose one has to quantify both the alternatives by the following parameters;

✓ The initial cost that includes purchase price, sales tax, cost of delivery and cost of assembly and installation.

- ✓ Annual operating cost.
- ✓ Annual profit which will depend on the productivity i.e. quantity of concrete prepared.
- ✓ The expected useful life.
- ✓ The expected salvage value.
- ✓ Other expenditure or income (if any) associated with the equipment.
- ✓ Income tax benefit

Then on the basis of the economic criteria, the best alternative is selected by calculating the present worth or future worth or the equivalent uniform annual worth of both alternatives by incorporating the appropriate interest rate per year and the number of years (i.e. the comparison must be made over same number of years for both alternatives). Then the concrete mixer with least cost or higher net income is considered for purchase. In addition to economic parameters as mentioned above, the non-economic parameters namely environmental, social, and legal and the related regulatory and permitting process must also be considered for the evaluation and selection of the best alternative. These non-economic parameters are essentially required (in addition to the economic factors) for the selection of the best alternative for the infrastructure and heavy construction projects like dams, bridges, roadways etc. and other publicly and privately funded projects namely office buildings, hospitals, apartment building and shopping malls etc. When the available alternatives exhibit the same equivalent cost or same net income, then the non-economic parameters may play a vital role in the selection of the best alternative. It may be noted here that the non-economic parameters cannot be expressed in numerical values.

Lecture-3

Cash flow diagram:

The graphical representation of the cash flows i.e. both cash outflows and cash inflows with respect to a time scale is generally referred as cash flow diagram.

A typical cash flow diagram is shown in Fig. 1.5. The cash flows are generally indicated by vertical arrows on the time scale as shown in Fig. 1.5.

The cash outflows (i.e. costs or expense) are generally represented by vertically downward arrows whereas the cash inflows (i.e. revenue or income) are represented by vertically upward arrows.

In the cash flow diagram, number of interest periods is shown on the time scale. The interest period may be a quarter, a month or a year. Since the cash flows generally occur at different time intervals within an interest period, for ease of calculation, all the cash flows are assumed to occur at the end of an interest period. Thus in Fig. 1.5, the numbers on the time scale represent the end of year (EOY).
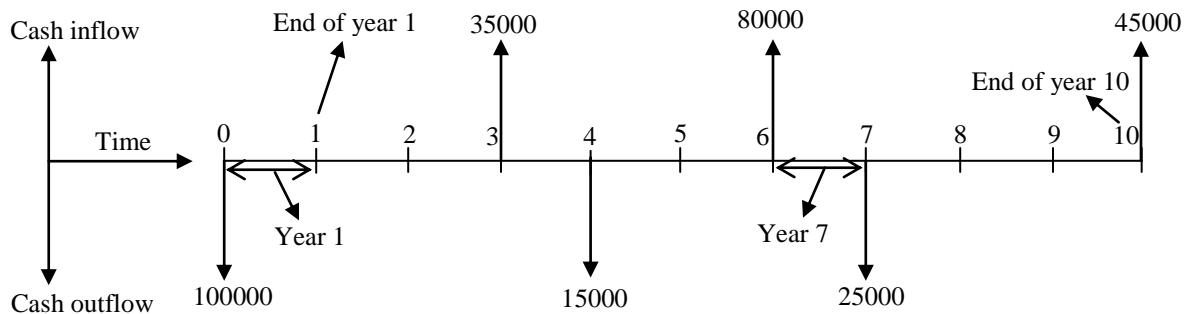

Fig 1.5 Cash flow diagram

In Fig. 1.5 the cash outflows are Rs.100000, Rs.15000 and Rs.25000 occurring at end of year (EOY) '0' i.e. at the beginning, EOY '4' and EOY '7' respectively. Similarly the cash inflows Rs.35000, Rs.80000 and Rs.45000 are occurring at EOY '3', EOY '6' and EOY '10' respectively.

Compound interest factors:

The compound interest factors and the corresponding formulas are used to find out the unknown amounts at a given interest rate continued for certain interest periods from the

known values of varying cash flows. The following are the notations used for deriving the compound interest factors.

P = Present worth or present value

F = Future worth or future sum

A = Uniform annual worth or equivalent uniform annual worth of a uniform series continuing over a specified number of interest periods

n = number of interest periods (years or months)

i = rate of interest per interest period i.e. % per year or % per month

Unless otherwise stated, *the rate of interest is compound interest* and is for the entire number of interest periods i.e. for 'n' interest periods.

The present worth (P), future worth (F) and uniform annual worth (A) are shown in Fig. 1.6.

In this figure the present worth, P is at the beginning and the uniform annual series with annual value 'A' is from end of year 1 till end of year 5. Both 'P' and 'A' are cash outflows. It may be noted that the uniform annual series with annual value 'A' may be also continued throughout the entire interest periods i.e. from beginning till end of year 10 or for some intermediate interest periods like commencing from end of year 3 till end of year 8.

The future worth 'F' is occurring at end of year 4 (cash outflow), at end of year 6 (cash inflow) and at the end of year 10 (cash inflow).
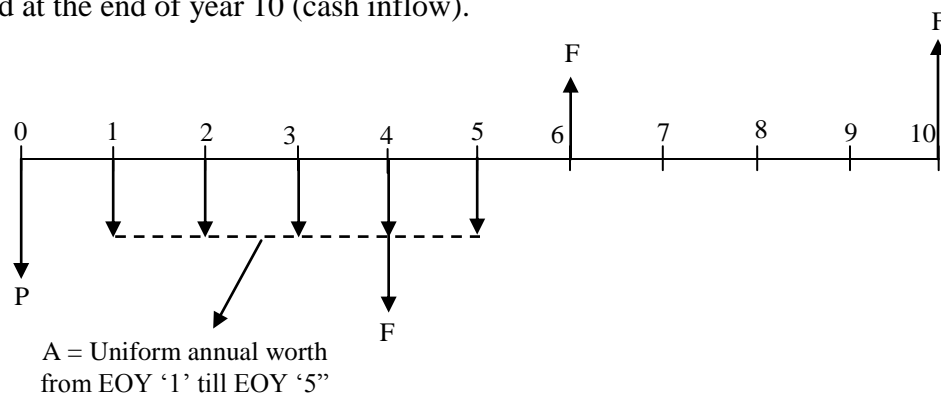

Fig 1.6 Cash flow diagram showing P, F and A

While deriving the different compound interest factors, it is assumed that the interest is *compounded once per interest period i.e. discrete compounding.* Further the cash flows are assumed to be discrete i.e. *they occur at the end of interest period.*

Single payment compound amount factor (SPCAF)

The single payment compound amount factor is used to compute the future worth (F) accumulated after "n" years from the known present worth (P) at a given interest rate *'i'* per interest period. It is assumed that the interest period is in years and the interest is compounded once per interest period.

The known present worth (P), unknown future worth (F) and the total interest period 'n' years are shown in Fig. 1.7.
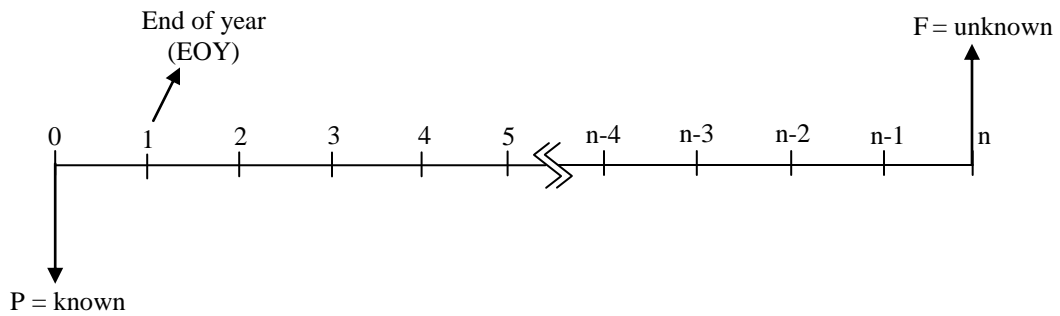

Fig 1.7 Cash flow diagram for 'known P' and 'unknown F'

The future worth (F_1) accumulated at the end of year 1 i.e. 1st year is given by;

$$F_1 = P + P \times i = P(1+i) \qquad \text{.......................................} (1)$$

The future worth accumulated at the end of year 2 i.e. F_2 will be equal to the amount that was accumulated at the end of 1st year i.e. F_1 plus the amount of interest accumulated from end of 1st year to the end of 2nd year on F_1 and is given by;

$$F_2 = F_1 + F_1 \times i = F_1(1+i) \qquad \text{...........................} (2)$$

Putting the value of F_1 from equation (1) in equation (2), the value of F_2 is given by;

$$F_2 = F_1(1+i) = P(1+i)(1+i) = P(1+i)^2 \qquad \text{..........................} (3)$$

Similarly, the future worth accumulated at the end of year 3 i.e. F_3 is equal to the amount that was accumulated at the end of 2nd year i.e. F_2 plus the amount of interest accumulated from end of 2nd year to the end of 3rd year on F_2 and is given by;

$$F_3 = F_2 + F_2 \times i = F_2(1+i) \qquad \ldots\ldots\ldots\ldots \text{ (4)}$$

Putting the value of F_2 from equation (3) in equation (4), the value of F_3 is given by;

$$F_3 = F_2(1+i) = P(1+i)^2(1+i) = P(1+i)^3 \quad \ldots\ldots\ldots\ldots \text{ (5)}$$

Similarly, the future worth accumulated at the end of year 4 i.e. F_4 is given by;

$$F_4 = F_3(1+i) = P(1+i)^3(1+i) = P(1+i)^4 \quad \ldots\ldots\ldots\ldots \text{ (6)}$$

Thus the generalized formula for the future worth at the end of 'n' years is given by;

$$F = P(1+i)^n \qquad \ldots\ldots\ldots\ldots\ldots \text{ (7)}$$

The factor $(1+i)^n$ in equation (7) is known as the ***single payment compound amount factor (SPCAF)***. Thus if the value of 'P' is known, one can easily calculate the future worth F at the end of 'n' years at interest rate of 'i' (per year) by multiplying the present worth with the single payment compound amount factor.

Lecture-4

Single payment present worth factor (SPPWF):

The single payment present worth factor is used to determine the present worth of a known future worth (F) at the end of "n" years at a given interest rate *'i'* per interest period.

The present worth (P), future worth (F) and the total interest period 'n' years are shown in Fig. 1.8.

From equation (7), the expression for the present worth (P) can be written as follows;

$$P = F\left[\frac{1}{(1+i)^n}\right] \quad \ldots\ldots\ldots\ldots\ldots \text{ (8)}$$
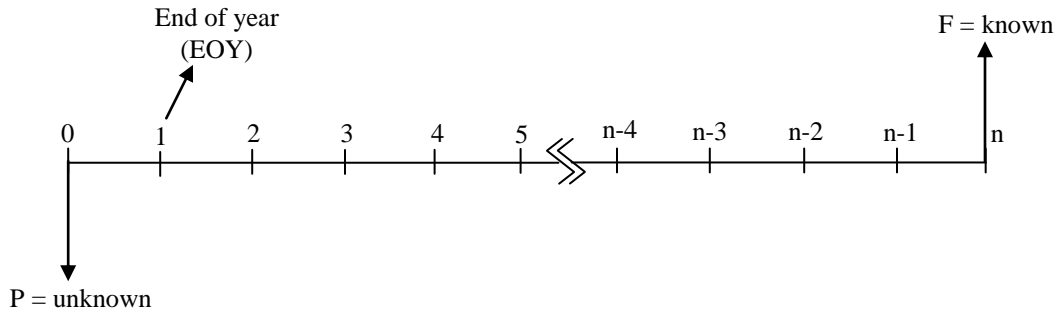

Fig 1.8 Cash flow diagram for 'known F' and 'unknown P'

The factor $1/(1+i)^n$ in equation (8) is known as *single payment present worth factor (SPPWF).* Thus if future worth (F) at the end of 'n' years is known, the present worth (P) at interest rate of 'i' (per year) can be calculated by multiplying the future worth with the single payment present worth factor.

Uniform series present worth factor (USPWF):

The uniform-series present worth factor is used to determine the present worth of a known uniform series. Let 'A' be the uniform annual amount at the end of each year, beginning from end of year '1' till end of year 'n'.

The known 'A', unknown 'P', and the total interest period 'n' years are shown in Fig. 1.9. This cash flow diagram refers to the case; if a person wants to get the known uniform amount of return every year, how much he has to invest now.

The present worth (P) of the uniform series can be calculated by considering each 'A' of the uniform series as the future worth. Then by using the formula in equation (8), the present worth of these future worth can be calculated and finally taking the sum of these present worth values.
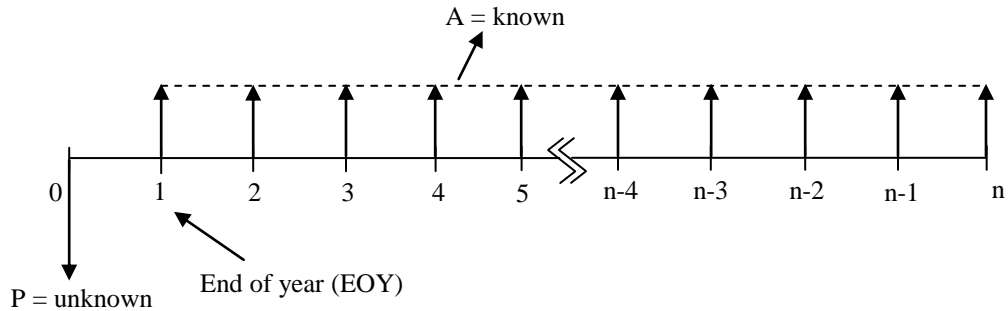

Fig 1.9 Cash flow diagram for 'known A' and 'unknown P'

The present worth (P) of the uniform series is given by;

$$P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \frac{A}{(1+i)^4} + \cdots\cdots + \frac{A}{(1+i)^{n-2}} + \frac{A}{(1+i)^{n-1}} + \frac{A}{(1+i)^n}$$

.... (9)

$$P = A\left[\frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4} + \cdots\cdots + \frac{1}{(1+i)^{n-2}} + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right]$$

.... (10)

$$P = A\left\{(1+i)^{-1} + (1+i)^{-2} + (1+i)^{-3} + (1+i)^{-4} + \cdots\cdots + (1+i)^{-(n-2)} + (1+i)^{-(n-1)} + (1+i)^{-n}\right\}$$
.. (11)

The expression in the bracket is a geometric sequence with first term equal to $(1+i)^{-1}$ and common ratio equal to $(1+i)^{-1}$. Then the present worth (P) is calculated by taking the sum of the first 'n' terms of the geometric sequence (at $i \neq 0$) and is given by;

$$P = A\left[(1+i)^{-1} \times \frac{1-(1+i)^{-n}}{1-(1+i)^{-1}}\right] \quad \cdots\cdots\cdots\cdots\cdots \quad (12)$$

The simplification of equation (12) results in the following the expression;

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad \text{...........................} \quad (13)$$

The factor within the bracket in equation (13) is known as *uniform **series present worth factor (USPWF).*** Thus if the value of 'A' in the uniform series is known, then the present worth P at interest rate of 'i' (per year) can be calculated by multiplying the uniform annual amount 'A' with uniform series compound amount factor.

The present worth (P) of a uniform annual series of known 'A' can also be calculated in the following manner;

Dividing both sides of equation (10) by (1+i) results in the following equation;

$$\frac{P}{(1+i)} = A\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4} + \frac{1}{(1+i)^5} + \cdots\cdots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{n+1}}\right]$$
$$\text{... (14)}$$

Subtracting equation (14) from equation (10) results in the following expression;

$$P - \frac{P}{(1+i)} = A\left[\frac{1}{(1+i)} - \frac{1}{(1+i)^{n+1}}\right] \quad \text{................} \quad (15)$$

Equation (14) can be rewritten as;

$$\frac{P \times i}{(1+i)} = A\left[\frac{1}{(1+i)} - \frac{1}{(1+i)^{n+1}}\right] \quad \text{....................} \quad (16)$$

Multiplying both sides of equation (15) by (1+i)/i and further simplification results in the following expression for the present worth 'P';

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Capital recovery factor (CRF):

The capital recovery factor is generally used to find out the uniform annual amount 'A' of a uniform series from the known present worth at a given interest rate *'i'* per interest period.

The cash flow diagram is shown in Fig. 1.10. This cash flow diagram indicates, if a person invests a certain amount now, how much he will get as return by an equal amount each year.

Fig 1.10 Cash flow diagram for 'known P' and 'unknown A'

From equation (12), the expression for the uniform annual amount (A) can be written as follows;

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

The factor within bracket in equation (17) is known as the ***capital recovery factor (CRF).*** Thus the uniform annual amount 'A' at interest rate of 'i' (per year) can be determined by multiplying the known present worth 'P' with the capital recovery factor.

It may be noted here that the expressions for uniform series present worth factor and capital recovery factor are derived with present worth 'P' located one interest period before the occurrence of first 'A' of the uniform series.

Lecture-5

Uniform series compound amount factor:

The uniform series compound amount factor is used to determine the future sum (F) of a known uniform annual series with uniform amount 'A'. The cash flow diagram is shown in Fig. 1.11. This cash flow diagram states that, if a person invests a uniform amount at the end of each year continued for 'n' years at interest rate of 'i' per year, how much he will get at the end of 'n' years.

Putting the value of present worth (P) from equation (8) in equation (13) results in the following;

$$F\left[\frac{1}{(1+i)^n}\right] = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad \text{.......................} \quad (18)$$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right] \quad \text{...............................} \quad (19)$$
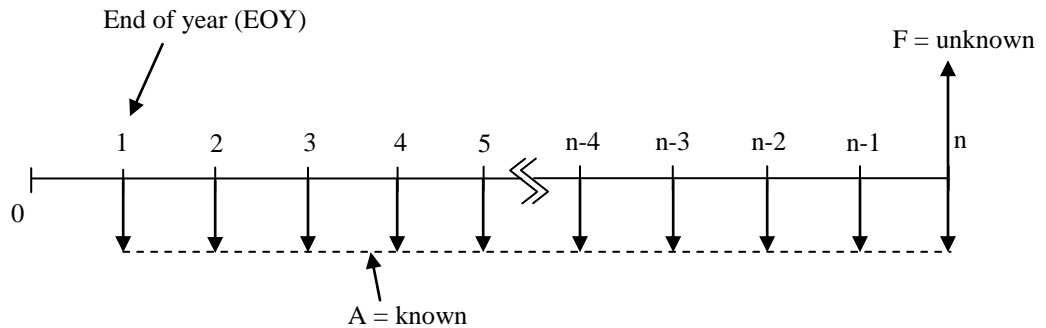
Fig 1.11 Cash flow diagram for 'known A' and 'unknown F'

The factor within bracket in equation (19) is known as *uniform series compound amount factor (USCAF).* Hence the future worth 'F' can be computed by multiplying the uniform annual amount 'A' with the uniform series compound amount factor.

Sinking fund factor:

The sinking fund factor is used to calculate the annual amount 'A' of a uniform series from the known future sum 'F'. The cash flow diagram is shown in Fig. 1.12. This cash flow diagram indicates that, if a person wants to get a known future sum at the end of 'n'

years at interest rate of 'i' per year, how much he has to invest every year by an equal amount.
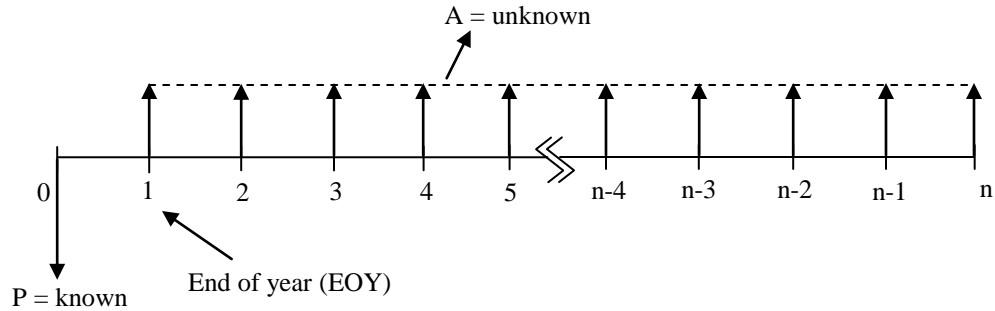

End of year (EOY)
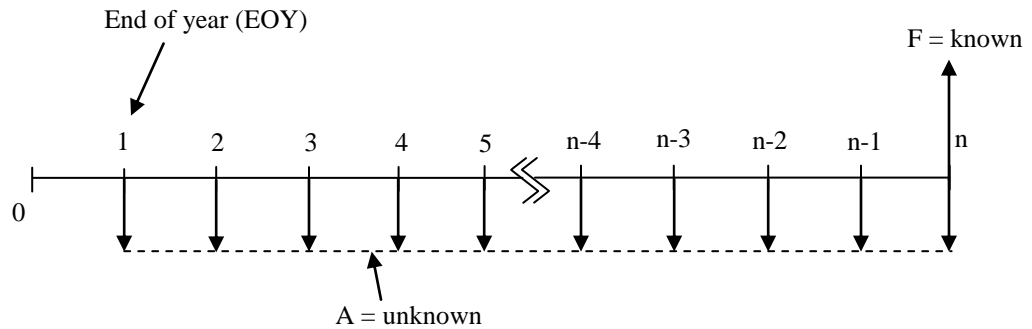
F = known

A = unknown

Fig 1.12 Cash flow diagram for 'known F' and 'unknown A'

From equation (19), the expression for the uniform annual amount (A) can be written as follows;

$$A = F\left[\frac{i}{(1+i)^n - 1}\right] \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (20)$$

The factor within bracket in equation (20) is known as ***sinking fund factor (SFF).*** Thus one can find out the annual amount 'A' of a uniform series by multiplying the future worth 'F' with the sinking fund factor.

The derivation of expressions for both uniform series compound amount factor and sinking fund factor is based on the fact that, the future sum 'F' occurs at the same time as the last 'A' of the uniform series.

The compound interest factors with details are presented Table 1.3.

Table 1.3 Interest factors (Discrete compounding) with rate of interest 'i' (%) and number of interest periods 'n'

Name of the factor (1)	Abbreviation (2)	Functional representation (3)	Mathematical expression (4)	Given (5)	To find out (6) = (4) x (5)
Single payment compound amount factor	SPCAF	$(F/P, i, n)$	$(1+i)^n$	P	F
Single payment present worth factor	SPPWF	$(P/F, i, n)$	$\dfrac{1}{(1+i)^n}$	F	P
Uniform series present worth factor	USPWF	$(P/A, i, n)$	$\dfrac{(1+i)^n - 1}{i(1+i)^n}$	A	P
Capital recovery factor	CRF	$(A/P, i, n)$	$\dfrac{i(1+i)^n}{(1+i)^n - 1}$	P	A
Uniform series compound amount factor	USCAF	$(F/A, i, n)$	$\dfrac{(1+i)^n - 1}{i}$	A	F
Sinking fund factor	SFF	$(A/F, i, n)$	$\dfrac{i}{(1+i)^n - 1}$	F	A

Using this information, one can easily find out the required expenditure or income i.e. may be 'P', 'F' or 'A' from the know cash flows. For example if it is required to find out the uniform annual amount 'A' of a uniform series from the know value of present worth 'P', then multiply the capital recovery factor (CRF) i.e. $\dfrac{i(1+i)^n}{(1+i)^n - 1}$ with 'P' to get the required uniform annual amount 'A' (highlighted in the above table).

In addition, the use of functional representations of the compound interest factors presented in this table can be used to find out the unknown amounts. For illustration, if it is required to find out 'A' from known 'P', then expression for 'A' can be written as follows;

$$A = P\left(\frac{A}{P}, i, n\right)$$

Further the functional representation of a compound interest factor can be obtained by multiplying the other relevant compound interest factors at a given rate of interest 'i' and number of interest periods 'n' and is given as follows;

$$\left(\frac{A}{P}, i, n\right) = \left(\frac{A}{F}, i, n\right) \times \left(\frac{F}{P}, i, n\right) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (21)$$

Lecture-6

Cash flow involving arithmetic gradient payments or receipts:

Some cash flows involve the payments or receipts in gradients by same amount. In other words, the expenditure or the income increases or decreases by same amount. The cash flow involving such payments or receipts is known as uniform gradient series. For example, if the cost of repair and maintenance of a piece of equipment increases by same amount every year till end of its useful life, it represents a cash flow involving positive uniform gradient. Similarly if the profit obtained from an investment decreases by an equal amount every year for a certain number of years, it indicates a cash flow involving negative uniform gradient. The cash flow diagrams for positive gradient and negative gradient are shown in Fig. 1.3 and Fig. 1.4 respectively.
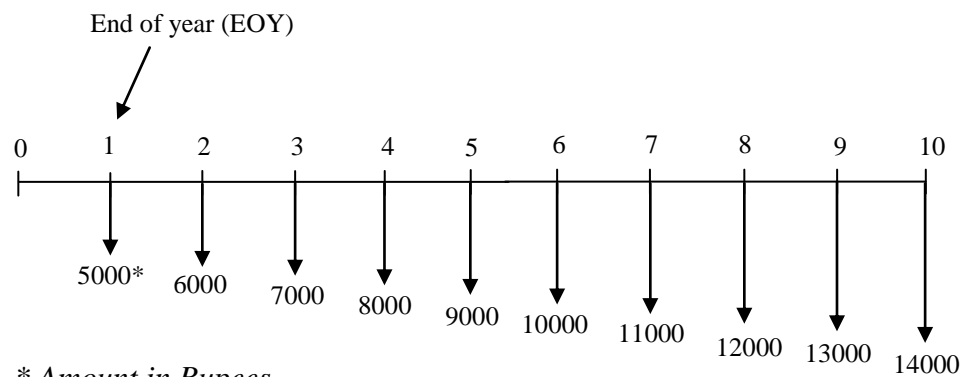
Amount in Rupees.

A positive gradient of Rs.1000.

Fig 1.13 Cash flow diagram involving a positive uniform gradient
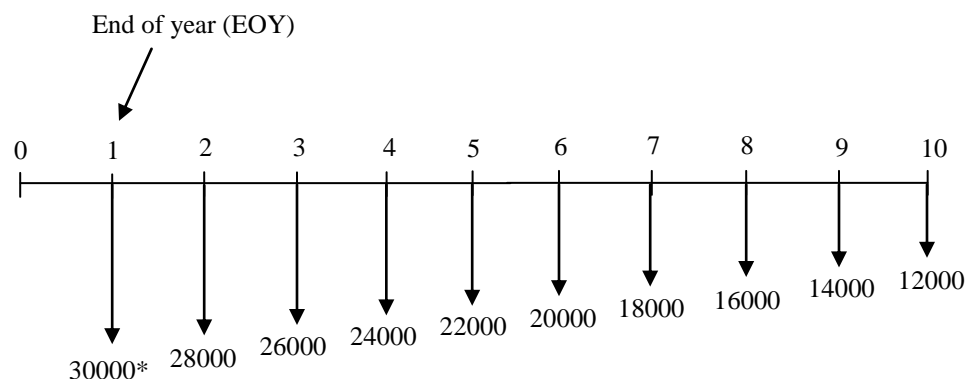
Amount in Rupees.

A negative gradient of Rs.2000.

Fig 1.14 Cash flow diagram involving a negative uniform gradient

The present worth, future worth and the equivalent uniform annual worth of the uniform gradient can be derived using the compound interest factors. The generalized cash flow diagram involving a positive uniform gradient with base value 'B' and the gradient 'G' is shown in Fig. 1.15a. The cash flow shown in Fig. 1.15a can be split into two cash flows; one having the uniform series with amount 'B' and the other having the gradient series with values in multiples of gradient amount 'G' and is shown in Fig. 1.15b. This gradient series is also know as the arithmetic gradient series as the expense or the income increases by the uniform arithmetic amount 'G' every year.
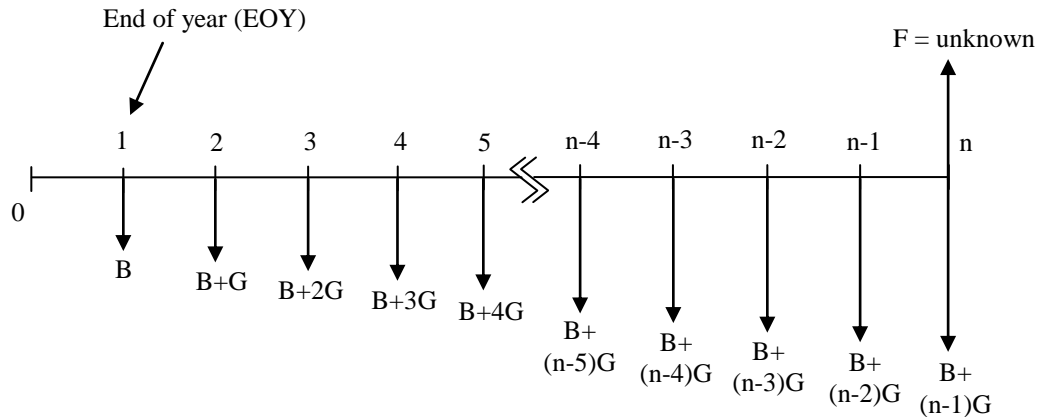

Fig 1.15a Generalized Cash flow diagram involving a positive uniform gradient with 'unknown F'
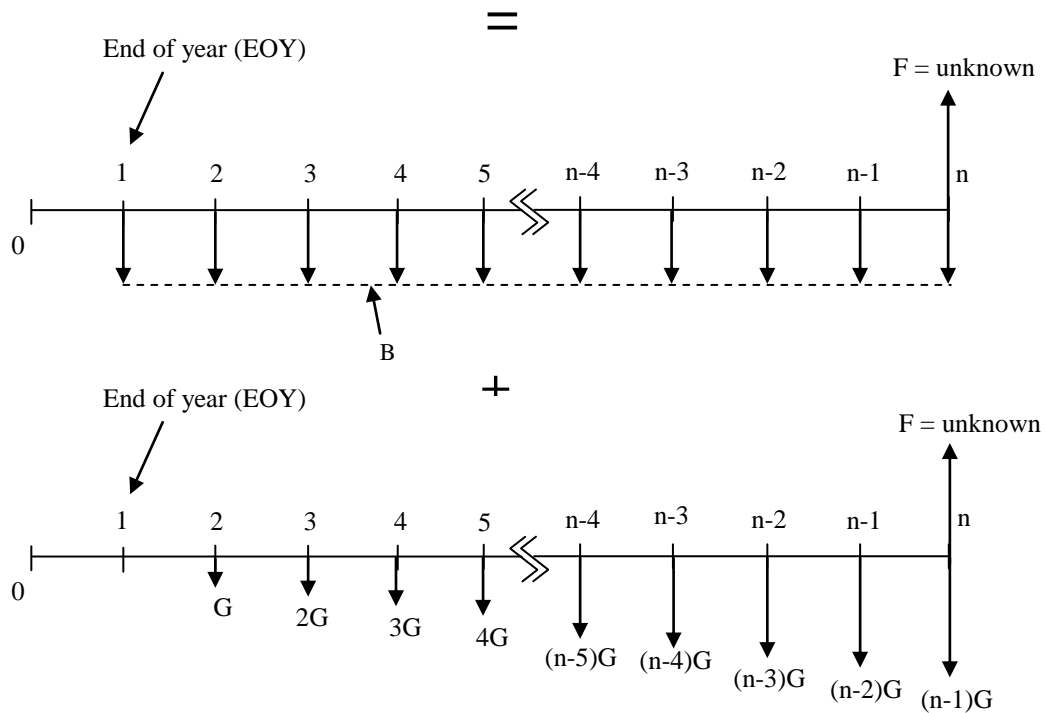

Fig 1.15b Cash flow diagram (positive uniform gradient) for 'unknown F'

The future worth of the cash flow shown in Fig. 1.15a can be obtained by finding the out the individual future worth of the cash flows shown in Fig. 1.15b and then taking their sum. As already stated, the future worth of the cash flow involving an uniform series can be determined by multiplying the uniform annual amount 'B' with the uniform series compound amount factor. The future worth (F) of the gradient series shown in Fig 1.15b can be determined by finding out the individual future worth of the gradient values (i.e. in multiples of gradient amount 'G') at the end of different years at interest rate of 'i' per year and then taking the sum of these individual futures values. Then 'F' is given as follows;

$$F = \left\{ \begin{array}{l} G(1+i)^{n-2} + 2G(1+i)^{n-3} + 3G(1+i)^{n-4} + 4G(1+i)^{n-5} + \cdots \cdots \\ \cdots \cdots \cdots \cdots \cdots + (n-4)G(1+i)^3 + (n-3)G(1+i)^2 + (n-2)G(1+i) + (n-1)G \end{array} \right\} \cdots \cdots$$

(22)

Dividing both sides of equation (22) by $(1+i)^n$ results in the following equation;

$$\frac{F}{(1+i)^n} = G \left\{ \begin{array}{l} \dfrac{1}{(1+i)^2} + \dfrac{2}{(1+i)^3} + \dfrac{3}{(1+i)^4} + \dfrac{4}{(1+i)^5} + \cdots \cdots \cdots \\ \\ \cdots \cdots \cdots + \dfrac{n-4}{(1+i)^{n-3}} + \dfrac{n-3}{(1+i)^{n-2}} + \dfrac{n-2}{(1+i)^{n-1}} + \dfrac{n-1}{(1+i)^n} \end{array} \right\} \cdots$$

…. (23)

Multiplying both sides of equation (23) by (1+i) results in the following expression;

$$\frac{F(1+i)}{(1+i)^n} = G \left\{ \begin{array}{l} \dfrac{1}{(1+i)} + \dfrac{2}{(1+i)^2} + \dfrac{3}{(1+i)^3} + \dfrac{4}{(1+i)^4} + \cdots \cdots \cdots \\ \\ \cdots \cdots \cdots + \dfrac{n-4}{(1+i)^{n-4}} + \dfrac{n-3}{(1+i)^{n-3}} + \dfrac{n-2}{(1+i)^{n-2}} + \dfrac{n-1}{(1+i)^{n-1}} \end{array} \right\}$$

……… (24)

Subtracting equation (23) from equation (24) results in the following expression;

$$F\left[\frac{(1+i)-1}{(1+i)^n} \right] = G \left\{ \begin{array}{l} \dfrac{1}{(1+i)} + \dfrac{1}{(1+i)^2} + \dfrac{1}{(1+i)^3} + \dfrac{1}{(1+i)^4} + \cdots \cdots \cdots \\ \\ \cdots \cdots + \dfrac{1}{(1+i)^{n-4}} + \dfrac{1}{(1+i)^{n-3}} + \dfrac{1}{(1+i)^{n-2}} + \dfrac{1}{(1+i)^{n-1}} + \dfrac{1}{(1+i)^n} - \dfrac{n}{(1+i)^n} \end{array} \right\}$$

… (25)

The equation (25) can be rewritten as follows;

$$F\left[\frac{i}{(1+i)^n}\right] = G\left\{\begin{array}{l} \dfrac{1}{(1+i)} + \dfrac{1}{(1+i)^2} + \dfrac{1}{(1+i)^3} + \dfrac{1}{(1+i)^4} + \cdots\cdots \\[2mm] \cdots\cdots + \dfrac{1}{(1+i)^{n-4}} + \dfrac{1}{(1+i)^{n-3}} + \dfrac{1}{(1+i)^{n-2}} + \dfrac{1}{(1+i)^{n-1}} + \dfrac{1}{(1+i)^n} \end{array}\right\} - \frac{nG}{(1+i)^n}$$

….. (26)

The expression in the bracket is a geometric sequence with first term equal to $1/(1+i)$ and common ratio equal to $1/(1+i)$. Thus equation (26) is rewritten by taking sum of the first 'n' terms of the geometric sequence (at $i \neq 0$) and is given by;

$$F\left[\frac{i}{(1+i)^n}\right] = G\left[\frac{1}{(1+i)} \times \frac{1 - \dfrac{1}{(1+i)^n}}{1 - \dfrac{1}{(1+i)}}\right] - \frac{nG}{(1+i)^n} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The expression for future worth 'F' can be written as follows:

$$F = G\left[\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}\right] \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The expression in the bracket is known as the uniform gradient future worth factor (UGFWF). With functional representation equation (28) can be rewritten as follows;

$$F = G\left(\frac{F}{G}, i, n\right)$$

Thus the functional representation of uniform gradient future worth factor (UGFWF) can be written as follows;

$$\left(\frac{F}{G}, i, n\right) = \left[\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}\right] \quad\ldots\ldots\ldots\ldots\ldots (29)$$

It may be noted here that the future worth 'F' occurs in the same year as that of the last gradient amount. Further the functional representations of other gradient factors can be found out by multiplying the relevant gradient factor with the compound interest factor at the given rate of interest 'i' per interest period and number of interest periods 'n'.

Lecture-7

Arithmetic gradient:

The present worth factor of a uniform gradient series (i.e. arithmetic gradient series) with values in multiples of gradient amount 'G' can be obtained by multiplying the uniform gradient future worth factor (UGFWF) with the single payment present worth factor (SPPWF) and the functional representation is given as follows

$$\left(\frac{P}{G},i,n\right) = \left(\frac{F}{G},i,n\right) \times \left(\frac{P}{F},i,n\right) \quad \text{.................. (30)}$$

$\left(\dfrac{P}{G},i,n\right)$ is known as the uniform gradient present worth factor (UGPWF).

Now putting the expressions for uniform gradient future worth factor (UGFWF) and single payment present worth factor (SPPWF) in equation (30) results in the following expressions;

$$\left(\frac{P}{G},i,n\right) = \left[\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}\right] \times \left[\frac{1}{(1+i)^n}\right] \quad \text{............. (31)}$$

On further simplification results in the following equation;

$$\left(\frac{P}{G},i,n\right) = \frac{1}{i(1+i)^n}\left[\frac{(1+i)^n - 1}{i} - n\right] \quad \text{................. (32)}$$

Thus the expression of the present worth 'P' can be written as follows

$$P = G\left(\frac{P}{G},i,n\right)$$

The cash flow diagram is shown in Fig. 1.16. It may be noted here that the gradient starts at the end of year '2' whereas the present worth occurs at the beginning i.e. in year 0.
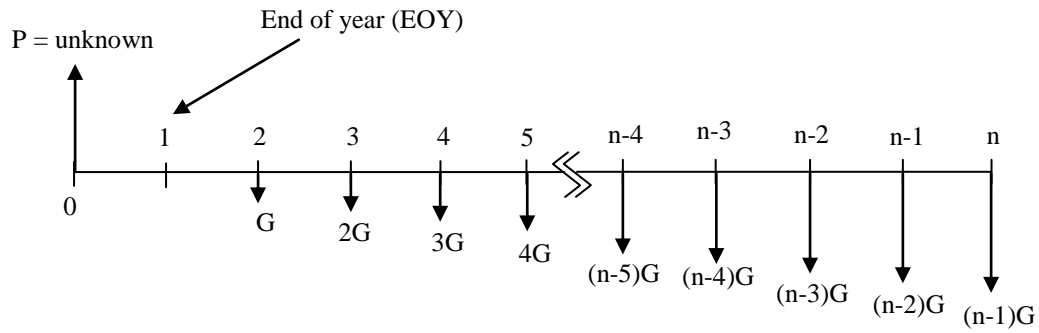
Fig. 1.16 Cash flow diagram involving a uniform gradient with 'unknown P'

Similarly the annual worth factor of a uniform gradient series can be obtained by multiplying the uniform gradient future worth factor (UGFWF) with the sinking fund factor (SFF) and the functional representation is presented as follows;

$$\left(\frac{A}{G}, i, n\right) = \left(\frac{F}{G}, i, n\right) \times \left(\frac{A}{F}, i, n\right) \dots\dots\dots\dots\dots (33)$$

$\left(\dfrac{A}{G}, i, n\right)$ is known as the uniform gradient annual worth factor (UGAWF).

Thus the expression for the uniform gradient annual worth factor is obtained by putting the expressions for uniform gradient future worth factor (UGFWF) and sinking fund factor (SFF) in equation (33) and is given as follows;

$$\left(\frac{A}{G}, i, n\right) = \left[\frac{(1+i)^n - 1}{i^2} - \frac{n}{i}\right] \times \left[\frac{i}{(1+i)^n - 1}\right] \dots\dots\dots (34)$$

$$\left(\frac{A}{G}, i, n\right) = \left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right] \dots\dots\dots\dots\dots (35)$$

The expression of the equivalent uniform annual worth 'A' of the uniform gradient series can be written as follows;

$$A = G\left(\frac{A}{G}, i, n\right)$$

The cash flow diagram is shown in Fig. 1.17. It may be noted here that, the equivalent uniform annual worth 'A' occurs from end of year '1' to end of year 'n' whereas the gradient starts at the end of year '2'.
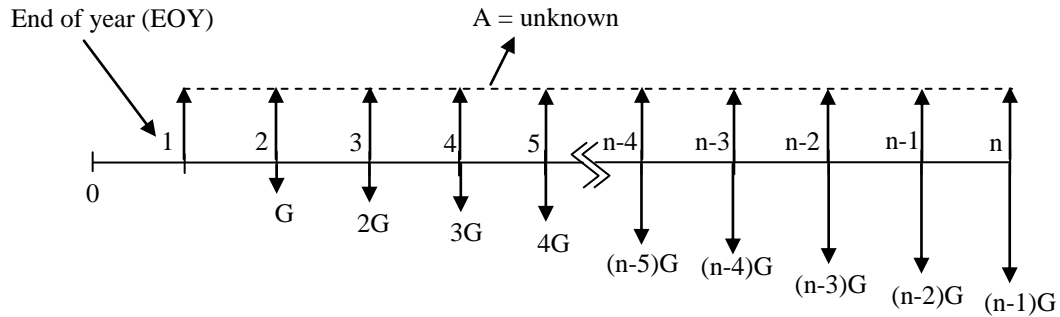
Fig. 1.17 Cash flow diagram involving a uniform gradient with 'unknown A'

Now one cane easily find out the present worth, future worth and equivalent uniform annual worth of the cash flow shown in Fig. 1.15a (a positive uniform gradient with base value 'B' and the gradient 'G') by taking the sum of the corresponding values from the uniform series with uniform annual amount 'B' i.e. base value and the gradient series with values in multiples of gradient amount 'G'. Similarly the present worth, future worth and equivalent uniform annual worth of a cash flow involving a negative uniform gradient can be calculated by subtracting the corresponding values of the gradient series (with values in multiples of gradient amount 'G') from the uniform series with uniform annual amount 'B' i.e. base value as shown Fig. 1.18a and Fig. 1.18b.
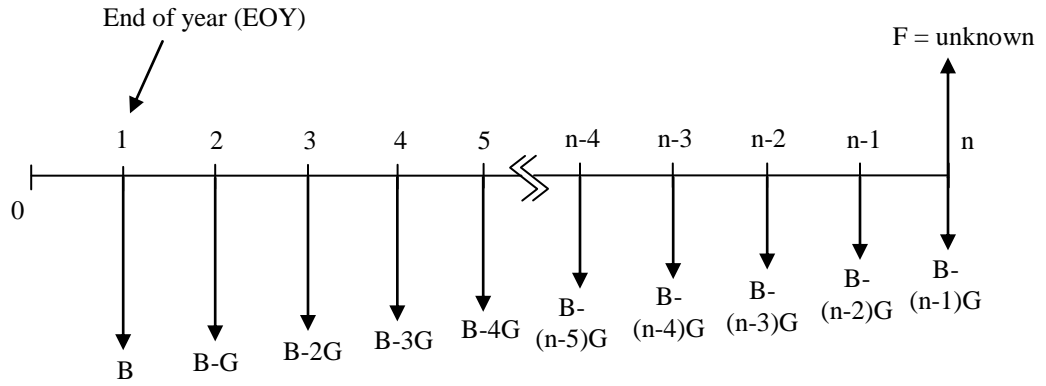
Fig 1.18a Generalized Cash flow diagram involving a negative uniform gradient with 'unknown F'
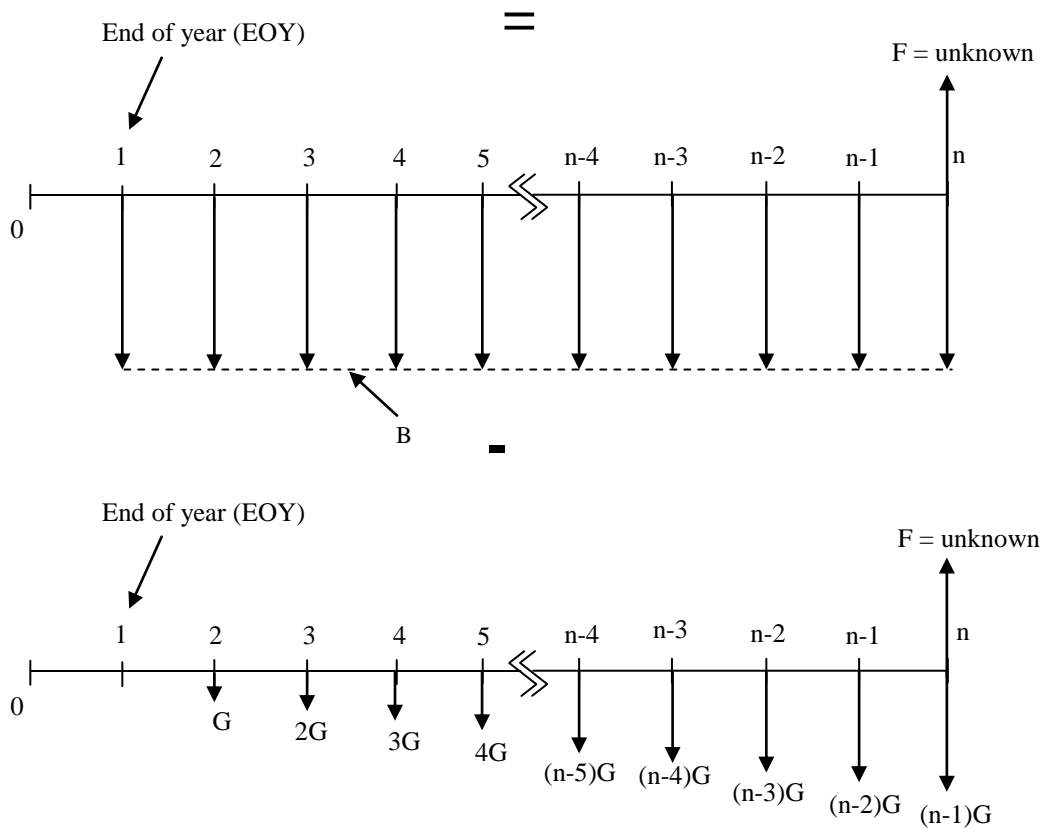
Fig 1.18b Cash flow diagram (negative uniform gradient) for 'unknown F'

Lecture-8

Cash flow involving geometric gradient series:

Sometimes the cash flows may have expenses or incomes being increased by a constant percentage in the successive time periods i.e. in successive years. Such kind of cash flow is known as geometric gradient series. The generalized cash flow diagram involving geometric gradient series with expense or receipt 'C' at the end of year '1" and geometric percentage increase 'g' is shown in Fig. 1.19.
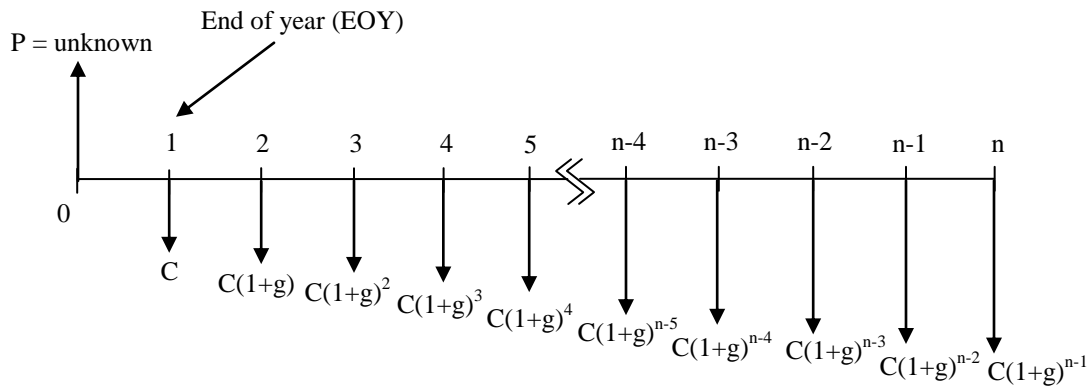
Fig 1.19 Cash flow diagram involving geometric gradient with 'unknown P'

The present worth (P) of the geometric gradient series can be calculated by considering each amount as the future worth and then taking sum of these present worth values.

$$P = \left\{ \frac{C}{(1+i)} + \frac{C(1+g)}{(1+i)^2} + \frac{C(1+g)^2}{(1+i)^3} + \frac{C(1+g)^3}{(1+i)^4} + \cdots\cdots + \frac{C(1+g)^{n-3}}{(1+i)^{n-2}} + \frac{C(1+g)^{n-2}}{(1+i)^{n-1}} + \frac{C(1+g)^{n-1}}{(1+i)^n} \right\}$$

………… (36)

Equation (36) can be rewritten as follows;

$$P = C\left\{ \frac{1}{(1+i)} + \frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \cdots\cdots + \frac{(1+g)^{n-3}}{(1+i)^{n-2}} + \frac{(1+g)^{n-2}}{(1+i)^{n-1}} + \frac{(1+g)^{n-1}}{(1+i)^n} \right\}$$

………… (37)

Dividing both sides of the equation (37) by (1+i) results in the following expression;

$$\frac{P}{(1+i)} = C\left\{\frac{1}{(1+i)^2} + \frac{(1+g)}{(1+i)^3} + \frac{(1+g)^2}{(1+i)^4} + \frac{(1+g)^3}{(1+i)^5} + \cdots\cdots + \frac{(1+g)^{n-3}}{(1+i)^{n-1}} + \frac{(1+g)^{n-2}}{(1+i)^n} + \frac{(1+g)^{n-1}}{(1+i)^{n+1}}\right\}$$

…………… (38)

Now multiplying both sides of equation (38) by (1+g) results in the following expression;

$$\frac{P(1+g)}{(1+i)} = C\left\{\frac{(1+g)}{(1+i)^2} + \frac{(1+g)^2}{(1+i)^3} + \frac{(1+g)^3}{(1+i)^4} + \frac{(1+g)^4}{(1+i)^5} + \cdots\cdots + \frac{(1+g)^{n-2}}{(1+i)^{n-1}} + \frac{(1+g)^{n-1}}{(1+i)^n} + \frac{(1+g)^n}{(1+i)^{n+1}}\right\}$$

…………… (39)

Subtracting equation (37) from equation (39) results in the following;

$$\frac{P(g-i)}{(1+i)} = C\left\{\frac{(1+g)^n}{(1+i)^{n+1}} - \frac{1}{(1+i)}\right\}$$ ………………… (40)

Equation (40) can be rewritten as follows;

$$P = \frac{C\left\{\frac{(1+g)^n}{(1+i)^n} - 1\right\}}{(g-i)}$$ …………….. (41)

The expression in equation (41) is valid when g ≠ i. When 'g' is equal to 'i' then right hand side of the equation (41) takes the indeterminate form of $\frac{0}{0}$. In this case the expression for the present worth 'P' can be obtained by applying L'Hospital's rule as follows;

$$\lim_{g \to i} \frac{C\left\{\frac{(1+g)^n}{(1+i)^n} - 1\right\}}{(g-i)} = \lim_{g \to i} \frac{d\left[C\left\{\frac{(1+g)^n}{(1+i)^n} - 1\right\}\right]}{d\left[(g-i)\right]} = \lim_{g \to i} \frac{\frac{Cn(1+g)^{n-1}}{(1+i)^n}}{1} = \frac{Cn}{(1+i)}$$ …….. (42)

Then the expression for the present worth 'P' is given as follows;

$$P = \frac{Cn}{(1+i)}$$ …………………………………………………… (43)

Similarly the future worth and equivalent uniform annual worth of the geometric gradient series can be obtained by multiplying its present worth 'P' with the compound interest factors namely single payment compound amount factor (SPCAF) and capital recovery factor (CRF) respectively at the given rate of interest 'i' per interest period and the number of interest periods.

Now using the above expressions, one can easily calculate the present worth, future worth and equivalent uniform annual worth of the cash flow involving either expenditure or income or both increasing in the form of geometric gradient.

It may be noted here that the expressions for compound interest factors can also be obtained by using beginning of year (interest period) convention.

The values of discrete compound interest factors at different values of interest rate and interest period can be calculated by using the expressions of these factors as mentioned earlier and the interest tables can thus be generated. The interest tables are available in the relevant texts (cited in the list of references for this course). The values of different discrete compound interest factors from these interest tables can directly be used in the engineering economy calculations.

The use of different compound interest factors discussed so far will be illustrated by solving various examples for comparison of different alternatives in Module 2. Further the use of discrete compound interest factors from interest tables will also be illustrated in Module 2.

Construction Economics & Finance

Module 3
Lecture-1

Depreciation:-

It represents the reduction in market value of an asset due to age, wear and tear and obsolescence. The physical deterioration of the asset occurs due to wear and tear with passage of time. Obsolescence occurs to due to availability of new technology or new product in the market that is superior to the old one and the new one replaces the old even though the old one is still in working condition. The tangible assets for which the depreciation analysis is carried out are construction equipments, buildings, electronic products, vehicles, machinery etc. Depreciation amount for any asset is usually calculated on yearly basis. Depreciation is considered as expenditure in the cash flow of the asset, although there is no physical cash outflow. Depreciation affects the income tax to be paid by an individual or a firm as it is considered as an allowable deduction in calculating the taxable income. Generally the income tax is paid on taxable income which is equal to gross income less the allowable deductions (expenditures). Depreciation reduces the taxable income and hence results in lowering the income tax to be paid.

Before discussing about different methods of depreciation, it is necessary to know the common terms used in depreciation analysis. These terms are initial cost, salvage value, book value and useful life. **Initial cost** is the total cost of acquiring the asset. **Salvage value** represents estimated market value of the asset at the end of its useful life. It is the expected cash inflow that the owner of the asset will receive by disposing it at the end of useful life. **Book value** is the value of asset recorded on the accounting books of the firm at a given time period. It is generally calculated at the end of each year. Book value at the end of a given year equals the initial cost less the total depreciation amount till that year. **Useful life** represents the expected number of years the asset is useful in terms of generating revenue. The asset may still be in working condition after the useful life but it may not be economical. Useful life is also known as depreciable life. The asset is depreciated over its useful life.

The commonly used depreciation methods are straight-line depreciation method, declining balance method, sum-of-years-digits method and sinking fund method.

Straight-line (SL) depreciation method:-

It is the simplest method of depreciation. In this method it is assumed that the book value of an asset will decrease by same amount every year over the useful life till its salvage value is reached. In other words the book value of the asset decreases at a linear rate with the time period.

The expression for annual depreciation in a given year 'm' is presented as follows;

$$D_m = \frac{P - SV}{n} \quad \text{.......................................} \quad (3.1)$$

Where

D_m = depreciation amount in year 'm' (m = 1, 2, 3, 4, ..., n, i.e. $1 \le m \le n$)

P = initial cost of the asset

n = useful life or depreciable life (in years) over which the asset is depreciated.

In equation (3.1), $\frac{1}{n}$ is the constant annual depreciation rate which is denoted by the term 'd_m'.

Since the depreciation amount is same every year, $D_1 = D_2 = D_3 = D_4 = \ldots\ldots\ldots = D_m$

The book value at the end of 1st year is equal to initial cost less the depreciation in the 1st year and is given by;

$$BV_1 = P - D_m \quad \text{...................................} \quad (3.2)$$

Book value at the end of 2nd year is equal to book value at the beginning of 2nd year (i.e. book value at the end of 1st year) less the depreciation in 2nd year and is expressed as follows;

$$BV_2 = BV_1 - D_m \quad \text{...................................} \quad (3.3)$$

As already stated depreciation amount is same in every year.

Now putting the expression of 'BV_1' from equation (3.2) in equation (3.3);

$$BV_2 = (P - D_m) - D_m = P - 2D_m \quad \text{.................} \quad (3.4)$$

Similarly the book value at the end of 3rd year is equal to book value at the beginning of 3rd year (i.e. book value at the end of 2nd year) less the depreciation in 3rd year and is given by;

$$BV_3 = BV_2 - D_m \quad \text{.............................} \quad (3.5)$$

$$BV_3 = (P - 2D_m) - D_m = P - 3D_m \quad \text{...............} \quad (3.6)$$

In the same manner the generalized expression for book value at the end of any given year 'm' can be written as follows;

$$BV_m = P - mD_m \quad \text{.............................} \quad (3.7)$$

Declining balance (DB) depreciation method:-

It is an accelerated depreciation method. In this method the annual depreciation is expressed as a fixed percentage of the book value at the beginning of the year and is calculated by multiplying the book value at the beginning of each year with a fixed percentage. Thus this method is also sometimes known as fixed percentage method of depreciation. The ratio of depreciation amount in a given year to the book value at the beginning of that year is constant for all the years of useful life of the asset. When this ratio is twice the straight-line depreciation rate i.e. $\frac{2}{n}$, the method is known as double-declining balance (DDB) method. In other words the depreciation rate is 200% of the straight-line depreciation rate. Double-declining balance (DDB) method is the most commonly used declining balance method. Another declining balance method that uses depreciation rate equal to 150% of the straight-line depreciation rate i.e. $\frac{1.5}{n}$ is also used for calculation of depreciation. In declining balance methods the depreciation during the early years is more as compared to that in later years of the asset's useful life.

In case of declining balance method, for calculating annual depreciation amount, the salvage value is not subtracted from the initial cost. It is important to ensure that, the asset is not depreciated below the estimated salvage value. In declining-balance method the calculated book value of the asset at the end of useful life does not match with the salvage value. If the book value of the asset reaches its estimated salvage value before the end of useful life, then the asset is not depreciated further.

Representing constant annual depreciation rate in declining balance method as 'd_m', the expressions for annual depreciation amount and book value are presented as follows;

The depreciation in 1st year is calculated by multiplying the initial cost (i.e. book value at beginning) with the depreciation rate and is given by;

$$D_1 = P \times d_m = P(1 - d_m)^0 \times d_m \quad \text{.............................} \quad (3.8)$$

Where

D_1 = depreciation amount in 1st year

P = initial cost of the asset as already mentioned

d_m = constant annual depreciation rate

The book value at the end of 1st year is equal to initial cost less the depreciation in the 1st year and is calculated as follows;

$$BV_1 = P - D_1$$

Now putting the expression of 'D_1' from equation (3.8) in the above expression;

$$BV_1 = P - D_1 = P - P \times d_m = P(1 - d_m) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.9)$$

The depreciation in 2nd year i.e. 'D_2' is calculated by multiplying the book value at the beginning of 2nd year (i.e. book value at the end of 1st year) with the depreciation rate 'd_m' and is given as follows;

$$D_2 = BV_1 \times d_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.10)$$

Now putting the expression of 'BV_1' from equation (3.9) in equation (3.10) results in the following;

$$D_2 = BV_1 \times d_m = P(1 - d_m) \times d_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.11)$$

Book value at the end of 2nd year is equal to book value at the beginning of 2nd year (i.e. book value at the end of 1st year) less the depreciation in 2nd year and is expressed as follows;

$$BV_2 = BV_1 - D_2$$

Now putting the expressions of 'BV_1' and 'D_2' from equation (3.9) and equation (3.11) respectively in above expression results in the following;

$$BV_2 = BV_1 - D_2 = [P(1 - d_m)] - [P(1 - d_m) \times d_m] = P(1 - d_m)(1 - d_m) = P(1 - d_m)^2 \ldots \quad (3.12)$$

Similarly the depreciation in 3rd year i.e. 'D_3' is calculated as follows;

$$D_3 = BV_2 \times d_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.13)$$

Now putting the expression of 'BV_2' from equation (3.12) in equation (3.13);

$$D_3 = BV_2 \times d_m = P(1 - d_m)^2 \times d_m \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.14)$$

Book value at the end of 3rd year is calculated as follows;

$$BV_3 = BV_2 - D_3$$

$$BV_3 = BV_2 - D_3 = \left[P(1-d_m)^2\right] - \left[P(1-d_m)^2 \times d_m\right] = P(1-d_m)^2(1-d_m) = P(1-d_m)^3$$

$$\ldots\ldots\ldots\ldots \quad (3.15)$$

In the same manner the generalized expression for depreciation in any given year 'm' can be written as follows (referring to equations (3.8), (3.11) and (3.14));

$$D_m = P(1-d_m)^{m-1} \times d_m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.16)$$

Similarly the generalized expression for book value at the end of any year 'm' is given as follows (referring to equations (3.9), (3.12) and (3.15));

$$BV_m = P(1-d_m)^m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.17)$$

The book value at the end of useful life i.e. at the end of 'n' years is given by;

$$BV_n = P(1-d_m)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.18)$$

The book value at the end of useful life is theoretically equal to the salvage value of the asset. Thus equating the salvage value (SV) of the asset to its book value (BV_n) at the end of useful life results in the following;

$$SV = BV_n = P(1-d_m)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.19)$$

Thus for calculating the depreciation of an asset using declining balance method, equation (3.19) can be used to find out the constant annual depreciation rate, if it is not stated for the asset. From equation (3.19), the expression for constant annual depreciation rate 'd_m' from known values of initial cost 'P' and salvage value ($SV > 0$) is obtained as follows;

$$SV = P(1-d_m)^n$$

$$\frac{SV}{P} = (1-d_m)^n$$

$$d_m = 1 - \left(\frac{SV}{P}\right)^{1/n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.20)$$

In double-declining balance (DDB) method, for calculating annual depreciation amount and book value at the end of different years, the value of constant annual depreciation rate 'd_m' is replaced by '$2/n$' in the above mentioned equations.

Example -1

The initial cost of a piece of construction equipment is Rs.3500000. It has useful life of 10 years. The estimated salvage value of the equipment at the end of useful life is Rs.500000. Calculate the annual depreciation and book value of the construction equipment using straight-line method and double-declining balance method.

Solution:

Initial cost of the construction equipment $= P = $ Rs.3500000

Estimated salvage value $= SV = $ Rs.500000

Useful life $= n = 10$ years

For straight-line method, the depreciation amount for a given year is calculated using equation (3.1).

$$D_m = D_1 = D_2 = \cdots\cdots = D_9 = D_{10} = \frac{3500000 - 500000}{10} = \text{Rs.}300000$$

The book value at the end of a given year is calculated by subtracting the annual depreciation amount from previous year's book value.

Book value at the end of 1st year $= BV_1 = $ Rs.3500000 $-$ Rs.300000 $=$ Rs.3200000

Book value at the end of 2nd year $= BV_2 = $ Rs.3200000 $-$ Rs.300000 $=$ Rs.2900000

Similarly the book values at the end of other years have been calculated in the same manner. The annual depreciation amount and book values at end of years using straight-line depreciation method are presented in Table 3.1. For straight-line method, the book value at the end of different years can also be calculated by using equation (3.7). For example, the book value at the end of 2nd year is given by;

$$BV_2 = 3500000 - 2 \times 300000 = \text{Rs.}2900000$$

For double-declining balance method, the constant annual depreciation rate 'd_m' is given by;

$$d_m = \frac{2}{n} = \frac{2}{10} = 0.2$$

The depreciation amount for a given year is calculated using equation (3.16).

Depreciation for 1st year $= D_1 = 3500000 \times (1 - 0.2)^{1-1} \times 0.2 = \text{Rs.}700000$

Depreciation for 2nd year $= D_2 = 3500000 \times (1 - 0.2)^{2-1} \times 0.2 = \text{Rs.}560000$

The book value at the end of a given year is calculated by subtracting the annual depreciation amount from previous year's book value.

Book value at the end of 1st year = BV_1 = Rs.3500000 – Rs.700000 = Rs.2800000

Book value at the end of 2nd year = BV_2 = Rs.2800000 – Rs.560000 = Rs.2240000

Similarly the annual depreciation and book value at the end of other years have been calculated in the same manner and are presented in Table 3.1.

The book value at the end of different years can also be calculated by using equation (3.17). Using this equation, the book value at the end of 2nd year is given by;

$$BV_2 = 3500000(1-0.2)^2 = Rs.2240000$$

Table 3.1 Depreciation and book value of the construction equipment using straight-line method and double-declining balance method

Year	Straight-line method		Double-declining balance method	
	Depreciation (Rs.) D_m	Book value (Rs.) BV_m	Depreciation (Rs.) D_m	Book value (Rs.) BV_m
0	-	3500000	-	3500000
1	300000	3200000	700000	2800000
2	300000	2900000	560000	2240000
3	300000	2600000	448000	1792000
4	300000	2300000	358400	1433600
5	300000	2000000	286720	1146880
6	300000	1700000	229376	917504
7	300000	1400000	183500.80	734003.20
8	300000	1100000	146800.64	587202.56
9	300000	800000	117440.51	469762.05
10	300000	500000	93952.41	375809.64

From Table 3.1 it is observed that the book value at the end of useful life i.e. 10 years in double-declining balance method is less than salvage value. Thus the equipment will be depreciated fully before the useful life. In other words the book value reaches the estimated salvage value before the end of useful life. As the book value of the equipment is not depreciated further after it has reached the estimated salvage value, the depreciation

amount is adjusted accordingly. The book value of the construction equipment at the end of 8th, 9th, and 10th years are Rs.587202.56, Rs.469762.05 and Rs.375809.64 respectively (from Table 3.1, Double-declining balance method). Similarly the depreciation amounts for 9th and 10th years are Rs.117440.51 and Rs.93952.41 respectively. Thus to match the book value with the estimated salvage value, the depreciation in 9th year will be Rs.87202.56 (Rs.587202.56 – Rs.500000) and that in 10th year will be zero. This will lead to a book value of Rs.500000 (equal to salvage value) at the end of 9th year and 10th year. For this case, the switching from double-declining balance method to straight-line method can be done to ensure that the book value does not fall below the estimated salvage value and this procedure of switching is presented in Lecture 3 of this module.

Lecture-2

Depreciation:-

Sum-of-years-digits (SOYD) depreciation method:-

It is also an accelerated depreciation method. In this method the annual depreciation rate for any year is calculated by dividing the number of years left (from the beginning of that year for which the depreciation is calculated) in the useful life of the asset by the sum of years over the useful life.

The depreciation rate 'd_m' for any year 'm' is given by;

$$d_m = \frac{(n - m + 1)}{SOY} \quad \text{..} \quad (3.21)$$

Where n = useful life of the asset as stated earlier

SOY = sum of years' digits over the useful life = $\dfrac{n(n + 1)}{2}$

Rewriting equation (3.21);

$$d_m = \frac{(n - m + 1)}{\dfrac{n(n + 1)}{2}} \quad \text{...} \quad (3.22)$$

The depreciation amount in any year is calculated by multiplying the depreciation rate for that year with the total depreciation amount (i.e. difference between initial cost 'P' and salvage value 'SV') over the useful life.

Thus the expression for depreciation amount in any year 'm' is represented by;

$$D_m = d_m \times (P - SV) \quad \text{...............................} \quad (3.23)$$

Putting the value of 'd_m' from equation (3.21) in equation (3.23) results in the following;

$$D_m = \frac{(n - m + 1)}{SOY} \times (P - SV) \quad \text{...........................} \quad (3.24)$$

The depreciation in 1st year i.e. 'D_1' is obtained by putting 'm' equal to '1' in equation (3.24) and is given by;

$$D_1 = \frac{(n - 1 + 1)}{SOY} \times (P - SV) = \frac{(P - SV)}{SOY} \times n \quad \text{..............} \quad (3.25)$$

The book value at the end of 1st year is equal to initial cost less the depreciation in the 1st year and is given by;

$$BV_1 = P - D_1$$

Now putting the expression of 'D_1' from equation (3.25) in the above expression results in following;

$$BV_1 = P - \frac{(P-SV)}{SOY} \times n \quad \text{...............................} \quad (3.26)$$

The depreciation in 2nd year i.e. 'D_2' is given by;

$$D_2 = \frac{(n-2+1)}{SOY} \times (P-SV) = \frac{(P-SV)}{SOY} \times (n-1) \quad \text{........} \quad (3.27)$$

Book value at the end of 2nd year is equal to book value at the beginning of 2nd year (i.e. book value at the end of 1st year) less the depreciation in 2nd year and is given by;

$$BV_2 = BV_1 - D_2$$

Now putting the expressions of 'BV_1' and 'D_2' from equation (3.26) and equation (3.27) respectively in above expression results in the following;

$$BV_2 = BV_1 - D_2 = \left[P - \frac{(P-SV)}{SOY} \times n \right] - \left[\frac{(P-SV)}{SOY} \times (n-1) \right]$$

$$BV_2 = P - \left[\frac{(P-SV)}{SOY} \times (n+n-1) \right] = P - \frac{(P-SV)}{SOY} \times (2n-1) \quad \text{............} \quad (3.28)$$

Similarly the expressions for depreciation and book value for 3rd year are presented below.

$$D_3 = \frac{(n-3+1)}{SOY} \times (P-SV) = \frac{(P-SV)}{SOY} \times (n-2) \quad \text{..........................} \quad (3.29)$$

$$BV_3 = BV_2 - D_3 = \left[P - \frac{(P-SV)}{SOY} \times (2n-1) \right] - \left[\frac{(P-SV)}{SOY} \times (n-2) \right]$$

$$BV_3 = P - \left[\frac{(P-SV)}{SOY} \times (2n-1+n-2) \right] = P - \frac{(P-SV)}{SOY} \times (3n-3) \quad \text{........} \quad (3.30)$$

The expressions for depreciation and book value for 4th year are given by;

$$D_4 = \frac{(n-4+1)}{SOY} \times (P-SV) = \frac{(P-SV)}{SOY} \times (n-3) \quad \text{.............................} \quad (3.31)$$

$$BV_4 = BV_3 - D_4 = \left[P - \frac{(P-SV)}{SOY} \times (3n-3) \right] - \left[\frac{(P-SV)}{SOY} \times (n-3) \right]$$

$$BV_4 = P - \left[\frac{(P-SV)}{SOY} \times (3n-3+n-3) \right] = P - \frac{(P-SV)}{SOY} \times (4n-6) \quad \text{........ (3.32)}$$

Similarly the expressions for depreciation and book value for 5th year are given by;

$$D_5 = \frac{(n-5+1)}{SOY} \times (P-SV) = \frac{(P-SV)}{SOY} \times (n-4) \quad \text{........................ (3.33)}$$

$$BV_5 = BV_4 - D_5 = \left[P - \frac{(P-SV)}{SOY} \times (4n-6) \right] - \left[\frac{(P-SV)}{SOY} \times (n-4) \right]$$

$$BV_5 = P - \left[\frac{(P-SV)}{SOY} \times (4n-6+n-4) \right] = P - \frac{(P-SV)}{SOY} \times (5n-10) \quad \text{...... (3.34)}$$

The expressions for book value in different years are presented above to find out the generalized expression for book value at end of any given year. Now referring to the expressions of book values BV_1, BV_2, BV_3, BV_4, BV_5 in above mentioned equations, it is observed that the variable terms are 'n', '$(2n-1)$', '$(3n-3)$', '$(4n-6)$' and '$(5n-10)$' respectively. These variable terms can also be written '$(n+1-\mathbf{1})$', '$(2n+1-\mathbf{2})$', '$(3n+1-\mathbf{4})$', '$(4n+1-\mathbf{7})$' and '$(5n+1-\mathbf{11})$' respectively. The numbers 1, 2, 4, 7, and 11 in these variable terms follow a series and it is observed that value of each term is equal to value of previous term plus the difference in the values of current term and previous term. On this note, the general expression for value of 'm' term of this series is given by;

$$\text{value of } 'm' \text{ term} = 1 + \frac{m(m-1)}{2} \quad \text{.. (3.35)}$$

Now the generalized expression for book value for any year 'm' is given by;

$$BV_m = P - \frac{(P-SV)}{SOY} \times \left(mn+1 - \left[1 + \frac{m(m-1)}{2} \right] \right)$$

$$BV_m = P - \frac{(P-SV)}{SOY} \times \left[mn - \frac{m(m-1)}{2} \right] \quad \text{.......................... (3.36)}$$

In this method also, the annual depreciation during early years is more as compared to that in later years of the asset's useful life.

Sinking fund (SF) depreciation method:-

In this method it is assumed that money is deposited in a sinking fund over the useful life that will enable to replace the asset at the end of its useful life. For this purpose, a fixed amount is set aside every year from the revenue generated and this fixed sum is considered to earn interest at an interest rate compounded annually over the useful life of the asset, so that the total amount accumulated at the end of useful life is equal to the total depreciation amount i.e. initial cost less salvage value of the asset. Thus the annual depreciation in any year has two components. The first component is the fixed sum that is deposited into the sinking fund and the second component is the interest earned on the amount accumulated in sinking fund till the beginning of that year.

For this purpose, first the uniform depreciation amount (i.e. fixed amount deposited in sinking fund) at the end of each year is calculated by multiplying the total depreciation amount (i.e. initial cost less salvage value) over the useful life by sinking fund factor. After that the interest earned on the accumulated amount is calculated. The calculations are shown below.

The first component of depreciation i.e. uniform depreciation amount 'A' at the end of each year is given by;

$$A = (P - SV) \times (A/F, i, n) = (P - SV) \times \frac{i}{(1+i)^n - 1} \qquad \dots\dots\dots\dots\dots\dots . \ (3.37)$$

Where i = interest rate per year

Depreciation amount for 1st year is equal to only 'A' as this is the amount (set aside every year from the revenue generated) to be deposited in sinking fund at the end of 1st year and hence there is no interest accumulated on this amount.

Therefore $D_1 = A = A(1+i)^0$ $\qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .\ (3.38)$

Now book value at the end of 1st year is given by;

$BV_1 = P - D_1 = P - A(1+i)^0$ $\qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots .\ (3.39)$

Depreciation amount for 2nd year is equal to uniform amount 'A' to be deposited at the end of 2nd year plus the interest earned on the amount accumulated till beginning of 2nd year i.e. on depreciation amount for 1st year. Thus depreciation for amount for 2nd year is given by;

$$D_2 = A + D_1 \times i$$

Now putting 'D_1' equal to 'A' in the above expression results in;

$$D_2 = A + A \times i = A(1+i) \qquad \text{...} \qquad (3.40)$$

Book value at the end of 2nd year is given by;

$$BV_2 = BV_1 - D_2$$

Now putting the expressions of 'BV_1' and 'D_2' from equation (3.39) and equation (3.40) respectively in above expression results in the following;

$$BV_2 = BV_1 - D_2 = P - A(1+i)^0 - A(1+i) = P - A\left[(1+i)^0 + (1+i)^1\right] \quad \text{......} \quad (3.41)$$

Depreciation amount for 3rd year is equal to uniform amount 'A' to be deposited at the end of 3rd year plus the interest earned on the amount accumulated till beginning of 3rd year i.e. on sum of depreciation amounts for 1st year and 2nd year. Thus depreciation for amount for 3rd year is given by;

$$D_3 = A + (D_1 + D_2) \times i$$

Putting the expressions of 'D_1' and 'D_2' from equation (3.38) and equation (3.40) respectively in above expression results in the following;

$$D_3 = A + (D_1 + D_2) \times i = A + [A + A(1+i)] \times i = A + A \times i + A(1+i) \times i = A(1+i) + A(1+i) \times i$$

On simplifying the above expression;

$$D_3 = A(1+i) \times (1+i) = A(1+i)^2 \qquad \text{..................................} \qquad (3.42)$$

Now book value at the end of 3rd year is given by;

$$BV_3 = BV_2 - D_3$$

Putting the expressions of 'BV_2' and 'D_3' from equation (3.41) and equation (3.42) respectively in above expression results in;

$$BV_3 = BV_2 - D_3 = P - A\left[(1+i)^0 + (1+i)^1\right] - A(1+i)^2 = P - A\left[(1+i)^0 + (1+i)^1 + (1+i)^2\right]$$

$$\text{...................} \qquad (3.43)$$

Now the generalized expression for depreciation in any given year 'm' can be written as follows (referring to equations (3.38), (3.40) and (3.42));

$$D_m = A(1+i)^{m-1} \qquad \text{...} \qquad (3.44)$$

Putting the value of 'A' from equation (3.37) in equation (3.44) results in the following;

$$D_m = A(1+i)^{m-1} = (P - SV) \times \frac{i}{(1+i)^n - 1} \times (1+i)^{m-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.45)$$

Similarly the generalized expression for book value at the end of any year 'm' is given by (referring to equations (3.39), (3.41) and (3.43));

$$BV_m = P - A\left[(1+i)^0 + (1+i)^1 + (1+i)^2 + (1+i)^3 + \cdots\cdots\cdots + (1+i)^{m-2} + (1+i)^{m-1}\right]$$

$$\ldots\ldots\ldots\ldots \quad (3.46)$$

The expression in the square bracket of equation (3.46) is in the form of a geometric series and its sum represents the factor known as uniform series compound amount factor (stated in Module-1). Therefore replacing the expression in square bracket of equation (3.46) with uniform series compound amount factor results in;

$$BV_m = P - A(F/A, i, m) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.47)$$

Putting the value of 'A' from equation (3.37) and expression for uniform series compound amount factor in equation (3.47) results in the following generalized expression for book value at the end of any year 'm' ($1 \leq m \leq n$).

$$BV_m = P - A(F/A, i, m) = P - (P - SV) \times \frac{i}{(1+i)^n - 1} \times \left[\frac{(1+i)^m - 1}{i}\right]$$

$$BV_m = P - (P - SV) \times \left[\frac{(1+i)^m - 1}{(1+i)^n - 1}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.48)$$

In this method, the depreciation during later years is more as compared to that in early years of the asset's useful life.

Example -2

Using the information provided in previous example, calculate the annual depreciation and book value of the construction equipment using sum-of-years-digits method and sinking fund method. The interest rate is 8% per year.

Solution:

$P = $ Rs.3500000

$SV = $ Rs.500000

$n = 10$ years

For sum-of-years-digits method, the depreciation amount for a given year is calculated using equation (3.24).

SOY = sum of years' digits over the useful life = $\dfrac{10 \times (10 + 1)}{2} = 55$

Depreciation for 1st year $= D_1 = \dfrac{(10 - 1 + 1)}{55} \times (3500000 - 500000) = Rs.545454.55$

Book value at the end of 1st year:

$BV_1 = Rs.3500000 - Rs.545454.55 = Rs.2954545.45$

Depreciation for 2nd year $= D_2 = \dfrac{(10 - 2 + 1)}{55} \times (3500000 - 500000) = Rs.490909.09$

Book value at the end of 2nd year:

$BV_2 = Rs.2954545.45 - Rs.490909.09 = Rs.2463636.36$

Similarly the annual depreciation and book value at the end of other years for the construction equipment have been calculated and are presented in Table 3.2.

The book value at the end of different years can also be calculated by using equation (3.36). Using this equation, the book value at the end of 2nd year is calculated as follows;

$BV_2 = 3500000 - \dfrac{(3500000 - 500000)}{55} \times \left[2 \times 10 - \dfrac{2 \times (2 - 1)}{2} \right] = Rs.2463636.36$

For sinking fund method, the depreciation amount for a given year is calculated using equation (3.45).

The interest rate per year $= i = 8\%$

Depreciation amount for 1st year:

$D_1 = (3500000 - 500000) \times \dfrac{0.08}{(1 + 0.08)^{10} - 1} \times (1 + 0.08)^{1-1} = Rs.207088.47$

Book value at the end of 1st year:

$BV_1 = Rs.3500000 - Rs.207088.47 = Rs.3292911.53$

Depreciation amount for 2nd year:

$D_2 = (3500000 - 500000) \times \dfrac{0.08}{(1 + 0.08)^{10} - 1} \times (1 + 0.08)^{2-1} = Rs.223655.55$

Book value at the end of 2nd year:

$BV_2 = Rs.3292911.53 - Rs.223655.55 = Rs.3069255.98$

Similarly the annual depreciation and book value at the end of other years are calculated in the same manner and are given in Table 3.2.

The book value at the end of a given year can also be calculated by using equation (3.48). Using this equation, the book value at the end of 2nd year is given by;

$$BV_2 = 3500000 - (3500000 - 500000) \times \left[\frac{(1+0.08)^2 - 1}{(1+0.08)^{10} - 1}\right] = \text{Rs.}3069255.99$$

Table 3.2 Depreciation and book value of the construction equipment using sum-of-years-digits method and sinking fund method

Year	Sum-of-years-digits method		Sinking fund method	
	Depreciation (Rs.) D_m	Book value (Rs.) BV_m	Depreciation (Rs.) D_m	Book value (Rs.) BV_m
0	-	3500000	-	3500000
1	545454.55	2954545.45	207088.47	3292911.53
2	490909.09	2463636.36	223655.55	3069255.98
3	436363.64	2027272.73	241547.99	2827707.99
4	381818.18	1645454.55	260871.83	2566836.16
5	327272.73	1318181.82	281741.58	2285094.58
6	272727.27	1045454.55	304280.90	1980813.68
7	218181.82	827272.73	328623.38	1652190.30
8	163636.36	663636.36	354913.25	1297277.06
9	109090.91	554545.45	383306.31	913970.75
10	54545.45	500000	413970.81	499999.94* = 500000

* This minor difference is due to effect of decimal points in the calculation.

The book value at the end of all years over the useful life of the construction equipment obtained from different depreciation methods (Example 1 and Example 2) are shown in Fig. 3.1.
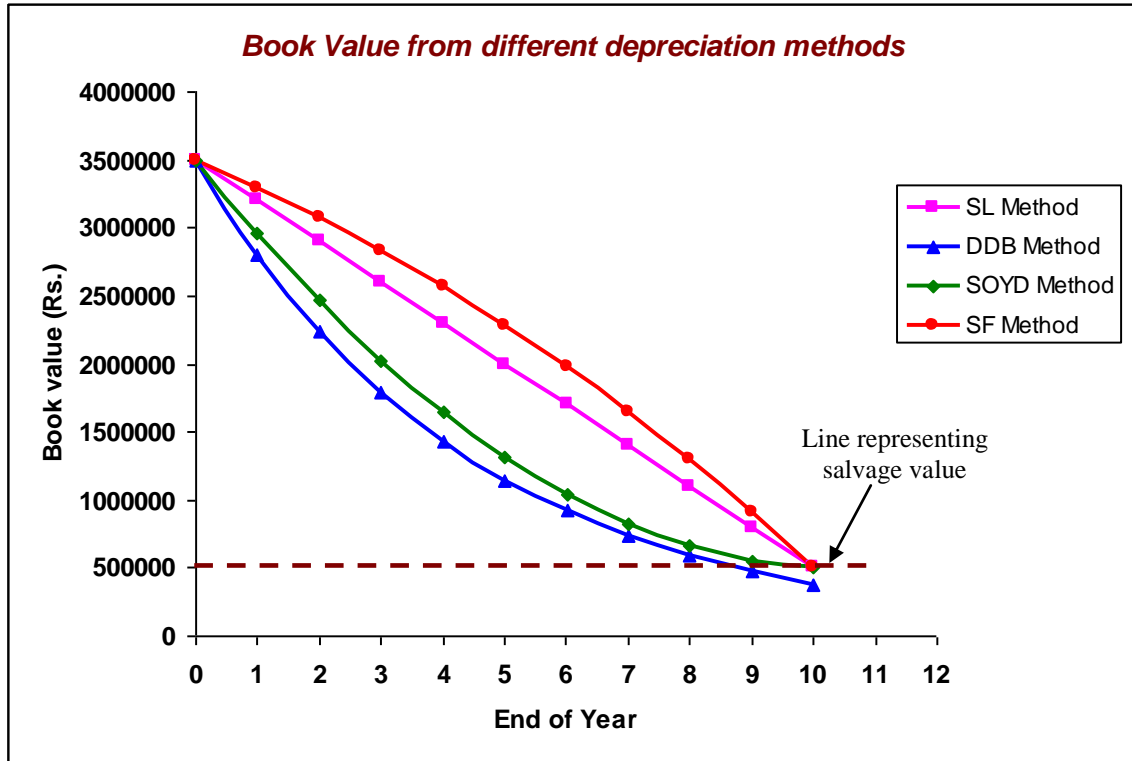
Fig. 3.1 Book value of the construction equipment using different depreciation methods

In the above figure, the line representing the salvage value is also shown. In double-declining balance method only, the calculated book value at the end of useful life i.e. 10th year is not same as the estimated salvage value.

Lecture-3

Switching between different depreciation methods:-

Switching from one depreciation method to another is done to accelerate the depreciation of book value of the asset and thus to have tax benefits. Switching is generally done when depreciation amount for a given year by the currently used method is less than that by the new method. The most commonly used switch is from double-declining balance (DDB) method to straight-line (SL) method. In double-declining balance method, the book value as calculated by using equation (3.17) never reaches zero. In addition the calculated book value at the end of useful life does not match with the salvage value. Switching from double-declining balance method to straight-line method ensures that the book value does not fall below the estimated salvage value of the asset.

In the following examples the procedure of switching from double-declining balance method to straight-line method is illustrated.

Example -3

The initial cost of an asset is Rs.1000000. It has useful life of 9 years. The estimated salvage value of the asset at the end of useful life is zero. Calculate the annual depreciation and book value using double-declining balance method and find out the year in which the switching is done from double-declining balance method to straight-line method.

Solution:

Initial cost of the asset $= P =$ Rs.1000000

Useful life $= n = 9$ years

Salvage value $= SV = 0$

For double-declining balance (DDB) method, the constant annual depreciation rate 'd_m' is given by;

$$d_m = \frac{2}{n} = \frac{2}{9} = 0.222$$

The annual depreciation and the book value at the end different years are calculated using the respective equations stated earlier and are presented in Table 3.3.

Table 3.3 Depreciation and book value from double-declining balance method

Year	Depreciation (Rs.)	Book value (Rs.)
0	-	1000000
1	222000	778000
2	172716	605284
3	134373.05	470910.95
4	104542.23	366368.72
5	81333.86	285034.86
6	63277.74	221757.12
7	49230.08	172527.04
8	38301	134226.04
9	29798.18	**104427.86**

From the above table it is observed that the book value at the end of useful life is Rs.104427.86, which is more than the estimated salvage value i.e. 0. The asset is not completely depreciated. Thus switching is done from double-declining balance method to straight-line method and is shown in Table 3.4.

Table 3.4 Double-declining balance method and switching to straight-line method

Year	Depreciation amount (Rs.) (DDB method)	Depreciation amount (Rs.) (SL method)	Selected depreciation amount (Rs.)	Book value (Rs.)
0	-	-	-	1000000
1	222000	111111.11	222000	778000
2	172716	97250	172716	605284
3	134373.05	86469.14	134373.05	470910.95
4	104542.23	78485.16	104542.23	366368.72
5	81333.86	73273.74	81333.86	285034.86
6**	**63277.74**	**71258.72**	**71258.72**	213776.15
7	49230.08	71258.72	71258.72	142517.43
8	38301	71258.72	71258.72	71258.72
9	29798.18	71258.72	71258.72	0

** Switching from DDB method to SL method

In the above table, annual depreciation values from double-declining balance (DDB) method are also presented to compare with those obtained from straight- line method. For straight-line (SL) method the depreciation amount for a given year '*m*' is calculated by dividing the difference of the book value (at the beginning of that year) and salvage value by the number of years remaining from beginning of that year till the end of useful life and is shown below;

$$D_m = \frac{BV_{m-1} - SV}{(n-m+1)}$$

For illustration, the straight-line depreciation in 4th year (*m* = 4) is calculated as follows;

$$D_4 = \frac{470910.95 - 0}{(9-4+1)} = \text{Rs.}78485.16$$

From the annual depreciation values by straight-line method as shown in 3rd column of above table, it is observed that, the annual values are not uniform. This is because when switching is done from DDB method to SL method, the larger depreciation amount between the two the methods for a given year is subtracted from the previous year's book value to calculate the book value at the end of desired year and this book value is used for calculating the depreciation amount for next year in straight-line method. The larger annual depreciation values (i.e. selected depreciation amount) between DDB method and SL method are provided in 4th column of Table 3.4. The book value at the end of a given year presented in 5th column of above table is obtained by subtracting 'selected depreciation amount' of that year from the previous year's book value. From the above table it is observed that the annual depreciation amount in DDB method is greater than that in SL method from 1st year to 5th year and from 6th year onwards the annual depreciation is greater in SL method than that in DDB method. Thus switching from DDB method to SL method takes place in 6th year. With switchover from DDB method to SL method the book value at the end of useful life i.e. 9th year is equal to zero (same as the estimated salvage value) as compared to Rs.104427.86 without switching to straight-line method. In addition the total depreciation amount over the useful life of the asset (initial cost minus salvage value) with switchover from DDB method to SL method is Rs.1000000 (i.e. sum of values in 4th column of Table 3.4) as desired whereas the total

depreciation amount without switchover is Rs.895572.14 (sum of values in 2nd column of Table 3.4).

Example -4

A piece of construction equipment has initial cost and estimated salvage value of Rs.1500000 and Rs.200000 respectively. The useful life of equipment is 10 years. Find out the year in which the switching from double-declining balance method to straight-line method takes place.

Solution:

Initial cost of the asset = P = Rs.1500000, Salvage value = SV = Rs.200000

Useful life = n = 10 years

The constant annual depreciation rate 'd_m' for double-declining balance (DDB) method is calculated as follows;

$$d_m = \frac{2}{n} = \frac{2}{10} = 0.2$$

The annual depreciation amount and the book value at the end different years using DDB method are presented in Table 3.5.

Table 3.5 Depreciation and book value using double-declining balance method

Year	Depreciation (Rs.)	Book value (Rs.)
0	-	1500000
1	300000	1200000
2	240000	960000
3	192000	768000
4	153600	614400
5	122880	491520
6	98304	393216
7	78643.20	314572.80
8	62914.56	251658.24
9	50331.65	201326.59
10	40265.32	**161061.27**

From the above table it is noted that the book value at the end of useful life i.e. 10th year is Rs.161061.27, which is less than the estimated salvage value i.e. Rs.200000. Thus the total depreciation amount over the useful life is more than the desired. Hence switching is done from double-declining balance method to straight-line method and is presented in Table 3.6.

In Table 3.6, annual depreciation by SL method, selected depreciation amount and book values are calculated in the same manner as that in the previous example. The annual depreciation for 10th year in DDB method is Rs.40265.32 and subtracting this amount from 9th year book value results in a book value (at the end of 10th year) less than the estimated salvage value (As seen from Table 3.5). Further the book value at the end of 9th year (i.e. Rs.201326.59) is greater than salvage value. Thus switching from DDB method to SL method takes place in 10th year. With switching from DDB method to SL method, the book value at the end of 10 years is equal to Rs.200000 (same as the estimated salvage value) as compared to Rs.161061.27 without switching to straight-line method.

Table 3.6 Double-declining balance method with switching to straight-line method

Year	Depreciation amount (Rs.) (DDB method)	Depreciation amount (Rs.) (SL method)	Selected depreciation amount (Rs.)	Book value (Rs.)
0	-	-	-	1500000
1	300000	130000	300000	1200000
2	240000	111111.11	240000	960000
3	192000	95000	192000	768000
4	153600	81142.86	153600	614400
5	122880	69066.67	122880	491520
6	98304	58304	98304	393216
7	78643.20	48304	78643.20	314572.80
8	62914.56	38190.93	62914.56	251658.24
9	50331.65	25829.12	50331.65	201326.59
10**	**40265.32**	**1326.59**	**1326.59**	200000

** Switching from DDB method to SL method

Further the total depreciation amount (i.e. initial cost less estimated salvage value) with switchover from DDB method to SL method is Rs.1300000 (i.e. sum of values in 4th

column of Table 3.6) as desired whereas the total depreciation amount without switchover is Rs.1338938.73 (sum of values in 2nd column of Table 3.6).

Lecture-4

Inflation:

Inflation is defined as an increase in the amount of money required to purchase or acquire the same amount of products and services that was acquired without its effect. Inflation results in a reduction in the purchasing power of the monetary unit. In other words, when prices of the products and services increase, we buy less quantity with same amount of money i.e. the value of money is decreased. For example, the quantity of items we purchase today at a cost of Rs.1000 is less than that was purchased 5 years ago. This is due to a general change (increase) in the price of the goods and services with passage of time. On the other hand deflation results in an increase in the purchasing power of the monetary unit with time and it rarely occurs. Due to effect of deflation, more can be purchased with same amount of money in future time period than that can be purchased today. The inflation rate (f) is measured as the rate of increase (per time period) in the amount of money required to obtain same amount of products and services. Till now, the interest rate *'i'* that was used in the economic evaluation of a single alternative or between alternatives by different methods as mentioned in earlier lectures was assumed to be inflation-free i.e. the effect of inflation on interest rate was excluded. This interest rate '*i*' is also known as also real interest rate or inflation-free interest rate. It represents the real gain in money of the cash flows with time without the effect of inflation. However if inflation is there in the general market, then effect of inflation on the interest rate needs to be taken into account for the economic analysis. The interest rate that includes the effect of price inflation which is occurring in general economy is known as the market interest rate (i_c). It takes into account the adjustment for the price inflation in the market. Market interest rate is also known as inflated interest rate or combined interest rate as it combines the effect of both real interest rate and the inflation.

In addition to above parameters, it is also required to define two parameters namely actual monetary units and constant value monetary units while considering the effect of inflation in the cash flow of the alternatives. The monetary units can be Rupees, Dollars, Euros etc. The actual monetary units are also referred as future or inflated monetary units. The purchasing power of the actual monetary units includes the effect of inflation on the

cash flows at the time it occurs. The constant value monetary units are also called as real or inflation-free monetary units. The constant value monetary units are expressed in terms of the same purchasing power for the cash flows with reference to a base period. Mostly in engineering economic studies, the base period is taken as '0' i.e. now. But it can be of any time period as required.

The effect of inflation on cash flows is demonstrated in the following example.

Example -5

The present (today's) cost of an item is Rs.20000. The inflation rate is 5% per year. How does the inflation affect the cost of the item for the next five years?

Solution:

The calculations are shown in Table 3.7.

<div align="center">Table 3.7 Effect of inflation on future cost of the item</div>

Year	Increase in cost of item due to inflation @ 5% per year (Rs.)	Cost of the item in inflated monetary units i.e. inflated cost (Rs.)	Cost of the item in inflated monetary units expressed as function of today's cost (i.e. in inflation-free monetary units) and inflation rate, (Rs.)
0	-	20000	20000
1	20000 x 0.05 = 1000	20000 + 1000 = 21000	$20000 \times (1 + 0.05)^1 = 21000$
2	21000 x 0.05 = 1050	21000 + 1050 = 22050	$20000 \times (1 + 0.05)^2 = 22050$
3	22050 x 0.05 = 1102.50	22050 + 1102.5 = 23152.50	$20000 \times (1 + 0.05)^3 = 23152.50$
4	23152.5 x 0.05 = 1157.63	23152.5 + 1157.62 = 24310.13	$20000 \times (1 + 0.05)^4 = 24310.13$
5	24310.13 x 0.05 = 1215.50	24310.13 + 1215.5 = 25525.63	$20000 \times (1 + 0.05)^5 = 25525.63$

From above table, the relationship between cost in actual monetary units (inflated) in time period 'n' and cost in constant value monetary units (inflation-free) can be represented as follows;

Cost in actual monetary units (inflated) = Cost in constant value monetary units $\times (1+f)^n$

$$\ldots\ldots\ldots\ldots (3.49)$$

From equation (3.49), the expression for cost in constant value monetary units (inflation-free) is written as follows

$$Cost\ in\ constant\ value\ monetary\ units\ (inflation\ \text{-}\ free) = \frac{Cost\ in\ actual\ monetary\ units\ (inflated)}{(1+f)^n}$$
……………. (3.50)

In the above expressions, 'f' is the inflation rate per year i.e. 5% and the base or reference period is considered as '0'. However the above relationship between constant value monetary units and actual monetary units can also be written for any base period 'b'.

$$Cost\ in\ constant\ value\ monetary\ units\ (inflation\ \text{-}\ free) =$$
$$Cost\ in\ actual\ monetary\ units\ (inflated) \times \left(\frac{1}{1+f}\right)^{n-b}$$ ……….. (3.51)

In the above relationship, the base period 'b' defines the purchasing power of constant value monetary units. As shown in Table 3.7, the actual cost (inflated) of the item in years 1, 2, 3, 4 and 5 are Rs.21000, Rs.22050, Rs.23152.50, 24310.13 and Rs.25525.63 respectively. However the cost of the item in inflation-free or constant value rupees in all the years is always Rs.20000 [i.e. $21000/(1+0.05)^1$, $22050/(1+0.05)^2$, etc.] i.e. equal to the cost at the beginning. In this example, effect of interest rate i.e. effect of time value of money is not considered.

Lecture-5

Equivalent worth calculation including the effect of inflation:-

For calculations of equivalent worth of cash flows of alternatives with price inflation in action, the interest rate to be adopted (i.e. real interest rate or combined interest rate) depends on whether the cash flows are expressed in inflated rupees or inflation-free rupees. When the cash flows are calculated in constant value monetary units, inflation-free interest rate (real interest rate) *'i'* is used. Similarly when the cash flows are estimated in actual monetary units, combined interest rate (inflated interest rate) *'i_c'* is used.

The relationship between present worth and future worth considering the time value of money is given by;

$$P = \frac{F_c}{(1+i)^n} \qquad\qquad \text{................................ (3.52)}$$

P = present worth (at base period '0')

F_c = future worth in constant value monetary units (inflation-free)

In the above equation, future worth in constant value monetary units (inflation-free) can be represented in terms of future worth in actual monetary units (inflated) by using the relationship stated in equation (3.50). Equation (3.52) can be rewritten by including the inflation rate.

$$P = \frac{F_a}{(1+f)^n} \times \frac{1}{(1+i)^n} \qquad\qquad \text{.............................. (3.53)}$$

In equation (3.53), F_a is the future worth in actual monetary units (inflated) and *'f'* is the inflation rate as stated earlier.

Equation (3.53) can be rewritten as follows;

$$P = F_a \times \frac{1}{(1+i+f+f \times i)^n} \qquad\qquad \text{.......................... (3.54)}$$

$$P = F_a \times \frac{1}{[1+(i+f+f \times i)]^n} \qquad\qquad \text{......................... (3.55)}$$

The expression $(i + f + f \times i)$ in the above equation that combines the effect of real interest rate 'i' and inflation rate 'f' is termed as combined interest rate or market interest rate (i_c) as stated earlier.

$$i_c = (i + f + f \times i) \qquad \text{.......................} \quad (3.56)$$

Equation (3.55) can be rewritten as follows;

$$P = F_a \times \frac{1}{(1 + i_c)^n} \qquad \text{.........................} \quad (3.57)$$

Using equation (3.57), the future worth in inflated monetary units can be calculated from known value of present worth 'P'.

Equation (3.57) can also be written in functional representation as follows;

$$P = F_a (P/F, i_c, n) \qquad \text{.........................} \quad (3.58)$$

Similarly the relationship between annual worth 'A' and present worth 'P' and that between 'A' and future worth 'F' of the cash flows considering the effect of inflation can be obtained.

From known values of combined interest rate 'i_c' and inflation rate 'f', the real interest rate 'i' can be obtained from equation (3.56) and is given as follows;

$$i_c = (i + f + f \times i) = f + i(1 + f)$$

$$i = \frac{(i_c - f)}{(1 + f)} \qquad \text{.........................} \quad (3.59)$$

The real interest rate 'i' (inflation-free) in equation (3.59) represents the equivalence between cash flows occurring at different periods of time with same purchasing power.

In the following examples the effect of inflation on equivalent worth of cash flows is illustrated.

Example -6

A person has now invested Rs.500000 at a market interest rate of 14% per year for a period of 8 years. The inflation rate is 6% per year.

i) What is the future worth of the investment at the end of 8 years?

ii) What is the accumulated amount at the end of 8 years in constant value monetary unit i.e. with the same buying power when the investment is made?

Solution:

The market or combined interest rate per year (i_c) and inflation rate per year (f) are 14% and 6% respectively.

i) The future worth i.e. amount of money accumulated at the end of 8 years will include the effect of inflation and the interest amount accumulated will be calculated using market or combined interest rate.

$$FW = P(F/P, i_c, n)$$

Putting 'P', 'i_c', 'n' equal to Rs.500000, 14% and 8 years respectively in the above expression;

$$FW = 500000(F/P, 14\%, 8)$$

$$FW = 500000 \times 2.8526 = 1426300$$

Thus the future worth of the investment at the end of 8 years in actual monetary unit (inflated) is Rs.1426300.

ii) The future worth in constant value monetary unit i.e. with same buying power when the investment is made (i.e. now) can be obtained by excluding the effect of inflation from the inflated future worth. Thus the future worth in constant value monetary unit is calculated by putting the values of time period 'n', base period 'b', inflation rate 'f' and the future worth (inflated) as calculated above in equation (3.51). In this case, the value of base period 'b' is zero.

FW in constant value monetary unit (inflation - free) =

$$'FW' \text{ in actual monetary unit (inflated)} \times \left(\frac{1}{1+f}\right)^{n-b}$$

$$FW \text{ in constant value monetary unit (inflation - free)} = 1426300 \times \left(\frac{1}{1+0.06}\right)^{8}$$

FW in constant value monetary unit (inflation - free) = Rs.894861

Thus the future worth of the investment at the end of 8 years in constant value monetary unit (inflation-free) is Rs.894861.

The future worth of the investment with same buying power as now i.e. when the investment is made can also be calculated by using the real interest rate. The real interest rate can be calculated from market interest rate and inflation rate using equation (3.59).

$$i = \frac{(i_c - f)}{(1+f)}$$

$$i = \frac{(0.14 - 0.06)}{(1 + 0.06)} = 0.07547$$

$i = 7.547\%$

Now the future worth of the investment with same buying power as now is calculated as follows;

$$FW = P(F/P, i, n).$$

$$FW = 500000(F/P, 7.547\%, 8) = 500000 \times 1.7897 = 894850$$

The calculated future worth of investment is same as that calculated earlier. The minor difference between the values is due to the effect of decimal points in the calculations.

Example -7

The cash flow of an alternative consists of payment of Rs.100000 per year for 6 years (uniform annual series). The real interest rate is 9% per year. The inflation rate per year is 5%. Find out the present worth of the uniform series when the annual payments are in actual monetary units (inflated).

Solution:

For determination of present worth of the uniform annual series payment, which is in actual monetary units (inflated), first the combined interest rate (i_c) is calculated from the known values of real interest rate (i) and inflation rate (f) using equation (3.56).

$i = 9\%, f = 5\%$

$$i_c = (i + f + f \times i)$$

$$i_c = (0.09 + 0.05 + 0.05 \times 0.09) = 0.1445$$

$i_c = 14.45\%$

Now the present worth of the uniform series payment (in actual monetary units) with uniform annual amount 'A' (Rs.100000) is given by;

$$PW = A(P/A, i_c, n)$$

$$PW = 100000(P/A, 14.45\%, 6) = 100000 \times 3.8412 = 384120$$

$PW = Rs.384120$

The present worth of the uniform series payment (in actual monetary unit) can also be calculated by converting the uniform annual amount 'A' occurring in different years into constant value monetary units (inflation-free) and then determining the present worth of the series at real interest rate (i) by using 'P/F' factor.

The values of uniform annual amount '*A*' (Rs.100000) occurring in different years in constant value monetary units (inflation-free) are calculated as follows;

In year '1'

$$\frac{100000}{(1+f)^1} = \frac{100000}{(1+0.05)^1} = \text{Rs.95238.1}$$

In year '2'

$$\frac{100000}{(1+f)^2} = \frac{100000}{(1+0.05)^2} = \text{Rs.90702.9}$$

In year '3'

$$\frac{100000}{(1+f)^3} = \frac{100000}{(1+0.05)^3} = \text{Rs.86383.8}$$

In year '4'

$$\frac{100000}{(1+f)^4} = \frac{100000}{(1+0.05)^4} = \text{Rs.82270.2}$$

In year '5'

$$\frac{100000}{(1+f)^5} = \frac{100000}{(1+0.05)^5} = \text{Rs.78352.6}$$

In year '6'

$$\frac{100000}{(1+f)^6} = \frac{100000}{(1+0.05)^6} = \text{Rs.74621.5}$$

Now the present worth of above amounts at real interest rate i.e. 9% per year is calculated as follows;

$$PW = 95238.1(P\!/\!F, i, 1) + 90702.9(P\!/\!F, i, 2) + 86383.8(P\!/\!F, i, 3) + 82270.2(P\!/\!F, i, 4) + 78352.6(P\!/\!F, i, 5) + 74621.5(P\!/\!F, i, 6)$$

$$PW = 95238.1(P\!/\!F, 9\%, 1) + 90702.9(P\!/\!F, 9\%, 2) + 86383.8(P\!/\!F, 9\%, 3) + 82270.2(P\!/\!F, 9\%, 4) + 78352.6(P\!/\!F, 9\%, 5) + 74621.5(P\!/\!F, 9\%, 6)$$

$$PW = 95238.1 \times 0.9174 + 90702.9 \times 0.8417 + 86383.8 \times 0.7722 + 82270.2 \times 0.7084 + 78352.6 \times 0.6499 + 74621.5 \times 0.5963$$

$$PW = \text{Rs.384120}$$

Thus the present worth of the uniform series payment (in actual monetary unit) is same as that calculated earlier.

Taxes:

In the economic comparison of mutually exclusive alternatives, it is possible to select the best alternative without considering the effect of taxes, as all the alternatives are evaluated on the same basis of not taking into account the effect of taxes on the cash flows. However the inclusion of taxes in the economic evaluation of alternatives results in improved estimate of cash flows and the estimated return on the investment. There are different types of taxes which are imposed by central/state governments on the firms. These are income taxes (on taxable income), property taxes (on value of assets owned), sales taxes (on purchase of goods/services), excise taxes (on sale of specific goods/services), etc. Among these taxes, income tax is the most important one, which is considered in the engineering economic analysis. Usually property taxes, excise taxes and sales taxes are not directly associated with income of a firm.

The calculation of income taxes is based on taxable income of the firm and the income tax to be paid by a firm is equal to taxable income multiplied with the applicable income tax rate. The taxable income is calculated at the end of each financial year. For obtaining the taxable income, all allowable expenses (excluding capital investments) and depreciation cost are subtracted from the gross income of the firm. It may be noted here that depreciation is not the actual cash outflow but it represents the decline in the value of the asset (depreciable) and is considered as an allowable deduction for calculating the taxable income. The taxable income is also known as net income before taxes. The net income after taxes is obtained by subtracting the income taxes from taxable income (i.e. net income before tax).

The expression for taxable income is shown below.

$$Taxable\ income = gross\ income - all\ expenses\ (excluding\ capital\ expenditur\ es) - depreciati\ on$$

The details about revenues, expenses and profit or loss of a firm are presented in Lecture 2 of Module 6. The capital investments are not included in the expenses for calculating the taxable income, as this does not affect the income of the firm directly; however the mode of financing the capital investment may have an effect on the taxes. The income tax to be paid is calculated as a percentage of the taxable income and is given as follows;

$$Income\ tax = taxable\ income \times income\ tax\ rate$$

The applicable annual tax rate is usually based on range or bracket of taxable income and the taxable income is charged as per the graduated tax rates wherein higher tax rates are applied to higher taxable income. A different marginal tax rate is applied to each of the taxable income bracket. Marginal tax rate indicates the rate that is applied on each additional unit of currency (Rupees, Dollars or Euros) of the taxable income. Higher marginal tax rate is applied to higher taxable income range. The taxable income in a certain bracket is charged at the specified marginal tax rate that is fixed for that particular bracket. Sometimes depending upon the taxable income, a single tax rate (i.e. flat tax rate) is applied to calculate the income tax. Capital gain and loss which represent the gain and loss on the disposal or sale of depreciable assets/property are also considered in the income tax analysis. The after-tax economic analysis of an alternative can be carried out by calculating the annual cash flows after taxes and after-tax rate of return. The after-tax rate of return depends on the before-tax rate of return and the tax rate. The annual cash flows after taxes is obtained by subtracting the taxes from annual cash flows before taxes.

The calculation of income tax using marginal tax rates for different taxable income brackets is presented in the following example.

Example -8

For a given financial year, the gross income of a construction contractor is Rs.11530000. The expenses excluding the capital investment and the depreciation are Rs.4170500 and Rs.2080000 respectively. Calculate the taxable income and income tax for the financial year with the following assumed marginal tax rates for different taxable income ranges.

Taxable income range (Rs.)	Income tax rate (%)
1 - 1000000	10
1000001 - 3000000	20
3000001 - 5000000	30
Over 5000000	35

Solution:

For the financial year, the taxable income of the construction contractor is calculated using the expression stated earlier.

$$\text{Taxable income} = \text{gross income} - \text{expenses (excluding capital expenditures)} - \text{depreciation}$$

$$\text{Taxable income} = \text{Rs.}11530000 - \text{Rs.}4170500 - \text{Rs.}2080000 = \text{Rs.}5279500$$

The income tax will be equal to the sum of income taxes calculated over the taxable income ranges at the specified tax rates. The construction contractor will pay 10% on first Rs.1000000 of taxable income, 20% on next Rs.2000000 (i.e. Rs.3000000 – Rs.1000000) of taxable income, 30% on next Rs.2000000 (i.e. Rs.5000000 – Rs.3000000) of taxable income and 35% on the remaining Rs.279500 (Rs.5279500 – Rs.5000000) of taxable income.

$$\text{Income tax} = 0.1 \times 1000000 + 0.2 \times 2000000 + 0.3 \times 2000000 + 0.35 \times 279500 = \text{Rs.}1197825$$

The taxable income and income tax for the financial year are Rs.5279500 and Rs.1197825 respectively. For the construction contractor with the above taxable income, the average tax rate fore the financial year is calculated as follows;

$$\text{Average tax rate} = \frac{\text{taxes paid}}{\text{taxable income}} = \frac{\text{Rs.}1197825}{\text{Rs.}5279500} = 0.2269 = 22.69\%$$

Construction Economics & Finance

Module 4
Lecture-1

Equipment costs:-

For construction firms, it is important to accurately estimate the equipment cost as part of the total cost of the construction project. Inaccurate estimation of construction equipment cost may adversely affect the profit margin of the firms especially engaged in projects with more involvement of different types of construction equipments. The total cost of a piece of construction equipment consists of two components namely ownership cost and operating cost. This is also referred as O&O cost of the construction equipment. The selection of a piece of equipment in a construction project depends on the total cost associated with that equipment. The details about equipment ownership cost and operating cost are presented below.

Ownership cost:-

Ownership cost is the total cost associated with the construction equipment for owning it irrespective of the equipment is employed or not in the project. The ownership cost consists of the following;

a) Initial cost, b) Salvage value, c) Interest cost or cost of capital investment, d) Taxes, e) Insurance cost, and f) Storage cost

a) Initial cost

Initial cost is the capital investment required to own the equipment. It includes purchase cost, sales tax, transportation cost (or freight charges) to bring the equipment to company's storage yard or construction site and cost of assembly and installation of the equipment. If the equipment is mounted on rubber tires (pneumatic tires), then the tire cost is deducted from the initial cost for calculating ownership cost. This is because the expected service life of pneumatic tires is less than that of the remaining of the equipment. Accordingly the rate of deprecation of tires is different from that of the equipment. Cost of tires is considered as a part of operating cost of the equipment. As the equipment is used in the project, there is depreciation in value of the equipment with time

due to wear and tear and obsolescence. The total amount of depreciation for the construction equipment over the useful life is equal to the initial cost less the estimated salvage value. The details about different methods for calculation of depreciation of the asset are already presented in Module 3.

b) Salvage value

As already mentioned in Module 3, salvage value represents expected cash inflow that will be received by disposing of equipment at the end of its useful life. The estimation of expected salvage value of the equipment can be carried out by referring to the data obtained from past projects wherein same (or similar) equipment was used or information obtained from other relevant sources.

c) Interest cost or cost of capital investment

It is the annual cost of interest charged on the borrowed money or that of capital investment to acquire the ownership of the equipment. If the equipment is purchased by borrowing money from a lender, then interest cost is the interest charged (at interest rate charged by lender) on the borrowed amount. On the other hand if the equipment is purchased using construction firm's own funds, then cost of capital investment is the interest charged on capital investment at interest rate equal to construction firm's rate of return. Even though the construction firm uses its own funds to purchase the equipment, cost of capital investment is charged as part of the ownership cost because the construction firm could have invested the funds elsewhere to earn the return instead of purchasing the equipment.

The interest cost on borrowed money or cost of capital investment can be exactly calculated by considering time value of money and using appropriate compound interest factors. However the interest cost or cost of capital investment can also be calculated approximately as percentage of constant average annual investment (cost) over the useful life of the equipment. The annual interest rate or the rate of return is multiplied to the average annual investment to find out the annual interest cost or cost of capital investment. The average annual investment can be calculated by finding out the average value of the equipment over the useful life of the equipment. It may be noted here that the value of the equipment depreciates with time. By considering straight-line depreciation,

the average annual investment can be found out by calculating average of the book value at the beginning of 1st year and that at beginning of last year of useful life i.e. at the beginning of 'n' year.

From module 3, the expression for constant annual depreciation from straight-line method (equation (3.1)) is given as follows;

$$D_m = \frac{P - SV}{n}$$

Where, P is the initial cost of equipment, n is the useful life and SV is the estimated salvage value of the equipment.

The book value at the beginning 1st year is equal to the initial cost i.e. 'P'. Now the book value at beginning of last year (i.e. 'n'), which is equal to book value at the end of '$(n-1)$' year, can be calculated by using equation (3.7) from module 3.

$$BV_{n-1} = P - (n-1)D_m = P - (n-1)\left(\frac{P - SV}{n}\right)$$

Now the average annual investment (AAI) is calculated by taking the average of the book value at the beginning of 1st year and that at beginning of last year of useful life i.e. BV_{n-1}.

$$AAI = \frac{P + BV_{n-1}}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.1)}$$

Putting the expressions of BV_{n-1} in equation (4.1) results in;

$$AAI = \frac{P + P - (n-1)\left(\dfrac{P - SV}{n}\right)}{2} = \frac{2nP - (nP - n \times SV - P + SV)}{2n}$$

On simplifying the above expression;

$$AAI = \frac{P(n+1) + SV(n-1)}{2n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.2)}$$

It may be noted here that while calculating average annual investment (AAI) for equipments mounted on pneumatic tires using the equation (4.2), cost of pneumatic tires is subtracted from initial cost of the equipment. The annual interest cost or cost of capital investment annual can be calculated by multiplying the average annual investment (AAI) as mentioned in equation (4.2) by the annual interest rate or rate of return.

d) Taxes

It represents the property taxes to be paid to the state or central government. It depends on the value of the equipment owned and the applicable tax rate for a given location. The property tax can be calculated as a percentage of the average annual investment or a percentage of the book value in a given year. Generally it ranges from 2 to 5% of the average annual investment or book value of equipment.

e) Insurance cost

It represents the annual premium to be paid to insurance companies to cover the cost incurred due to accident, fire, theft etc. for the construction equipment. In other words, it represents the cost that protects the owner of the equipment against these damages. Similar to taxes, the insurance cost can be calculated as a percentage of the average annual investment or the book value in a given year. It is generally about 1 to 3% of the average annual investment or book value of equipment.

f) Storage cost

It is the cost of keeping the equipment in storage yards when it is not operating at the work site. Storage cost includes the rental and maintenance charge for storage yards, wages of security guards and wages of workers employed for bringing in and out of the storage yards. It is around 0.5 to 1.5% of the average annual investment or book value of equipment. The annual storage cost can be calculated for the entire fleet of equipment and is then prorated to individual equipment requiring the storage facility.

Similar to storage cost, the tax and insurance cost can be calculated for the equipment fleet and then prorated to individual equipment. It may be noted here that the annual rates (%) mentioned above for taxes, insurance and storage costs are typical values. However the actual rates will vary depending on the type and size of equipment, place of purchase, location of project site etc.

After calculating the different components, the total annual ownership cost of the construction equipment is calculated by summing up depreciation cost, investment (or interest) cost, tax, insurance and storage costs. Then the hourly cost of ownership can be calculated by dividing the annual ownership cost by the number of operating hours the equipment will operate annually. The hourly ownership cost can also be calculated by

first determining the components of ownership cost individually on hourly basis followed by adding these hourly costs to get the total hourly ownership cost.

Lecture-2

Operating cost:-

Operating cost is incurred only when the equipment is operated. The operating cost of the equipment is influenced by various parameters namely number of operating hours, location of job site, operating conditions, category of equipment etc. The operating cost consists of the following;

a) Repair and maintenance cost, b) Fuel cost, c) Cost of lubricating oil, filter and grease, d) Tire cost e) Equipment operator wages, f) Cost of replacing high-wear items and g) Cost of mobilization, demobilization and assembly.

a) Repair and maintenance cost

Repair and maintenance cost is incurred as the construction equipment is subjected to wear and tear due to the operations it performs. The repair and maintenance cost includes the cost of replacement parts, labor charges and the cost of setting up and operating facilities to carry out major repair and maintenance operations. The repair and maintenance cost varies from one year to another over the service life of the equipment, however it increases with age of the equipment. This cost contributes a substantial portion of the operating cost. The increase in service life and decrease in repair and maintenance cost of construction equipment can be achieved by carrying out timely recommended maintenance and repair operations. The minor repairs can be carried out at the job site where the equipment is operating, however the major repairs can be carried out in the facilities set up by the equipment owner or in the workshop of authorized dealers for the equipment. The annual repair and maintenance cost can be calculated as a percentage of the annual depreciation cost of the equipment. Past information available from company records or from other relevant sources for similar equipment under similar working conditions can be used for estimating the repair and maintenance cost. If this data is not available, equipment manufacturer's guidelines can be used for calculating this cost. The hourly repair and maintenance cost can be calculated by dividing the annual cost by the number of operating hours per year.

b) Fuel cost

The construction equipments are generally powered by internal combustion engines which use either gasoline (petrol) or diesel as the fuel. The fuel consumption depends on the rated flywheel horsepower (fwhp) of the engine and the nature of working conditions. The flywheel horsepower represents the power available for operating a piece of equipment. A gasoline engine consumes about 0.06 gal (gallon) of fuel per flywheel horsepower hour whereas a diesel engine consumes about 0.04 gal of fuel per flywheel horsepower hour when operating under standard conditions. Historical data about the quantity of fuel consumed by similar equipment under similar working conditions can be used to estimate the hourly fuel consumption by the equipment. If these records are not available, equipment manufacturer's fuel consumption data can be used to estimate the fuel cost. The hourly fuel cost can be calculated by multiplying the hourly fuel consumption by its unit price.

c) Cost of lubricating oil, filter and grease

The quantity of lubricating oil, filter and grease required depends on operating hours, frequency of changes, engine characteristics and working conditions at the job site. The equipment manufacturer guidelines or past data from experience of similar equipment in similar working conditions can be used to estimate of time period between the changes.

The quantity of lubricating oil required by an engine is equal to the amount added during complete change plus the small amount added between the changes. The quantity of lubricating oil required by engine can be calculated by using the following relationship.

$$q_l = \frac{hp \times f \times 0.006\,lb\,/\,hp - h}{7.4\,lb\,/\,gal} + \frac{c}{t} \qquad \ldots\ldots\ldots\ldots\ldots \text{(4.3)}$$

Where

q_l = quantity of oil required in gal/h

hp = rated horsepower of the engine

f = operating factor

c = capacity of crankcase in gallons

t = number of hours (i.e. duration) between the oil changes

The term 0.006 lb/hp-h in the above equation is based on the assumption that the quantity of lubricating oil required per rated horsepower hour between the changes is 0.006 lb and

7.4 lb/gal is the conversion factor. The operating factor (*f*) in the above equation depends on throttle load factor and the efficiency factor i.e. operating efficiency. Manufacturer's guidelines or past experience from similar equipment under similar working conditions can be used to calculate the cost of these items. The hourly cost of lubricating oil, filter and grease can be calculated from hourly consumption quantities. It may be noted here that the quantity of oil required in SI unit system can also be obtained by using the appropriate conversion factors in the above equation.

d) Tire cost

The cost of pneumatic tires (rubber tires) is considered as a part of operating cost. The tire cost includes the tire repair and replacement charges. The service life of tires is generally shorter than that of the equipment as the tires wear out at a faster rate than the equipment. The life of tires varies according to extent of wear it is subjected to, which depends on the job site conditions. Tire repair charges can be calculated as a certain percentage of tire depreciation cost. The past data from similar operating and project site conditions and tire and equipment manufacturer guidelines can be used to estimate the expected life of tires. The hourly tire repair and replacement cost can be calculated by dividing the sum of the cost of a set of tires and repair charges by the life of tires in hours.

e) Equipment operator wages

The operator cost includes the hourly wages and benefits paid by the company to the operators. It includes normal wages, workmen's compensation insurance premium, fringe benefits, bonus etc. The operator wages vary from project to project. The operator cost is normally calculated as a separate cost category and is added to other components of operating cost.

f) Cost of replacing high-wear items

It represents the cost of high-wears items and these items have a shorter life as compared to the service life of the equipment. The high-wear items include blades, cutting edges, drill bits, bucket teeth etc. The expected life of these items can be estimated from past records or from manufacture guidelines. The hourly cost can be calculated by dividing the unit cost by estimated life (in hours).

g) Cost of mobilization, assembly and demobilization

This cost includes transportation charges from one project site to another, cost required for getting road permits, unloading charges, cost of assembly at the project site etc. The hourly cost can be calculated by dividing the total cost by the number of operating hours. In the following examples, the calculation of equipment ownership and operating costs is demonstrated.

Example -1

The initial cost of a piece of construction equipment (pneumatic tire mounted) is Rs.5500000. The estimated salvage value of the equipment is Rs.900000. The useful life of the equipment is 10 years. The equipment will operate 2000 hours per year. The cost of one set of tires is Rs.400000. A new set of tires will be replaced at the end of every 3 years of operation. In addition a repair work of cost Rs.450000 is expected at the end of year '6'. The interest rate is 8% per year. Find out the total cost per hour for the construction equipment considering time value of money.

Solution:

For calculating the hourly ownership and operating cost by considering time value of money, first the equivalent uniform annual cost is calculated at the given interest rate and is then divided by the number of operating hours per year.

As the initial cost includes cost of tires, tire cost is subtracted from initial cost of the equipment for calculating ownership cost, as cost of tires is considered as an element of operating cost.

Equivalent uniform annual worth of initial cost: A_1

Initial cost less cost of tires = Rs.5500000 – Rs.400000 = Rs.5100000

$$A_1 = Rs.5100000(A/P,i,n) = Rs.5100000(A/P,8\%,10)$$

$$A_1 = Rs.5100000 \times 0.1490$$

$$A_1 = Rs.759900$$

Equivalent uniform annual worth of salvage value: A_2

Salvage value = Rs.900000

$$A_2 = Rs.900000(A/F,i,n) = Rs.900000(A/F,8\%,10)$$

$A_2 = \text{Rs.}900000 \times 0.0690$

$A_2 = Rs.62100$

$$\text{Hourly ownership cost} = \frac{(A_1 - A_2)}{\text{Yearly operating hours}} = \frac{\text{Rs.}759900 - \text{Rs.}62100}{2000\,\text{hours}} = \text{Rs.}348.90/\text{h}$$

In the above expression, (A_1-A_2) is the equivalent uniform annual worth of the depreciation cost of the equipment considering time value of money.

Equivalent uniform annual worth of tire cost: A_3

The equipment will require 3 sets of replacement tires in addition to the original set. The first replacement set will be required at the end of 3rd year, second set at the end of 6th year and third set at the end of 9th year. In order to find out the equivalent uniform annual worth of tire cost, first the equivalent present worth of cost of replacement tires is calculated using the appropriate compound interest factor followed by adding it to the cost of original set (initial set) of tires and then converting the equivalent present worth to equivalent uniform annual cost over the entire useful life of 10 years using the appropriate compound interest factor.

$A_3 = [400000 + 400000(P/F,8\%,3) + 400000(P/F,8\%,6) + 400000(P/F,8\%,9)](A/P,8\%,10)$
$A_3 = [400000 + 400000 \times 0.7938 + 400000 \times 0.6302 + 400000 \times 0.5002](0.1490)$

$A_3 = Rs.174282$

Equivalent uniform annual worth of repair cost: A_4

Cost of repair work at the end of year '6' = Rs.450000.

$A_4 = Rs.450000(P/F,8\%,6)(A/P,8\%,10)$

$A_4 = \text{Rs.}450000 \times 0.6302 \times 0.1490 = \text{Rs.}42255$

$$\text{Hourly operating cost} = \frac{A_3 + A_4}{\text{Yearly operating hours}} = \frac{\text{Rs.}174282 + \text{Rs.}42255}{2000\,\text{hours}} = \text{Rs.}108.3/\text{h}$$

The total hourly cost for the construction equipment:

Hourly ownership cost + hourly operating cost = Rs.348.90/h + Rs.108.3/h

$$= \textbf{Rs.457.2/h}$$

Example -2

For a piece of construction equipment mounted on pneumatic tires, find out the hourly ownership cost using average annual investment method with straight line depreciation from the following data;

Initial cost = Rs.8000000, Estimated salvage value = Rs.1350000,

Useful life of the equipment = 11 years, Cost of a set of tires = Rs.600000.

The equipment will operate 1800 hours per year. The interest rate on investment is 7.5% per year. The annual rates for taxes, insurance and storage cost are 3%, 2.5% and 1% respectively. For this equipment, also calculate the hourly tire cost by not considering time value of money, if a new set of tires will be replaced at the end of every 4 years of operation and the estimated tire repair cost is 15% of straight-line depreciation.

Solution:

As the initial cost includes cost of tires, tire cost is subtracted from initial cost of the equipment for calculating ownership cost.

Initial cost less cost of tires (P) = Rs.8000000 – Rs.600000 = Rs.7400000

Salvage value (SV) = Rs.1350000, Useful life (n) = 11 years

The average annual investment (AAI) is calculated using equation (4.2) and is presented below.

$$AAI = \frac{P(n+1) + SV(n-1)}{2n}$$

$$AAI = \frac{Rs.7400000(11+1) + Rs.1350000(11-1)}{2 \times 11}$$

$$AAI = Rs.4650000$$

Annual interest or investment cost: A_1

The average annual interest or investment cost is calculated by multiplying average annual investment (AAI) by the annual interest rate (7.5%).

$$A_1 = AAI \times 0.075 = Rs.4650000 \times 0.075 = Rs.348750$$

Annual depreciation cost: A_2

The annual depreciation cost is calculated by straight-line depreciation method.

$$A_2 = \frac{P - SV}{n} = \frac{Rs.7400000 - Rs.1350000}{11} = Rs.550000$$

The annual expenses for taxes (A_3), insurance (A_4) and storage (A_5) are calculated by multiplying average annual investment (AAI) by the respective annual rates (%).

$$A_3 = AAI \times 0.03 = Rs.4650000 \times 0.03 = Rs.139500$$

$$A_4 = AAI \times 0.025 = Rs.4650000 \times 0.025 = Rs.116250$$

$$A_5 = AAI \times 0.01 = Rs.4650000 \times 0.01 = Rs.46500$$

Then the hourly ownership cost for the construction equipment is calculated as follows;

$$\text{Hourly ownership cost} = \frac{(A_1 + A_2 + A_3 + A_4 + A_5)}{\text{Annual operating hours}}$$

$$= \frac{Rs.348750 + Rs.550000 + Rs.139500 + Rs.116250 + Rs.46500}{1800 \, \text{hours}} = Rs.667.2/h$$

Hourly tire cost (by not considering time value of money)

The hourly tire cost is equal to the sum of hourly tire use (replacement) cost and hourly tire repair cost. The hourly tire use cost is obtained by dividing the cost of a set of tires by the life of tires in hours. The hourly tire repair cost is equal to 15% of the hourly depreciation (straight-line) cost of tires. The total depreciation cost of a set of tires over its estimated life is equal to its initial cost as its salvage value is assumed as zero. As mentioned in the question, a new set of tires will be replaced at the end of every 4 years of operation and the equipment will operate 1800 hours per year. Thus the life of a set of tires is equal to 7200 hours (1800×4).

Hourly tire cost = hourly tire use cost + hourly tire repair cost

$$\text{Hourly tire cost} = \frac{Rs.600000}{7200 \, \text{hours}} + \frac{Rs.600000 \times 0.15}{7200 \, \text{hours}} = Rs.95.8/h$$

Thus for the construction equipment, the hourly ownership cost is Rs.667.2/h and the hourly tire cost (by not considering time value of money) is Rs.95.8/h.

Similar to the calculation of hourly cost of equipment as mentioned in above examples, other components of equipment operating cost can also be calculated by using already mentioned guidelines and equations.

Lecture-3

Buy, Rent and Lease Options

There are different methods of acquiring the construction equipment required for a project. These are namely buying, renting and leasing. There are advantages and disadvantages associated with each of these methods. Before acquiring the equipment, it is essential to explore all the options by considering various parameters namely company's cash flow, working capital, equipment utilization and its maintenance, obsolescence and replacement of equipment, operating conditions etc.

Buying

Buying results in direct ownership of the equipment. Acquisition of equipment by buying is done either through cash purchase by using company funds or through financing purchase. The outright cash purchasing is done when sufficient funds are available. However cash purchase can have an adverse effect on company's cash flow as it reduces the liquid asset thus affecting company's working capital. When sufficient funds are not available for outright cash purchase, the equipment can be acquired by finance purchasing wherein the purchasing is done through loan arrangements from lenders i.e. banks or other financial institutions that includes the payment of loan through installments along with an initial down payment. One of the main advantages of owning the equipment by outright cash purchase is that it may result in lowest cost per operating hour as compared to renting or leasing. Other advantages associated with buying include, complete control of the owner over use of the equipment and its maintenance and replacement of equipment when it is no more economical. In addition, there is also income tax benefit associated with depreciation of the equipment. Acquiring the equipment through buying is an economically attractive option when there is more work load leading to higher utilization rate of the equipment over its useful life. Otherwise it will lead to the risk of not getting the required return on the capital investment if there is not enough utilization of the equipment. This is one of the disadvantages associated with buying. If the equipment is purchased through finance purchasing, the equipment owner has to pay the required loan installment to the lender even when the equipment is not

operational. Acquisition through buying may sometimes force the owner to use the obsolete equipment due to financial constraints.

Renting

Renting is a method of acquiring the equipment for a shorter duration. It is an alternative to direct ownership (i.e. through buying) of the equipment for a shorter period. Acquisition of equipment through renting is suitable when the contractor or the construction company requires the equipment for a project task of shorter duration. In addition through renting, the company can select the equipment that is exactly suited for the project task and it is possible to acquire the equipment based on latest technology which is more productive than older models. In these circumstances, renting of the equipment is more beneficial than direct ownership even though the rental charges are higher than the direct ownership charges. Since the equipment is not owned by the user, there is no tax benefit associated with depreciation of the equipment. However tax benefit is gained as the rental cost is considered as an expense that reduces the income of the company using the equipment. The capital that is tied up when the equipment is acquired through buying can be used in other investment if the equipment is rented. Among the three methods of acquiring the equipment, typically the hourly rental cost is higher as compared to that of lease or direct ownership. Rental period may a day, a week or a month and may go up to one year. The rental rates are generally established on daily basis, weekly basis or monthly basis. The daily rate on hourly basis is higher than the weekly rate on hourly basis and the weekly rate (on hourly basis) is higher than the monthly rate (on hourly basis). The repair and maintenance cost of the equipment to be paid either by the user or owner of the equipment (i.e. renting company) is stated in the rental contract. In case of major repair work, generally the cost is paid by the renting company whereas the cost of minor repair and maintenance that is incurred at the project site is usually paid by the user of the equipment. The cost fuel and lubricants is mostly paid by the user of the equipment. By renting, it is possible to reduce the downtime experienced due to breakdown of the equipment followed by repair as the equipment is replaced by the rental service. Further in order to check the suitability of specific equipment in actual job site conditions, the equipment can be rented and its performance

can be tested before taking a decision to purchase the equipment that involves a major capital investment. In addition, the user of the equipment can get rid of the cost of transporting the equipment from one project site to another by renting it. However the user mostly pays the transportation charges for bringing the equipment from renting company's yard to the work site and also pays the cost of assembly, loading etc.

Leasing

Leasing is another method of acquiring the equipment, for a longer period of time as compared to renting. It is a long term alternative to direct ownership of the equipment. The leasing company (i.e. owner of the equipment) is known as lessor whereas the user of the equipment is known as lessee. Lease is a contract between the lessor and the lessee wherein the lessee uses the equipment owned by lessor by paying the rentals over the lease period. Mostly the lease period is more than six months and may run up to years. It is important for the lessee to know about the details of past and ongoing leases in which lessor is involved and also to check the terms and conditions of the lease agreement before entering into lease contract with lessor. Most of the equipment leases are noncancellable. During lease period the lessor retains the ownership of the equipment and also gets the tax benefits from depreciation of the equipment. Thus there is no tax benefit to lessee from depreciation of the equipment. However similar to renting, lessee gains tax benefits as the lease payments are considered as an expense. Due to leasing, the capital of lessee is not tied up in purchasing the equipment and lessee can use it for other investment. Even if the equipment is purchased through loan arrangement, the owner has to pay an initial down payment. However leasing can provide 100% financing, including the cost delivery and installation of the equipment. Lease payments can be made monthly, yearly or at other time intervals as agreed upon between the two parties. The lease payments need not necessarily be uniform over the lease period and there is flexibility of coordinating the schedule of payment (to be made by the lessee) to that of revenue generated from use of the equipment by the lessee. Generally, the cash inflow due to use of the equipment at the beginning may be low and thus the lessee prefers to pay less amount at the beginning of lease period. This type of payment arrangement may

be accepted by the lessor from tax considerations as the lessor receives less payment at the beginning.

Two types of commonly offered leases through which construction equipments acquired are finance lease and operating lease. The finance lease is generally offered by a financial institution (usually a bank or a finance company) and the equipment is leased to the lessee. The lease period may extend up to the operating life of the equipment. The rental paid by the lessee over the lease period covers the cost of the equipment less the estimated residual value at the end of lease period, along with the profit margin of the lessor. The lessee has the option to purchase the equipment with a discounted price or a predetermined price at the end of the lease period. Usually the lease contract can not be cancelled till the lessor has recovered the investment cost at the end of the lease period. The finance lease is stated on lessee's balance sheet.

The operating lease is offered by the manufacturer or dealer of the equipment. In operating lease, payment charges are lower as compared to finance lease. In this type of lease, lessor (manufacturer or dealer of the equipment) provides the skilled service personnel required for carrying out the repair and maintenance operations and this type of arrangement is more suitable for sophisticated equipments requiring specialized repair and maintenance. Thus in this type of lease, lessee does not hire service personnel required for carrying out servicing and maintenance operations. Usually lessee returns the equipment to lessor at the end of lease period. Unlike finance lease, operating lease is not stated on lessee's balance sheet and is often referred to as off-balance-sheet financing.

Lecture-4

Replacement Analysis

Replacement analysis is carried out when there is a need to replace or augment the currently owned equipment (or any asset). There are various reasons that result in replacement of a given equipment. One of the reasons is the reduction in the productivity of currently owned equipment. This occurs due to physical deterioration of its different parts and there is decrease in operating efficiency with age. In addition to reduced productivity, there is also increase in operating and maintenance cost for the construction equipment due to physical deterioration. This necessitates the replacement of the existing one with the new alternative. Similarly if the production demands a change in the desired output from the equipment, then there is requirement of augmenting the existing equipment for meeting the required demand or replacing the equipment with the new one. Another reason for replacement of the existing equipment is obsolescence. Due to rapid change in the technology, the new model with latest technology is more productive than the currently owned equipment, although the currently owned equipment is still operational and functions acceptably. Thus continuing with the existing equipment may increase the production cost. The impact of rapid change in technology on productivity is more for the equipment with more automated facility than the equipment with lesser automation.

In replacement analysis, the existing (i.e. currently owned) asset is referred as **defender** whereas the new alternatives are referred as **challengers**. In this analysis the 'outsider perspective' is taken to establish the first cost of the defender. This initial cost of the defender in replacement analysis is nothing but the estimated market value from perspective of a neutral party. In other words this cost is the investment amount which is assigned to the currently owned asset (i.e. defender) in the replacement analysis. The current market value represents the opportunity cost of keeping the defender i.e. if the defender is selected to continue in the service. In other words, if the defender is selected, the opportunity to obtain its current market value is forgone. Sometimes the additional cost required to upgrade the defender to make it competitive for comparison with the new alternatives is added to its market value to establish the total investment for the defender.

Along with the market value, there will be revised estimates for annual operating and maintenance cost, salvage value and remaining service life of the defender, which are expected to be different from the original values those were estimated at the time of acquiring the asset. The past estimates of initial cost, annual operating and maintenance cost, salvage value and useful life of defender are not relevant in the replacement analysis and are thus neglected. The past estimates also incorporate a **sunk cost** which is considered irrelevant in replacement analysis. Sunk cost occurs when the book value (as determined using depreciation method) of an asset is greater than its current market value, when the asset (i.e. defender) is considered for replacement. In other words it represents the amount of past capital investment which can not be recovered for the existing asset under consideration for replacement. Sunk cost may occur due to incorrect estimates of different cost components and factors related productivity of the defender, those were made at the time of original estimates in the past with uncertain future conditions. Since sunk cost represents a loss in capital investment of the asset, the income tax calculations can be done accordingly by considering this capital loss. In replacement analysis the incorrect past estimates and decisions should not be considered and only the cash flows (both present and future) applicable to replacement analysis should be included in the economic analysis. For replacement analysis, it is important know about different lives of an asset, as this will assist in making the appropriate replacement decision. The different lives are physical life, economic life and useful life. **Physical life** of an asset is defined as the time period that is elapsed between initial purchase (i.e. original acquisition) and final disposal or abandonment of the asset. **Economic life** is defined as the time period that minimizes the total cost (i.e. ownership cost plus operating cost) of an asset. It is the time period that results in minimum equivalent uniform annual worth of the total cost of the asset. **Useful life** is defined as the time period during which the asset is productively used to generate profit. In replacement analysis the defender and challenger is compared over a study period. Generally the remaining life of the defender is less than or equal to the estimated life of the challenger. When the estimated lives of the defender and challenger are not equal, the duration of the study period has to be appropriately selected for the replacement analysis. When the estimated lives of defender

and challenger are equal, annual worth method or present worth method may be used for comparison between defender and the challengers (new alternatives).

In the following example, replacement analysis involving equal lives of defender and challenger is discussed.

Example -3

A construction company has purchased a piece of construction equipment 3 years ago at a cost of Rs.4000000. The estimated life and salvage value at the time of purchase were 12 years and Rs.850000 respectively. The annual operating and maintenance cost was Rs.150000. The construction company is now considering replacement of the existing equipment with a new model available in the market. Due to depreciation, the current book value of the existing equipment is Rs.3055000. The current market value of the existing equipment is Rs.2950000. The revised estimate of salvage value and remaining life are Rs.650000 and 8 years respectively. The annual operating and maintenance cost is same as earlier i.e. Rs.150000.

The initial cost of the new model is Rs.3500000. The estimated life, salvage value and annual operating and maintenance cost are 8 years, Rs.900000 and Rs.125000 respectively. Company's MARR is 10% per year. Find out whether the construction company should retain the ownership of the existing equipment or replace it with the new model, if study period is taken as 8 years (considering equal life of both defender and challenger).

Solution:

For the replacement analysis, initial cost (Rs.4000000), initial estimate of salvage value (Rs.850000) and remaining life (12 − 3 = 9 years) and current book value (Rs.3055000) of the existing equipment (i.e. defender) are irrelevant. Similarly sunk cost of Rs.105000 (Rs.3055000 − Rs.2950000) is also not relevant for the replacement analysis. For the replacement analysis the current revised estimates of the existing equipment will be used.

For existing equipment (defender),

Current market value (P) = Rs.2950000, Salvage value (F) = Rs.650000,

Annual operating and maintenance cost (A) = Rs.150000, Study period (n) = 8 years.

For new model (challenger),

Initial cost (P) = Rs.3500000, Salvage value (F) = Rs.900000,

Annual operating and maintenance cost (A) = Rs.125000, Study period (n) = 8 years.

Now the equivalent uniform annual worth of both defender (i.e. the existing equipment) and challenger (i.e. the new model) at MARR of 10% (i.e. i = 10%) are calculated as follows;

For defender;

$$AW_{def} = -2950000(A/P,i,n) - 150000 + 650000(A/F,i,n)$$

$$AW_{def} = -2950000(A/P,10\%,8) - 150000 + 650000(A/F,10\%,8)$$

$$AW_{def} = -2950000 \times 0.1874 - 150000 + 650000 \times 0.0874$$

$$AW_{def} = -646020$$

For challenger;

$$AW_{cha} = -3500000(A/P,i,n) - 125000 + 900000(A/F,i,n)$$

$$AW_{cha} = -3500000(A/P,10\%,8) - 125000 + 900000(A/F,10\%,8)$$

$$AW_{cha} = -3500000 \times 0.1874 - 125000 + 900000 \times 0.0874$$

$$AW_{cha} = -702240$$

From the above calculations, it is observed that equivalent uniform annual cost of the defender is less than that of the challenger. Thus the construction company should continue in retaining the ownership of the defender against the challenger with above details. Since the useful lives of defender and challenger are equal, the same conclusion will also be obtained by using present worth method for economic evaluation.

Lecture-5

Replacement Analysis

When useful lives of defender and challenger are not same i.e. remaining life of defender is not equal to useful life of the challenger (new alternative), mostly the duration of the longer life span alternative is selected as the study period. In other words the useful life of the challenger (which is generally greater than remaining life of defender) is taken as the study period. In this case it is assumed that the equivalent uniform annual cost of the defender (i.e. the shorter life span alternative) will be same after its remaining life and till the end of the study period. In other words the shorter life span alternative will function at the same equivalent annual cost throughout the study period. However if realistic estimate of the equivalent annual cost of the shorter life span alternative (i.e. defender) after its remaining life is available, the same can be used appropriately in the economic analysis over the study period.

Sometimes the use of longer study period may not be beneficial because of the fact that the rapid obsolescence may force the replacement of longer life span alternative, due to availability of new models with latest technology which are more productive. In addition inaccurate estimate of different cost components with uncertain future conditions for longer life span alternative is also another factor which may adversely affect the selection of longer study period. These reasons may force the management of the company to select a shorter study period for replacement analysis between defender and challenger. However the use of a shorter study period will force the recovery of initial capital investment of the longer life span alternative (i.e. challenger) at company's MARR in a shorter period of time which is less than its estimated useful life and this may affect the unbiased selection of the most economical alternative in the replacement analysis. When a shorter study period is used, the use of a realistic estimate of the salvage value or market value of the longer life span alternative (i.e. challenger) will result in an unbiased selection of the most economical alternative.

In the following example, the effect of longer and shorter study period on the selection of the most economical alternative between defender and challenger is discussed.

Example -4

For the replacement analysis of an asset of a construction company, the following information is available;

For defender (existing asset);

The original estimate includes, initial cost 4 years ago = Rs.5000000, Estimated salvage value and useful life at the time purchase = Rs.1200000 and 12 years respectively, Annual operating cost = Rs.115000.

The revised estimate includes, current market or trade-in value = Rs.3600000, remaining life = 7 years, Salvage value = Rs.780000, Annual operating cost = Rs.115000 (same as earlier).

For challenger (new alternative);

Initial cost = Rs.4500000, Estimated salvage value and useful life = Rs.1040000 and 14 years respectively, Annual operating cost = Rs.100000, Estimated value of the asset at the end of 7 years = Rs.2500000.

Carry out the replacement analysis using 14 year study period and 7 year study period at MARR of 8% per year.

Solution:

Replacement analysis using 14 year study period;

For defender (existing asset);

Current market or trade-in value = Rs.3600000, Salvage value = Rs.780000,

Annual operating cost = Rs.115000.

For challenger (new alternative);

Initial cost = Rs.4500000, Salvage value = Rs.1040000,

Annual operating cost = Rs.100000.

The equivalent uniform annual worth of defender is calculated at MARR of 8% over a period of 7 years (its remaining life) by using its revised estimates and it is assumed that, the calculated equivalent uniform annual worth of defender will be same after its remaining life and over the study period of 14 years. The equivalent uniform annual worth of challenger is calculated over the study period of 14 years (its estimated useful life) at MARR of 8%.

For defender;

$$AW_{def} = -3600000(A/P, i, n) - 115000 + 780000(A/F, i, n)$$

$$AW_{def} = -3600000(A/P, 8\%, 7) - 115000 + 780000(A/F, 8\%, 7)$$

$$AW_{def} = -3600000 \times 0.1921 - 115000 + 780000 \times 0.1121$$

$$AW_{def} = -719122$$

For challenger;

$$AW_{cha} = -4500000(A/P, i, n) - 100000 + 1040000(A/F, i, n)$$

$$AW_{cha} = -4500000(A/P, 8\%, 14) - 100000 + 1040000(A/F, 8\%, 14)$$

$$AW_{cha} = -4500000 \times 0.1213 - 100000 + 1040000 \times 0.0413$$

$$AW_{cha} = -602898$$

From the above calculation, it is observed that the new alternative is the most economical one over the study period of 14 years as it shows lower equivalent annual cost. Thus the construction company should replace the existing asset with the new one.

Replacement analysis using 7 year study period;

For defender (existing asset) the equivalent uniform annual cost will be same i.e. Rs.719122 as calculated above for the period of 7 years.

The equivalent uniform annual worth of challenger is calculated by considering its estimated value (i.e. Rs.2500000) at the end of 7 years.

For challenger;

$$AW_{cha} = -4500000(A/P, i, n) - 100000 + 2500000(A/F, i, n)$$

$$AW_{cha} = -4500000(A/P, 8\%, 7) - 100000 + 2500000(A/F, 8\%, 7)$$

$$AW_{cha} = -4500000 \times 0.1921 - 100000 + 2500000 \times 0.1121$$

$$AW_{cha} = -684200$$

The new alternative again shows lower equivalent annual cost as compared to the defender (existing asset) over the study period of 7 years.

If a study period of 6 years is considered and it is assumed the equivalent annual cost of the defender remains same i.e. Rs.719122 over the study period, then the equivalent annual cost of the challenger is calculated over this study period. Assuming the same estimated value of the challenger i.e. Rs.2500000 at the end of 6 years, its equivalent annual cost over the study period of 6 years is calculated as follows;

$$AW_{cha} = -4500000(A/P, i, n) - 100000 + 2500000(A/F, i, n)$$

$$AW_{cha} = -4500000(A/P, 8\%, 6) - 100000 + 2500000(A/F, 8\%, 6)$$

$$AW_{cha} = -4500000 \times 0.2163 - 100000 + 2500000 \times 0.1363$$

$$AW_{cha} = -732600$$

From this calculation it is observed the new alternative (challenger) is exhibiting higher equivalent annual cost as compared to the existing asset (defender) when the study period of 6 years is used. This is due to the fact that the challenger is allowed only 6 years to recover the same investment as compared to the study period of 7 or 14 years and thus resulting in an increased equivalent annual cost.

As already mentioned (in Lecture 4 of this module), if there is requirement of a change in the desired output from the existing equipment, then the enhanced production capacity can be achieved by augmenting the existing equipment or replacing it with a new one. In some cases construction firms plan for major overhaul or retrofitting of the existing equipment in order to met the required demand. In the following example replacement analysis involving the major overhaul of the currently owned asset i.e. defender and its comparison against challenger is presented.

Example -5

A construction firm has purchased an excavator 3 year ago at a cost of Rs.6000000 and the estimated life and salvage value at the time of purchase were 11 years and Rs.1600000 respectively. The annual operating cost was Rs.195000. The current market value of the equipment is Rs.4400000. The construction firm is planning for a major overhaul of the equipment now at a cost of Rs.1000000. After overhaul, the revised estimate of salvage value, annual operating cost and remaining life of the excavator are Rs.1250000, Rs.175000 and 9 years respectively.

However the construction firm has the option to replace the current excavator with a new model. The initial cost of the new model is Rs.6300000. The estimated life, annual operating cost and salvage value are 9 years, Rs.150000 and Rs.1800000 respectively. Determine whether the construction firm should continue with the existing excavator

with the planned overhaul or replace it with the new model if the firm's MARR is 10% per year.

Solution:

For the replacement analysis the revised estimates of the existing excavator (defender) are as follows.

Current market value (P) = Rs.4400000, Salvage value (F) = Rs.1250000, Cost of major overhaul, now = Rs.1000000, Annual operating cost (A) = Rs.175000.

For new model (challenger),

Initial cost (P) = Rs.6300000, Salvage value (F) = Rs.1800000,

Annual operating cost (A) = Rs.150000.

The estimated lives of both defender and challenger are same i.e. 9 years. Now the equivalent uniform annual cost of both the existing excavator and the new model at MARR of 10% per year (i.e. i = 10%) are calculated over the study period of 9 years and are presented as follows;

For existing excavator (defender);

$$AW_{def} = -4400000(A/P, i, n) - 1000000(A/P, i, n) - 175000 + 1250000(A/F, i, n)$$

$$AW_{def} = -4400000(A/P, 10\%, 9) - 1000000(A/P, 10\%, 9) - 175000 + 1250000(A/F, 10\%, 9)$$
$$AW_{def} = -4400000 \times 0.1736 - 1000000 \times 0.1736 - 175000 + 1250000 \times 0.0736$$

$$AW_{def} = -1020440$$

For new model (challenger);

$$AW_{cha} = -6300000(A/P, i, n) - 150000 + 1800000(A/F, i, n)$$

$$AW_{cha} = -6300000(A/P, 10\%, 9) - 150000 + 1800000(A/F, 10\%, 9)$$

$$AW_{cha} = -6300000 \times 0.1736 - 150000 + 1800000 \times 0.0736$$

$$AW_{cha} = -1111200$$

From the above calculations, it is noted that equivalent uniform annual cost of the existing excavator with the planned overhaul is less than that of the new model. Thus the construction firm should continue with the existing excavator. Similarly the replacement analysis involving comparison of defender with augmentation against the challenger can also be carried out.

Further in replacement analysis, the remaining economic life of the defender can be found out by determining the number of years, which minimizes the total cost of owning and operating the asset. For this purpose, the equivalent uniform annual cost of the defender is determined year by year by considering the time value of money. The remaining economic life of the defender is taken as the year which results in the minimum equivalent uniform annual cost. Similarly the economic life of the challenger can also be determined in the same manner by considering the time value of money. Further replacement analysis involving the comparison of total cost of defender for retaining it for one additional year in service with the minimum equivalent uniform annual cost of challenger can also be carried out and accordingly the decision of retaining or replacing the defender can be taken.

It may be noted that the calculation of equipment cost and replacement analysis using the information from real life construction projects can be carried out in the same manner.

Construction Economics & Finance

Module 5

Lecture-1

Cost Estimating:-

The purpose of cost estimating is to forecast the cost of a project prior to its actual construction. Cost estimating is a method of approximating the probable cost of a project before its construction. The exact cost of a project is known after completion of the project. Cost estimate is prepared at various stages during the life of a project on the basis of the information available during the time of preparation of the estimate. Generally for any construction project, three parties are involved namely owner, design professionals and construction professionals. In some cases the design professional and construction professional are from the same company or they form a team through a joint venture for providing service to the owner in the project. It is the responsibility of each party involved in the project to estimate the costs during various stages of the project. An early estimate helps the owner to decide whether the project is affordable within the available budget, while satisfying the project's objectives. For cost estimating, work breakdown structure (WBS) serves as an important framework for organized collection project cost data and preparing the cost estimates at different levels. It is a technique that involves the hierarchical breakdown of the project into different work elements at successive levels and defines the interrelationships between them. For preparation of cost estimates, the estimator performs quantity take-off to quantify each item of work by reviewing the contract drawings and specifications. In cost estimation, quantity take-off is an important task that is carried out before pricing each item of work and quantities should be represented in standard units of measure. Before bidding for a project, the estimator (along with his or her group) of a construction firm needs to determine the total cost of the project in accordance with contract documents consisting of drawings, specifications and all other technical documents and requirements. The total project cost consists of two components namely direct cost and indirect cost. Direct cost includes cost of materials, equipment and labor associated with each item of work and also includes cost of

subcontracted works. Indirect costs are the costs which are not associated with each item of work rather these costs are calculated for the entire construction work and includes overhead costs (both job office or site office overhead and general head office overhead), contingency i.e. cost for any unforeseen work and profit. It is important to accurately estimate all the cost components of the project before bidding. The material cost for each item of work can be calculated by multiplying required quantity of materials by its unit price. The material quantity take-off or the quantity of materials for an item of work can be easily calculated by using the information from contract drawings and specifications. The current unit price of materials can be obtained from the material suppliers. In addition, the estimator has to add a certain percentage for wastage of materials while calculating the material cost. The equipment cost consists of two components i.e. ownership cost and operating cost. Generally the equipment cost is expressed on hourly basis. As already mentioned (in Module 4), equipment operator wages which vary from project to project are normally calculated as a separate cost category and are added to other components of equipment operating cost. The procedure of calculating hourly equipment cost is already stated in Lecture-1 and Lecture-2 of Module 4. The hourly equipment cost and production rate of the equipment are used to calculate the cost per unit production of the equipment. The production rate of the equipment depends on its rated capacity, production cycle time, efficiency and also on job site conditions. The labour cost depends on the productivity of labourers which vary with the nature of work. The productivity of labourers depends on various factors namely their skill, number of working hours in a day, supervision, nature of job, job site conditions etc. The portion of work that is to be accomplished by the labourers can be found out form project documents i.e. from contract drawings and specifications. The labour cost is generally expressed on hourly basis. Historical data from past projects can also be referred for calculating the cost of materials, equipment and labor. For the contractors, the calculated bid price (sum of direct cost and indirect cost) for the project must have a balance with the associated profit margin such that the bid price should be low enough to be within the owner's budget and to win the bid and at the same time, it should be high enough to complete the project with the expected profit.

Lecture-2

Types of Estimates

As already stated in the previous lecture (Lecture-1 of this module), there are different types of estimates which are prepared at various stages during the life of a project starting from the initial phases to its final phase on the basis of the available information at the time of preparation of the estimates. The range of expected accuracy is more in the estimates which are prepared during initial stages and it narrows down as the project progresses with the availability of more detailed information and increase in the level of project definition. In addition to the project parameters, the degree accuracy of an estimate also depends on the experience, ability and judgment of the estimator. The construction cost estimate is broadly classified into two types approximate estimates and detailed estimates. The approximate estimates are prepared during initial stages of the project life cycle. These estimates are also known as preliminary, budget or order-of-magnitude estimates and are prepared to determine the preliminary cost of the project. From the approximate cost estimates, the owner of the project can be able to know whether the project can be undertaken within the available budget. There may be more than one design alternative for a project depending on the location, site conditions, type of structure etc. The estimator can determine the approximate cost of various alternatives for the project by taking preliminary design information from designer and can obtain the economical alternative that is affordable within the available budget for the project.

Detailed estimates are prepared in accordance with the complete set of contract documents. As already stated, the contractors prepare the detailed estimates before biding for the project by thoroughly reviewing the contract documents. Project site visit by the contractor personnel may be required to identify the parameters that can influence project cost and accordingly the estimate can be adjusted. The detailed cost estimate showing the bid price of the project is important to both the project owner and the contractor, as the bid price represents the amount the contractor will receive from the owner for completing the project in accordance with contract documents. The different types of cost estimates which are prepared during various phases of a project are described below.

Estimates during conceptual planning:

This estimate is prepared at the very initial stage i.e. during conceptual planning stage of a project. It is based on little information and on broad parameters namely size of the project, location and job site conditions and the expected construction quality of project as a whole. The size of the project may be expressed in terms of its capacity namely number of rooms for a hostel, number of beds for a hospital, length (km) of a highway etc. Owner of the project provides adequate input for defining scope of the project and this scope of the project forms the basis on which the conceptual estimate is prepared. This estimate is prepared to establish the preliminary budget of the project and accordingly project funding can be arranged. The degree of accuracy of this estimate is lowest among all the estimates those are prepared during various stages of a project.

Estimates during schematic design:

During this phase of the project, the cost estimate is prepared on the basis of preliminary design information along with required schematic documents. The designer may incorporate different design alternatives and the cost estimate is prepared for these design alternatives by the estimators depending on the available information. The cost estimates of different design alternatives are reviewed keeping in view the project scope and budget and the acceptable alternative(s) selected in this phase is analyzed in a detailed manner in the next phase of the project. This cost estimate is prepared by calculating the cost of major project elements by unit pricing from the available preliminary design information. Subcontractors or material suppliers may be asked to furnish information while pricing the major project elements. As the cost estimate is prepared using preliminary design information, a contingency may be added in the estimate to accommodate for the unknown design details. With the improved scope of the project, the expected degree of accuracy in this estimate is more as compared to that in conceptual estimate.

Estimates during design development:

During design development phase of the project, the cost estimate is prepared on the basis of more detailed design information and schematic documents. With the improved level of information, the most of the major project items namely volume of earthwork (m^3), volume of concrete (m^3), weight of steel (tons) etc. can be quantified and the cost

estimate is prepared using the known unit prices. Detailed information from subcontractors or material suppliers should be obtained and used in pricing the major project items. During this phase, all the identified major systems of the project namely structural systems (reinforced concrete vs. structural steel), masonry (clay brick units vs. concrete masonry units), pile foundation (concrete pile vs. steel pile) etc. are priced and then cost of each system is compared with that obtained from past similar projects. The project elements costing too high or too low as compared to past data should be reviewed and accordingly adjusted. With the availability of detailed design information and improved system definition, the expected degree of accuracy in this estimate is higher as compared to that in estimate prepared during schematic design phase of the project.

Estimates during procurement (i.e. estimates for construction of the project):
During this phase of the project, the cost estimate is prepared on the basis of complete set of contract documents that defines the project. The contractors bidding for the project prepare the cost estimate in accordance with contract documents by taking into consideration the estimated project duration. As already mentioned in the previous lecture (Lecture-1 of this module), the total cost of project can be divided into two categories namely direct cost and indirect cost. Direct cost includes cost of materials, equipment and labor associated with each item of work and cost of subcontracted works. Indirect costs are the costs which are not attributed to each item of work and are calculated for the entire project and include overhead cost, contingency and profit. The owner team also prepares the cost estimate to check the accuracy of the bid prices quoted by the contractors and negotiate a reasonable price with the contractor. As this cost estimate is prepared in accordance with complete set of contract documents of the project, the degree of accuracy of this estimate is extremely high.

Estimates for change orders during construction:
This estimate is prepared to cater the changes in the project scope as required by the owner during construction phase of the project.

Lecture-3

Approximate estimates

The different types of cost estimates, those are prepared during various phases of a project are already stated in Lecture 2 of this module. As already mentioned in Lecture 2 of this module, the approximate estimates (also known as order-of-magnitude estimates) are prepared during initial stages of the project life cycle. The different methods used in the preparation of approximate estimates are described below. In addition to use in the preparation of estimates during early stages of project development, these methods may also be sometimes used for cost estimating in the detailed design phase.

Unit estimate

This technique is used for preparing preliminary estimates (i.e. order-of- magnitude type estimate). This estimate is generally prepared during the conceptual planning phase of a project, with less information available with the estimator. In this method, the total estimate of cost is limited to a single factor. The examples of some of the 'per unit factor' used in construction projects are construction cost per square meter, housing cost per boarder of a hostel, construction cost per bed for a hospital, maintenance cost per hour, fuel cost per kilometer, construction cost per kilometer for a highway etc. The total cost is calculated by multiplying the cost per unit factor with the number of units of the corresponding factor. For example, a preliminary estimate is required to estimate the cost of constructing a new house with floor area of 170 square meter. If the cost per square meter is Rs.26,900 (assumed), then the cost of constructing the house will be Rs.45,73,000 (Rs.26,900×170). Similarly the preliminary cost estimate of constructing a given length of highway can be calculated by multiplying the unit cost i.e. construction cost per kilometer by the length of highway (in km).

The details of unit cost of different factors can be obtained from records of past projects. The 'per unit factor' obtained from past similar projects can be adjusted by taking into account different parameters namely location and size of the project, prevailing market conditions etc. The accuracy of this technique depends on the quality and reliability of the data available with the estimator.

Factor estimate

In this method, the estimate is divided into individual segments and then the cost of each segment is calculated followed by adding all the individual costs to estimate the total cost. Unlike unit technique, the factor technique includes separate factors for different cost items. Thus compared to unit technique, factor technique is a more detailed method of preparing the cost estimate and shows improved accuracy. The factor method is useful when several components are involved in the preparation of cost estimate.

Considering a simple example, a preliminary estimate is required to find out the cost of constructing a new multi-storey hostel of an educational institute. By unit method, the preliminary cost will be simply the cost per square meter multiplied with the total floor area (square meter) of the hostel. However a comparatively improved estimate can be prepared by using factor method wherein the total cost estimate is divided into different components. The hostel will consist of individual living rooms for students, hostel office, a kitchen cum dining room, a reading room, a common room, an indoor sports room, lavatories on each floor and other common amenities. The estimate of the total cost of the hostel will comprise of cost of these components. The cost of residing rooms for the students can be calculated by multiplying the cost of constructing one room (the information can be obtained from past similar projects with adjustment for different parameters as mentioned in unit estimate technique) by the required number of rooms. The estimate of the construction cost of the kitchen cum dining room can be calculated by multiplying its area in square meter by construction cost per square meter. Similarly the estimate of the construction cost of other common areas (i.e. reading room, common room etc.) can be calculated by multiplying the respective areas (in square meter) by construction cost per square meter. Then the total estimate of the construction cost of the hostel will be equal to the sum of all the cost components. As already mentioned the information about construction cost per square meter of common amenities can be obtained from past data on similar projects. In this example the estimate of construction cost of living rooms of students can also be calculated by multiplying the total area to be occupied by the rooms (square meter) by the cost per square meter.

Taking another example with more details, a preliminary cost estimate is required obtain the cost of materials to be used in constructing reinforced concrete sections of a structure. For reinforced concrete sections, the total cost estimate will comprise of cost of concrete and cost of steel reinforcement bars. For estimating the cost of concrete (considering normal strength concrete), the quantities of different ingredients namely cement, coarse aggregate and sand are calculated from known volume of concrete by using the mix proportion (either known or using data of similar grade of concrete from past projects). The cost of the individual ingredients of concrete can now be calculated by multiplying the quantities of the ingredients in the appropriate units with the respective unit cost. For example, the required quantity of cement can be expressed in kg or in terms of number of bags (i.e. total quantity of cement in kg divided by quantity of cement in kg per bag). Similarly the required quantities of both coarse aggregate and sand can be expressed in m^3 (i.e. quantity of aggregate in kg divided by bulk density in kg/m^3). The information about unit cost of different materials can be obtained from material suppliers. Similarly the cost of steel reinforcement can be obtained by multiplying the quantity of steel in tons with its unit price. Now the total cost of materials to be used in constructing reinforced concrete sections is calculated by summing the cost of concrete and cost of steel reinforcement. Similarly the preliminary estimate of other cost items can be obtained by using factor estimate method.

Cost indexes

The method involving cost indexes is used for obtaining preliminary cost estimate by considering the historical data of similar past projects. Cost indexes (or indices) are dimensionless numerical values which represent the price change of individual or multiple cost items over time with respect to a reference year. Cost indexes are used to update the historical cost figures and to obtain the current cost estimates. The relationship used for updating the historical cost figures to the cost at another point of time using cost index is presented below.

$$C_n = \frac{C_r I_n}{I_r}$$

Where

C_n = estimated cost of the item in year 'n'

C_r = cost of the item in year 'r' (at earlier point of time and $n > r$)

I_n = index value in year 'n'

I_r = index value in year 'r'

Here 'r' is the reference year (i.e. at earlier of point of time) at which the cost of the item known and 'n' is the year for which cost of the item is to be estimated. Cost indexes are periodically published by various public and private agencies.

Cost indexes are generated for an individual item or for wider variety of items. For a single item, cost index can be calculated as the ratio of cost of the item today (i.e. in current year) to the cost of the same item at some point of time in past (i.e. reference year) multiplied with cost index value of the reference year. Similarly for multiple items, a weighted-average index or composite index for a specific year can be obtained from known cost of the selected group of items in the given year and in reference year and the composite index of these items in the reference year by assigning the appropriate weightage factors to these items. In construction, the expected cost estimate of materials, equipment and labour for new projects can be obtained from the known cost of these items from similar past projects and cost indexes (in reference year and current year) using the above relationship.

Example -1

For estimating the expected cost of materials for a new construction project, the following information is available;

The material cost of a similar construction project that was completed four years ago was Rs.1,12,64,000 and the material cost index was 512. Calculate the approximate material cost for the new project, if material cost index now is 625.

Solution:

The approximate material cost for the new project 'C_n' is calculated using the above relationship and is presented below.

$$C_n = \frac{C_r I_n}{I_r}$$

Where

C_n = approximate material cost (present cost estimate)

C_r = = material cost of similar project four years ago = Rs.1,12,64,000

I_n = material cost index now = 625

I_r = material cost index four years ago = 512

$$C_n = \frac{\text{Rs.}1,12,64,000 \times 625}{512} = \text{Rs.}1,37,50,000$$

Thus the approximate material cost for the new construction project is Rs.1,37,50,000.

Lecture-4

Parametric estimate

Parametric cost estimate is prepared during early stages of project development. The cost estimate is based on various parameters which define characteristics of the project and includes physical attributes such as size, capacity, weight etc. The parametric cost estimate is based on cost-estimating relationships those use past cost data to obtain the current cost estimate. Cost estimating relationships are statistical models those relate the cost of a product or system to the physical attributes those define its characteristics. One of the most commonly used cost-estimating relationships is power-sizing model.

Power-sizing model

Power-sizing model is most commonly used for obtaining preliminary cost estimate of industrial plants and equipment. The power sizing model relates the cost of a plant or system to its capacity or size and uses the following relationship for cost estimate.

$$\frac{C_1}{C_2} = \left(\frac{Q_1}{Q_2}\right)^x$$

Where

C_1 = cost of Plant 1

C_2 = cost of Plant 2

Q_1 = capacity or size of Plant 1

Q_2 = capacity or size of Plant 2

x = power-sizing exponent or the cost-capacity exponent

In the above expression C_1 and C_2 are the costs at the same point in time, at which the estimate is required. Both Q_1 and Q_2 are in same physical units. The power-sizing exponent 'x' represents the economies of scale. The economies of scale indicate that the cost per unit output decreases as the size of operation increases. The decrease in cost per unit output occurs as the fixed cost is distributed over more number of output units as the scale of output increases and in addition the efficiency grows in production with increasing size of operation. Taking a simple example, the cost of constructing a four-storey hostel building will normally be less than twice the cost of constructing a similar

two-storey hostel building. One of the factors that gives rise to cost advantage in case of four-storey building is the cost of land (fixed cost), which is same for both buildings.

If the value of power-sizing exponent 'x' is less than 1, it indicates economies of scale whereas greater than 1 indicates diseconomies of scale. When the value of power-sizing exponent is equal to 1, it indicates a linear relationship between cost and capacity or size.

While using this technique to predict the current cost of an industrial plant of a given capacity (Q_1) from known past cost of the same plant of a different capacity (Q_2), first it is required to update the known past cost of the plant with capacity (Q_2) to the present time by using the relationship for cost index (as stated in previous lecture) from the known values of cost indexes in past and in current time. After that, the cost of the plant with capacity Q_1 is estimated using the power-sizing model (as stated above) from the updated cost of plant with capacity Q_2 (i.e. cost at current time) and value of the power-sizing exponent for this particular plant.

Learning curve is also another cost estimating relationship and it is based on the principle that with increase in number of repetitions, the performance becomes faster with increase in efficiency. In other words the time or cost of producing a unit output of a product/system reduces, each time it is produced. Learning curve relates to improvement in performance with increase in repetitions. A learning curve model is based on the assumption that, the time to produce a unit output reduces by a constant percentage as the number output unit is doubled. The reduction in time depends on the learning curve rate. Taking a simple example, if the time required to complete the production of first output unit is 20 hours, then the time required to complete the production of the second output unit at 80% learning curve rate will be 16 hours $(20\,\text{hours} \times 0.80)$. This 80% learning curve rate indicates a 20% reduction in production duration, each time the output is doubled Similarly the time required to produce the fourth output unit will be 12.8 hours $(16\,\text{hours} \times 0.80)$. The reduction in production time for fourth unit as compared to the second unit is 3.2 hours (16 hours – 12.8 hours) which is 20% of production time for the second output unit.

Life cycle cost

Life cycle cost is equal to sum of all the estimated costs associated with a product, service or system over its life span starting from conceptual planning at the beginning to schematic design, detailed design, construction or production, operation and maintenance till its disposal at the end of the life span. The concept of life cycle costing lies in designing/producing the products, services or systems with systematic identification of both recurring and nonrecurring costs during various phases of their life cycle and estimating the cash flows during these phases over the life cycle. The economic evaluation of an alternative on the basis of life cycle cost results in a detailed analysis of the of both present and future costs and thus helps in taking the right decision regarding the selection of the most economical alternative. For construction projects, it is important to accurately estimate all the cost components during various stages of the project life cycle.

The life span of a product, service or system depends on the different phases starting from conceptual planning to its disposal. The end of life cycle may be governed by economic requirement or by functional requirement and depends on specific product, service or system. The economic life of a product/system is normally shorter than its physical life. The product/system may still be functional (over physical life) but may not be economical for the entire life period and may be replaced. The details about the replacement analysis of an asset are already stated in Module 4.

The life cycle cost of a product, service or system increases, if the design changes are made during later stages of the life cycle. The cost of design change increases with each stage of life cycle with lower cost during the early stages. Further the flexibility in design changes during the early stages is more as compared to that in the later stages of life cycle. Thus the potential for cost savings is more during the early stages of the life cycle and thus the selection of the most economical alternative and the effective design procedure during the design stage results in higher cost savings. Thus it is essential to have a detailed design of the product/system (as per the required technical specifications and operational requirements) during the design stage of life cycle and to avoid or minimize the design changes during production and operation stages of life cycle to have a minimum impact on the life cycle cost of the product/system.

The life cycle cost analysis is more useful for selecting the alternatives for products, services or systems having longer life periods namely, infrastructure projects and projects involving more research and development components. The economic evaluation of alternatives using life cycle cost analysis can be carried out by finding out the equivalent worth of the each alternative by including all the cash flows occurring over various stages of life cycle by present worth analysis and selecting the most economical alternative that results in minimum life cycle cost.

Construction Economics & Finance

Module 6

Lecture-1

Financial management:

Financial management involves planning, allocation and control of financial resources of a company. Financial management is essential as it controls the financial operations of a company. For a construction company, the decision to bid for a project will depend on its financial status which in turn will be governed by financial management principles. The decision to bid for a project will depend on various factors namely whether the company have enough funds or require outside financing, whether to acquire the equipment through purchase or acquisition through renting or leasing, whether to carry out the entire work or subcontract a portion of the work etc. If the company uses its own funds for the project, it may have an adverse effect on its financial status as it will reduce the liquid asset thus affecting company's working capital. The construction industry differs from other industries because of its unique characteristics and accordingly the financial management principles are applied for using the financial resources of the company. Generally the construction companies receive the payments from the owners at specified time intervals as the construction work progresses and owners often retain certain amount subject to the satisfactory completion of the project. Thus the terms and conditions for receipt of payments from owners affect the cash flow of the construction companies and need the changes in allocation of financial resources. Further construction companies often subcontract some portion of the work (as required) to the subcontractors, which in turn affect the cash flow. The financial management decisions include the decisions for investment, financing and distribution of earnings. For construction companies the investment decisions relate to investment in the business i.e. investment of funds in acquiring the assets (both current assts and long-term assets) to be utilized in the projects for the expected return along with the risk of cash flows associated with uncertain future conditions. The financing decisions depend on decision to investment the funds and the resources possessed by the construction companies. In addition the financing decisions

are also controlled by other factors namely source of financing (from banks or other financial institutions), cost of financing i.e. interest cost on the loan and the financing duration. The decision for distribution of earnings or profits of the company depends on the dividends to be paid to the stockholders and the retained earnings to be reinvested in the business to increase the return. Thus for any company or organization, financial management has to ensure the supply of funds in acquiring the assets and their effective utilization in business activities, to ensure the expected return on the investment considering the risk associated and optimal distribution of the earnings.

Construction accounting

Construction accounting involves collecting, organizing, recording and reporting the financial data of a construction firm. It helps in identifying the problems encountered in effective management of financial resources of the firm and taking the corrective measures in time. The accounting system allows for the systematic recording and reporting the financial information form one period to another in accordance with the accounting principles. Financial data relates to assets, liabilities, net worth, cost incurred, revenue earned etc. and is required for preparing the financial statements of the construction firm. The most important financial statements which are used to represent the financial status and evaluate the performance of the firm are balance sheet and income statement. The financial statements are also used to communicate the financial status of the firm to the shareholders and financial institutions. The shareholders and financial institutions can get the useful information about the utilization of financial resources in the business activities, costs incurred, revenues earned and other financial information of the firm from period to period. In addition the financial information is also required for calculating the income tax to be paid by the firm.

Construction firms use different types of accounting methods for recognition of the revenues, costs and profit while preparing the financial statements. These accounting methods are cash method, accrual method, percentage-of-completion method and completed contract method. For obtaining the most accurate picture of financial status, the construction firm has to use the best suitable accounting practice for recognition of the revenue and expenses.

In **cash method**, revenue is recognized when construction firm actually receives the payments from the owner of the project for the services provided. Similarly the cost (expenses) is recognized when the construction firm actually pays the bills (to subcontractors, material suppliers etc.). Normally the cash method is not used for preparing the financial statements of the construction firm as the revenues or costs are recognized when these are actually received or paid rather than the revenues billed to date or the costs incurred to date. Due to this delay in the recognition of revenues and costs, this method does not provide accurate financial status of the firm.

In **accrual method**, revenue is recognized when construction firm bills the owner of the project i.e. before the actual receipt of payments from the owner and expenses are recognized as the construction firm receives the bills from subcontractors, material suppliers etc. i.e. before the bills are actually paid. This method of accounting is used for preparing the financial statements of the firm as it provides more accurate information about the financial status of the firm.

In **percentage-of-completion method**, the revenue and expenses are recognized as percentage of work that is complete during the course of the project. This method of accounting normally used for long-term construction projects. The revenue is recognized as the construction firm bills the owner of the project and expenses are recognized as the construction firm receives the bills from subcontractors, material suppliers etc. The percentage complete till date can be calculated by dividing the cost incurred to date by the total estimated cost at completion. Corresponding to this percentage completion, the revenue and profit to date are calculated**.** The percentage-of-completion method is used for preparing the financial statements and it provides the most accurate picture of the firm's financial status.

In **completed contract method**, the revenue and expenses are recognized when the project is complete. This method of accounting does not provide accurate information about the construction firm's financial condition as the recognition of revenue and expenses are deferred till the completion project rather than in the periods when the revenues are billed to date or the costs are incurred to date. The revenue recognized to date for the project at any stage within the project duration is zero, as the project is not complete. This method of accounting can be used for construction contracts having

higher uncertainty about the construction cost and revenue until the completion of the project, involving higher financial risks in the business. Further completed contract method causes large fluctuations in the profit as the cost and revenue are not recognized till the completion of the project.

Example -1

The following table presents the information about the financial data of a construction project.

Description of financial data	Amount (Rs.)
Contract amount	3,85,00000
Original estimated cost of the contract	3,46,50,000
Billed to date	2,57,95,000
Payments received to date	2,34,85,000
Costs incurred to date	2,07,90,000
Costs paid to date	1,84,03,000
Estimated or forecasted cost to complete	1,31,28,500

Calculate the revenue to date for the project using different methods of revenue recognition.

Solution:

Cash method

By using the data from above table, revenue to date for the project using cash method is calculated as follows;

$$\text{Revenue to date} = \text{Payments received to date} - \text{Costs paid to date}$$

$$\text{Revenue to date} = \text{Rs.}2,34,85,000 - \text{Rs.}1,84,03,000 = \text{Rs.}50,82,000$$

Accrual method

The revenue to date for the project by using accrual method is calculated as follows;

$$\text{Revenue to date} = \text{Billed to date} - \text{Costs incurred to date}$$

$$\text{Revenue to date} = \text{Rs.}2,57,95,000 - \text{Rs.}2,07,90,000 = \text{Rs.}50,05,000$$

Percentage-of-completion method

For calculating the revenue to date by using percentage-of-completion method, first the percentage of work that is complete is calculated.

$$\text{Percentage of completion} = \frac{\text{Costs incurred to date}}{\text{Total estimated cost at completion}} \times 100$$

Total estimated cost at completion is equal to costs incurred to date plus estimated or forecasted cost to complete.

$$\text{Percentage of completion} = \frac{\text{Costs incurred date}}{\text{Costs incurred to date} + \text{Estimated cost to complete}} \times 100$$

$$\text{Percentage of completion} = \frac{\text{Rs.}2,07,90,000}{\text{Rs.}2,07,90,000 + Rs.1,31,28,500} \times 100 = 61.29\%$$

The revenue to date for the project is now calculated by multiplying the contract amount with the percentage of completion and is given by;

$$\text{Revenue to date} = \text{Rs.}3,85,00000 \times 0.6129 = \text{Rs.}2,35,96,650$$

Now gross profit to date is calculated by subtracting costs incurred to date from revenue to date and is given by;

$$\text{Gross profit to date} = \text{Revenue to date} - \text{Costs incurred to date}$$

$$\text{Gross profit to date} = \text{Rs.}2,35,96,650 - \text{Rs.}2,07,90,000 = \text{Rs.}28,06,650$$

Completed contract method

By using completed contract method, revenue to date for the project would be zero as the project has not yet been completed.

Lecture-2

Chart of accounts:

Chart of accounts lists all the accounts which are required to organize the financial information of a construction firm. The accounts mentioned in chart of accounts are used to prepare the financial statements namely income statement and balance sheet of the firm. The accounts listed in chart of accounts are assets, liabilities, net worth, revenue, expenses and tax. In chart of accounts the components of balance sheet i.e. assets, liabilities and net worth are listed first followed by the components of income statement i.e. revenue, expenses and tax. Each account in the chart is assigned by a reference number for easy identification.

Financial statements

As already stated in Lecture 1 of this module, financial statements are used to indicate the financial status and evaluate the performance of a firm. The most important financial statements are income statement and balance sheet (already mentioned).

Income statement (or Profit and Loss statement)

Income statement is a summary of revenues, expenses and the resulting profit or loss of a firm for a stated period of time, usually the fiscal year. The income statement is also referred as profit and loss statement. It records the revenues and expenses in the interval between two balance sheets. It provides information about net profit or loss of the firm, thus indicating whether the firm is running the business with profit or loss. Profit is the excess of revenue over the expense. Through income statement, it is possible to track the performance of the firm during the stated period of time by analyzing the revenues and the expenses. Revenues represent the cash inflows earned in the exchange of services provided whereas the expenses represent the cash outflows, the firm spent for providing the services. In income statement the operating revenues and expenses represent the income and the expenditures those are associated with the core operations of the firm whereas the nonoperating revenue and expenses represent the income and expenditures from sources other than the core operations. For construction firms the operating revenues represent the earnings against the services provided i.e. earnings against the completion of portion of work or entire work of the projects. The operating expenses

represent the construction costs (both direct and indirect costs) i.e. material cost, equipment cost, labor cost, cost of subcontracted work and the overhead cost. As already mentioned in Lecture 1 of Module 5, the direct cost includes cost of materials, equipment and labor associated with each item of work, along with subcontracted costs whereas the indirect cost (overhead costs) is calculated for the entire construction project rather than allocating for each item of work in the project. In income statement, the nonoperating revenues include the interest earned and the revenue earned from sources other than the core operations (i.e. construction operations) of the firm. The nonoperating expenses include interest payments on loans and other expenditures which are not associated with the construction operations of the firm.

In the income statement, the operating revenues and expenses are shown first. From these records, the gross profit from operations is calculated. After that the net profit from operations is calculated by the subtracting the overhead cost from the gross profit (from operations). The nonoperating revenue and expenses are then recorded. The profit before tax is then equal to net profit from operations less the nonoperating revenue and expenses. It may be noted here that the nonoperating revenue is added whereas nonoperating expense is subtracted from net profit from operations. After that the net profit after tax is calculated by subtracting the income tax (at an appropriate income tax rate) from profit before tax.

A typical income statement of a construction firm is shown as follows.

XY Construction Firm Income Statement Year Ended March 31, 2012	
Operating Revenues	Rs.45260700
Operating Expenses	
Equipment	6338400
Materials	10304150
Labor	8067200
Subcontract	9185700
Total operating expenses	33895450
Gross profit from operations	**11365250**
General and administrative overhead cost	4661800
Net profit from operations	**6703450**
Nonoperating Revenues	
Interest receipts	769500
Nonoperating Expenses	
Interest payments	407850
Profit before tax	**7065100**
Income tax	2119530
Net profit after tax	**Rs.4945570**

For the above income statement, the calculations of gross profit from operations, net profit from operations, profit before tax and net profit after tax are presented below.

$$Gross\,profit\,from\,operations = operating\,revenues - operating\,expenses$$
$$= Rs.45260700 - Rs.33895450 = Rs.11365250$$

$$Net\,profit\,from\,operations = gorss\,profit\,from\,operations - general\,and\,adminstrative\,overhead\,cost$$
$$= Rs.11365250 - Rs.4661800 = Rs.6703450$$

$$Profit\,before\,tax = net\,profit\,from\,operations + nonoperating\,revenues - nonoperating\,expenses$$
$$= Rs.6703450 + Rs.769500 - Rs.407850 = Rs.7065100$$

$$Net\,profit\,after\,tax = profit\,before\,tax - income\,tax$$
$$= Rs.7065100 - Rs.2119530 = Rs.4945570$$

Lecture-3

Balance sheet

A balance sheet shows details about the financial status of a firm at a particular point in time. A balance sheet is a summary of the firm's assets, liabilities and owners' equity or net worth (i.e. worth that the firm owes to its owners i.e. stockholders or shareholders) at a given point in time. In other words for a firm, a balance sheet provides details about the assets it owns, liabilities it owes and the equity it owes to its owners. Balance sheet represents a snapshot of firm's financial position. It is commonly prepared at the end of financial year (i.e. fiscal year). But it may also be prepared at other time intervals i.e. quarterly, semiannually within a financial year or calendar year as required by the firm. The fundamental relationship between assets, liabilities and owners' equity on the balance sheet is as follows;

$$Assets = Liabilities + Owners'\,equity$$

As observed from above relationship, on the balance sheet the assets of a firm must be equal to the sum of its liabilities and owners' equity. The categories of assets, liabilities and owners' equity are presented as follows.

Assets are the resources of monetary value those are owned by the firm. The assets of a typical construction firm are cash, plant and equipment, land, buildings, construction materials, receivables etc. Assets are classified into three categories namely current assets, long-term assets and other assets.

Currents assets are defined as the assets which are likely to be converted into cash within one year. The categories of current assets are cash, accounts receivable, notes receivable, costs and profits in excess of billings, inventory, prepaid expenditures etc. **Cash** is the primary current asset. It includes cash in bank i.e. bank deposits (savings, term deposits with maturity less than one year) and petty cash. **Accounts receivable** include the amount that the owner of the project (i.e. client) owes to the firm for providing service or for the job done. For example the monthly bill to be paid by the client of a project to the construction firm is an account receivable till the payment against the bill is received. It also includes the retentions receivable. The retentions receivable represent the amount that is held up by the client of the project and is released

to the construction firm subject to fulfillment of requirements of the project. **Notes receivable** are the formal written notes that represent the amount that is expected to be received by the firm within a year. The formal note serves as written promise to receive the amount of money from the party obliged to make the payment. It includes bill payments to be received through a formal written promise within a year. **Costs and profits in excess of billings** are also known as underbillings and are recorded as current assets on the balance sheet. Costs and profits in excess of billings occur when the firm bills the clients (project owners) less than the sum of costs incurred and estimated profits for the portion of the work that is complete. The firms using percentage-of-completion method of accounting are required to recognize the revenues, expenses and estimated profits on the basis of percentage of project work that is complete and costs and profits in excess of billings are accordingly recorded on the balance sheet. Percentage-of-completion method is one of accounting methods that the construction firms use for preparing the financial statements. Underbillings may occur due to more cost incurred than the estimated on the completed work or due to non-proportionate distribution of profit. **Inventory** represents resources (mostly the materials) that are with the firm and are likely to be used in the project within a short period of time and its monetary value is entered in the balance sheet as current asset. For a construction firm, inventory includes the construction materials to be used in a project. The material suppliers (subcontractors) generally carry inventory of the construction materials which they supply to the construction firms. **Prepaid expenditures** represent the amount of money paid for future services and include prepaid insurance premium, prepaid taxes, advance rental payments etc.

Long-term assets represent the resources which can not readily be converted into money (cash) and these assets provide future benefits. **Fixed or tangible assets** (a category of long-term assets) include land, building, plant and equipment and other tangible assets with longer service life. Fixed assets are shown on the balance sheet at their original cost (i.e. purchase price) less the accumulated depreciation (except for land). The recorded accumulated depreciation on the balance sheet represents the loss in value of the fixed assets till date.

Other assets include other investments and equity in other enterprises.

Liabilities represent the obligations (i.e. debts) that the firm owes to other parties. Liabilities include bank loans, debts to subcontractors, advances from project owners (i.e. clients), taxes payable both current and deferred etc. Liabilities are classified into two categories namely current liabilities and long-term liabilities and are according shown on the balance sheet.

Current liabilities represent the obligations which are likely to be paid within one year. The categories of current liabilities are accounts payable, billings in excess of costs and profits, accrued payables, lease payments etc. Current liabilities are usually paid by using current assets. **Accounts payable** include the debts that the firm owes to other parties for services provided. For example the monthly bills received by the construction firm from subcontractors and suppliers represent the accounts payable till the payment of the bills. It also includes the retentions payable which represents the amount of money that is held up by the construction firm and is released to subcontractors or suppliers subject to the fulfillment of the requirements. **Billings in excess of costs and profits** also known as overbillings are recorded as current liabilities on the balance sheet. Billings in excess of costs and profits occur when the firm bills the clients (project owners) more than the sum of costs incurred and estimated profits. Overbillings may occur due to less cost incurred than the estimated on the completed project work or due to non-proportionate distribution of profit. **Accrued payables** are the expenses which the firm owe and include accrued wages, accrued taxes, accrued interest and accrued rents which are not paid. **Lease payments** are recorded as a current liability on balance sheet and for construction firms it represents rental payments for acquiring the construction equipment. In addition to these current liabilities, **notes payable** (include the debts which are likely to be paid through a formal written promise within a year) and **dividends payable** (portion of the profits or earnings that a firm pays to its stockholders) are also recorded as current liabilities on the balance sheet.

Long-term liabilities also known as non-current liabilities are the debts which are not expected to be paid within a year and normally refer to loans from banks or other financial institutions. Long-term liabilities include loans taken to purchase construction plant and equipment and other assets, payment of mortgage for buildings etc.

Owners' equity or *net worth* of the firm represents the excess of assets of a firm over its liabilities and is recorded on the balance sheet. For the companies, the owners' equity consists of three components namely common stock, preferred stock and retained earnings. Common stock represents the major source of investment by the stockholders in the company and provides the stockholders with voting rights to exercise in company decisions. Preferred stock that represents the ownership in a company usually do not provide the stockholders the voting rights to exercise in company affairs, however it ensures a dividend to be received by stockholders (holders of preferred stock) before any dividend is paid to holders of common stock. Retained earnings represent the profits or earnings which are reinvested in the company rather than being distributed as dividends to the stockholders. These earnings are retained by the company to reinvest in the business to increase the income and thus to increase the value of stock.

A typical balance sheet of a construction firm is shown below. In the balance sheet the total assets of the construction firm is equal to the sum of total liabilities and owners' equity.

AB Construction Firm Balance Sheet As of 31 March 2012			
Assets		**Liabilities**	
Current assets		**Current liabilities**	
Cash	Rs.1050000	Accounts payable including retentions	Rs.5981400
Accounts receivable including retentions	6860000	Billings in excess of costs and profits	2736500
Costs and profits in excess of billings	4263000	Accrued payables	1371600
Inventory	3925000	Notes payable	1264900
Total current assets	Rs.16098000	Dividends payable	981200
Fixed assets		Total Current liabilities	Rs.12335600
Land	Rs.2362000	Long-term debts	Rs.4610700
Building	10152000	Total liabilities	Rs.16946300
Construction and other equipment	10572000	**Owners' equity**	
Total fixed assets	Rs.23086000	Common stock	Rs.3150600
Less accumulated depreciation	7046160	Preferred stock	2569040
Net fixed asset	Rs.16039840	Retained earnings	9471900
Total assets	**Rs.32137840**	Total owners' equity	Rs.15191540
		Total liabilities and owners' equity	**Rs.32137840**

Lecture-4

Financial ratios

Financial ratios are used to evaluate the financial status of a company. These ratios provide information about the company's ability to meet the financial obligations (i.e. ability to pay the bills, debt payment ability), profitability, liquidity and effective use of its assets. Financial ratios represent the relationship between the financial data shown on the financial statements (i.e. balance sheet and income statement). These ratios are calculated by dividing a category of financial data by another category shown on financial statements and are then compared with other companies within the industry. Through financial ratios, it is easy to identify the changes in the financial status of the company which are not evident, by only referring to the values of different categories of assets, liabilities net worth, revenue and expenses shown on the financial statements. The different financial ratios which are used to assess the financial status of a company are presented below.

Current ratio

Current ratio indicates the ability of a company to meet the short-term financial obligations. In other words it is a measurement company's ability to pay the current liabilities by using the current assets and is given by;

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

Current assets and current liabilities shown on balance sheet are used to calculate this ratio and the details about the current assets and current liabilities are already stated in Lecture 3 (Balance sheet) of this module. This ratio gives information about working capital status of a company. A current ratio of less than 1.0 to 1 (i.e. 1.0:1) indicates that the current assets of the company is not sufficient to pay its current liabilities whereas a greater value of current ratio is a strong indicator of the ability of the company to pay its current liabilities. However on the other hand a considerably higher value of current ratio indicates that much of the assets of the company might have tied up in the current assets and can be invested in other business to increase the earnings of the firm.

Quick ratio (or acid-test ratio)

Quick ratio measures the ability of a firm to pay current liabilities using cash or other assets that can be readily converted into cash. Quick ratio is also referred as acid-test ratio. It is a measure of short-term liquidity of a firm and provides information about its immediate financial status. This ratio is calculated by dividing quick assets (categories of current asset) of a firm by its current liabilities. The quick assets include cash and accounts receivable. Quick ratio is given by the following relationship.

$$Quick\ ratio = \frac{Cash + accounts\ receivable}{Current\ liabilities}$$

Inventories and retentions receivable (of accounts receivable) are excluded from quick assets because these assets can not be converted into cash quickly. A quick ratio equal to or greater than 1.0:1 indicates the ability of a firm to pay the current liabilities with the quick assets. A quick ratio of less than 1.0:1 requires that the firm needs to convert inventories and other assets into cash for payment of current liabilities.

Debt to worth ratio

This ratio is calculated by dividing total liabilities of a company by its net worth or owners' equity. It is also referred as debt to equity ratio and is given by;

$$Debt\ to\ worth\ ratio = \frac{Total\ liabilities}{Net\ worth}$$

This ratio is a measure of a company's leverage and the desired value of this ratio should be less than or equal to 2.0:1. A debt to worth ratio less than 1.0:1 indicates that the firm is not utilizing the debt in the business to increase the return on the investment. Further a lower value of this ratio indicates that there is more protection to the creditors as the owners of the firm are contributing majority of the funds to the business of the firm. A debt to worth ratio of greater than 2.0:1 may result in a greater risk for the repayment of debt along with interest during recession in the construction industry.

Return on assets ratio

The return on assets ratio is calculated by dividing the net profit after taxes by the total assets of the company. This ratio is expressed as a percentage and is given by:

$$\text{Return on assets ratio} = \frac{\text{Net profit (after taxes)}}{\text{Total assets}}$$

The net profit after tax from income statement and total assets from balance sheet of a company are used to calculate this ratio. Return on assets ratio measures how efficiently a company uses its assets to earn the return and indicates profitability of a company. For a company, the efficient use of assets earns a higher return (higher value of this ratio) whereas there is low return on assets (lower value of this ratio) for a poorly run company.

Return on sales ratio

This ratio is calculated by dividing the net profit after taxes by the net sales revenue. Return on sales ratio is expressed as a percentage. It is given by:

$$\text{Return on sales ratio} = \frac{\text{Net profit (after taxes)}}{\text{Net sales revenue}}$$

The operating revenues (net sales revenue) and net profit after tax from income statement are used to calculate this ratio. This ratio indicates the profit margin for a company and measures how efficiently a company can meet the adverse market conditions such as decline in demand, falling prices and increased costs. An increasing value of this ratio (%) indicates higher profit margin due to increased revenue and lowered company expenses. Similarly a lower value of this ratio (%) indicates lower profit margin due to decline in revenue and increase in the costs.

There are also other financial ratios such as current liabilities to net worth ratio, fixed assets to net worth ratio, assets to revenues ratio etc. (calculated from data of financial statements) can be used to assess the financial status of a company. The calculation of some of the financial ratios is presented in the following example.

Example -2

Calculate the current ratio and debt to worth ratio for 'AB Construction Firm' (Balance Sheet, Lecture 3 of Module 6) and return on sales ratio for 'XY Construction Firm' (Income Statement, Lecture 2 of Module 6).

Solution:

From the balance sheet of 'AB Construction Firm', the values of current assets and current liabilities are Rs.1,60,98,000 and Rs.1,23,35,600 respectively. Further the values

of total liabilities and net worth (or owners' equity) are Rs.1,69,46,300 and Rs.1,51,91,540 respectively.

The current ratio for 'AB Construction Firm' is given by;

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\text{Current ratio} = \frac{\text{Rs.1,60,98,000}}{\text{Rs.1,23,35,600}} = 1.30$$

Thus the current ratio for the construction firm is 1.30.

The debt to worth ratio for 'AB Construction Firm' is given by;

$$\text{Debt to worth ratio} = \frac{\text{Total liabilities}}{\text{Net worth}}$$

$$\text{Debt to worth ratio} = \frac{\text{Rs.1,69,46,300}}{\text{Rs.1,51,91,540}} = 1.11$$

Thus the debt to worth ratio for the construction firm is 1.11.

From the income statement of 'XY Construction Firm', the values of operating revenues (net sales revenue) and net profit (after taxes) are Rs.4,52,60,700 and Rs.49,45,570 respectively.

The return on sales ratio for 'XY Construction Firm' is given by;

$$\text{Return on sales ratio} = \frac{\text{Net profit (after taxes)}}{\text{Net sales revenue}}$$

$$\text{Return on sales ratio} = \frac{\text{Rs.49,45,570}}{\text{Rs.4,52,60,700}} = 0.109 = 10.9\%$$

Thus the return on sales ratio for the construction firm is 10.9%.

<u>Lecture-5</u>

Working capital management

Working capital represents the excess of current assets over the current liabilities. The details about current assets and current liabilities are already presented in Lecture 3 of this module. Working capital includes the cash or near-cash assets which are required for the daily operations in the business. Adequate working capital is required for ensuring regular supply of raw materials and other consumables, timely payment of wages and for meeting other short-term expenses so as to have smooth running of the operations in the business. Working capital measures the sort-term financial strength and a company must have sufficient working capital to meet the short-term financial obligations. Insufficient working capital may result in difficulty in meeting the cash requirement for the operations in the business. On the other hand, much of assets tied up in working capital reduces the profit level of the company as excessive working capital makes the funds idle. Thus working capital management aims at establishing the optimum level of working capital for smooth running of the operations in the business by maintaining the desired liquidity and profitability of the company. For a company, the main objective lies in increasing the profit level so as to maximize the wealth of stockholders and increase the market value of stock and at the same time maintaining the liquidity for meeting the short-term financial obligations. Liquidity and profitability are inversely related as higher investment in current assets with lesser short-term borrowings increases the liquidity and reduces the profitability of the company whereas, for a company lesser investment in current assets with higher short-term borrowings decreases the liquidity and increases the profitability. Thus it is essential to have a balance between liquidity and profitability by maintaining the satisfactory level of working capital with the help of working capital management which assists in managing the desired level of current assets and current liabilities of the company.

Working capital is represented in a continuous cycle known as operating cycle and a typical operating cycle is shown in Fig. 6.1. The operating cycle is also referred to as working capital cycle or current assets and current liabilities cycle. The operating cycle starts with acquiring raw materials and consumables using cash (or on credit), maintaining their inventory, use in production, obtaining finished goods (or service)

followed by sales, receipt of payment against the bill and finally realizing the cash, which completes the operating cycle and the same process is again continued. In other words in the operating cycle, there is continuous flow from cash to suppliers, to inventory, to finished goods (or service), to accounts receivable and then to cash again. The length of an operating cycle (i.e. duration) depends on the number of stages involved (as mentioned above) and the average length (i.e. duration) of each stage. For a construction company undertaking a project, the operating cycle begins with procurement of construction materials and other consumables with cash (either available or from advance if any received from the owner of the project) or on credit from the suppliers. The quantities of these items to be purchased depend on how much to be used during a given period of time and how much to be kept in stock. Further cash is used for payment of wages to labourers employed in the project and payment of equipment rental charges to the equipment renting firm along with the payment of overhead expenses. If the equipment is owned by the construction company, then cash is utilized for meting the operating cost of the equipment employed in the project. The construction material suppliers and equipment renting firm are regarded as creditors if the company acquires these items on credit. After completion of a portion of work, the construction company raises the bill against the activities carried out during that period of time i.e. billing period which may be one month or two months (i.e. as per conditions of contract between the owner and the construction company). The billed amount becomes accounts receivable and owner of the project is regarded as debtor to the construction company till the receipt of payment against the bill. The receipt of payment against the bill and realization of cash completes the operating cycle and the process continues. The construction company must have sufficient working capital to meet expenses of materials, equipment, labour and overhead till the receipt of payment from the client (i.e. owner of the project). The estimation of working capital requirement is an important aspect of working capital management. The factors those affect the working capital requirement are nature and size of the business, type of raw materials, production process, credit policy, duration of operating cycle, volume of production and sales etc.
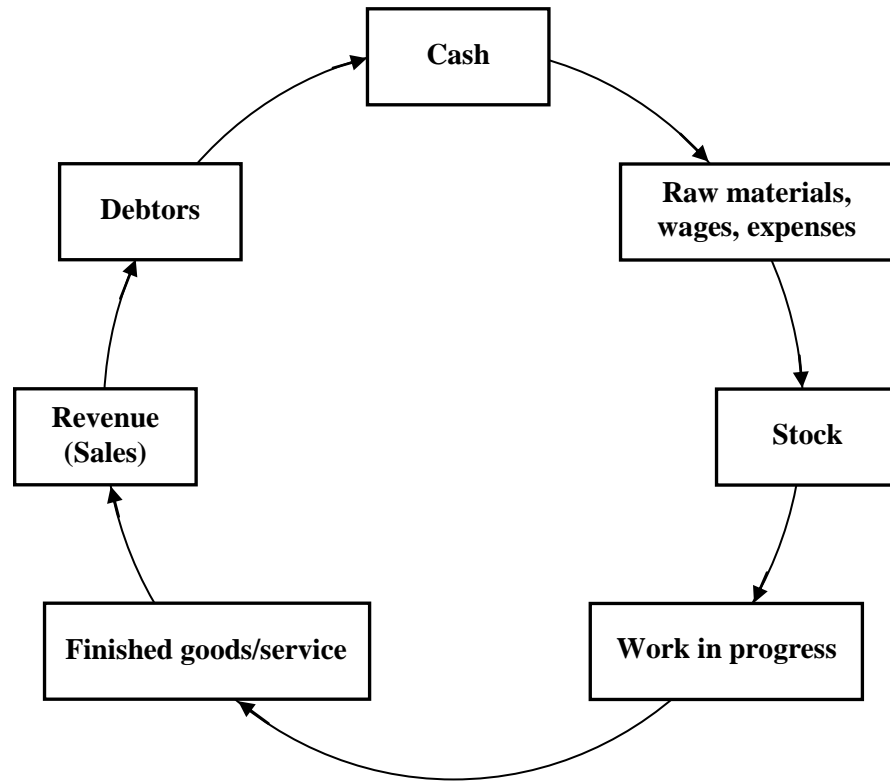
Fig. 6.1 Operating cycle

In the construction projects, there is usually a time lag between completion of a portion of work and the receipt of payment from owner of the project and thus production (sales) is not converted into cash instantaneously. Therefore working capital is required to meet the financial requirement in daily operations in the project. Often material suppliers and the equipment renting firm also allow a credit period to the construction company. Thus the credit period allowed by material suppliers and equipment renting firm to the construction company and that allowed by the construction company to the owner of the project are to be considered while calculating working capital requirement of the construction company.

The calculation of working capital requirement of a construction company is presented in the following example.

Example -3

The estimated annual revenue of a construction company from a project is Rs.51000000. In the project, the proportions of material cost, labour cost, equipment cost, overhead cost and profit are 37%, 23%, 16%, 14% and 10% of the estimated annual revenue

respectively. On an average two months' credit (i.e. average time period between commencement of a portion of work and receipt of payment from owner of the project) is allowed to the owner of the project. The materials are purchased from material suppliers and equipment is hired on rental charges from the equipment renting company. The credit period allowed by material suppliers and equipment renting firm to the construction company is one month. On average, two and half months' supply of materials is kept in stock at all the times. Calculate the working capital requirement of the construction company for the operations in the project.

Solution:

The working capital requirement will be calculated for meting the expenses of material, labour, equipment and overhead.

The annual material cost, labour cost, equipment cost and overhead with the corresponding percentages of annual revenue are Rs.18870000, Rs.11730000, Rs.8160000 and Rs.7140000 respectively.

Materials

Materials held in construction work (i.e. in work in progress) = 2 months

(i.e. average time period between the use of materials in the work and receipt of payment from owner of the project)

Materials in stock = 2.5 months

Credit from suppliers = 1 month

The construction company must have the working capital to meet the expense of 4.5 (i.e. 2 + 2.5) months' materials requirement. However the material suppliers allow 1 month credit to the company. Thus the construction company requires the working capital to meet the expense of 3.5 (i.e. 4.5 - 1) months' materials requirement. The working capital requirement for the materials is calculated as follows;

$$\text{Working capital requirement for materials} = \frac{(2 + 2.5 - 1)\,\text{months} \times \text{Rs.18870000}}{12\,\text{months}} = \text{Rs.5503750}$$

Similarly the working capital requirement for labour, equipment and overhead are calculated and are shown below.

Labour

In work in progress = 2 months

(i.e. average time period between employment of labourers in the work and receipt of payment from owner of the project)

$$\text{Working capital requirement for labour} = \frac{2\,\text{months} \times \text{Rs.11730000}}{12\,\text{months}} = \text{Rs.1955000}$$

Equipment

In work in progress = 2 months

(i.e. average time period between use of equipment in the work and receipt of payment from owner of the project)

Credit from renting company = 1 month

$$\text{Working capital requirement for equipment} = \frac{(2-1)\,\text{months} \times \text{Rs.8160000}}{12\,\text{months}} = \text{Rs.680000}$$

Overhead

In work in progress = 2 months

(i.e. average time period between commencement of work and receipt of payment from owner of the project)

$$\text{Working capital requirement for overhead} = \frac{2\,\text{months} \times \text{Rs.7140000}}{12\,\text{months}} = \text{Rs.1190000}$$

The working capital requirement of the construction company in the project till the receipt of payment from the owner of the project is given by;

$$\text{Working capital} = \text{Rs.5503750} + \text{Rs.1955000} + \text{Rs.680000} + \text{Rs.1190000} = \text{Rs.9328750}$$

Working capital financing

Financing of working capital is an important aspect of working capital management. The different sources of financing of working capital are short-term sources (bank financing such as loan facility, overdraft facility etc., trade credit, short-term sources other than banks etc.) and long-term sources (retained earnings, shares, debentures, long-term loans from banks and other sources, government development grants etc.). One of the important aspects in working capital financing lies in determining the suitable mix of

long-term and short-term finances as this is likely to affect the balance between liquidity and profitability of the company and the associated cost and risk of financing. The long-term finances are more costly as compared to short-term finances.

There are three basic approaches of financing working capital such as matching or hedging approach, conservative approach and aggressive approach. In matching or hedging approach, the permanent part of working capital requirement is financed by funds from long-term sources whereas the variable component is financed through short-term sources. Thus in this approach, the source of finance matches with the nature of the current assets. In conservative approach, the permanent part of working capital requirement and part of variable working capital are financed from long-term sources and short-term sources are used only to meet the emergency requirements. As majority of working capital financing is from long-term sources, this approach makes the financing more costly, less risky and leading to more liquidity and less profitability. In aggressive approach, the variable part of working capital requirement and even part of permanent working capital requirement are financed from short-term sources. As major sources of finances are short-term, this approach makes the financing less costly, more risky and results in low liquidity and high profitability.

It may be noted that, for further reading of various topics presented in lectures of different modules, the texts mentioned in the list of references for this course can be referred.