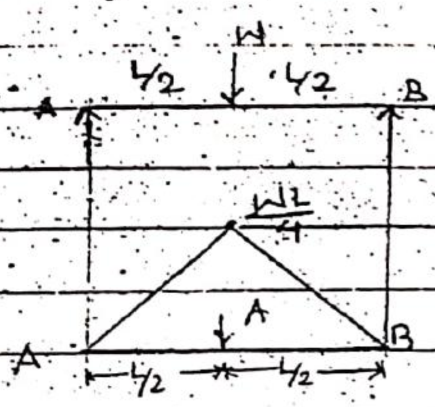
Analysis of a Continuous Beam

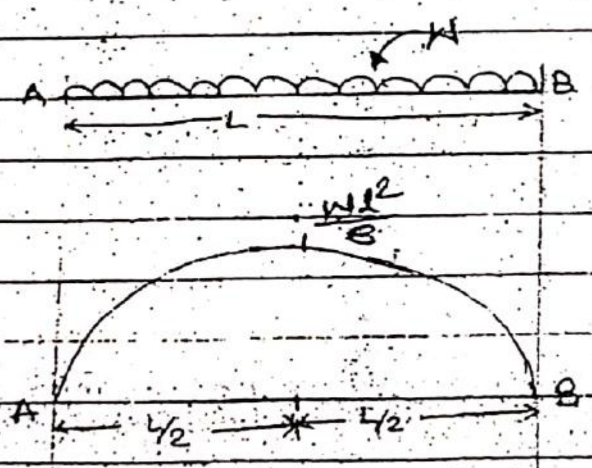
Three Moment Theorem (T.M.T)

⊙ Standard cases for BMD.

1) A beam subjected to a point load at centre.



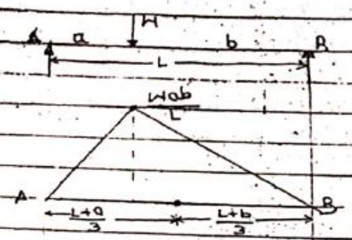
2) Beam of span 'l' subjected to an UDL.



area of parabola

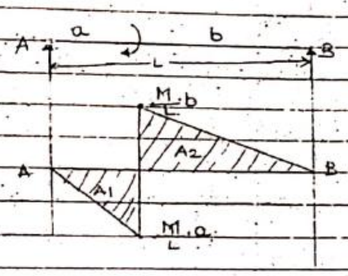
$= \frac{2}{3} BH$

3) A beam subjected to point load 'W'



$A = \frac{1}{2} BH$

4) A beam is subjected to clockwise moment 'M' acting at 'a' & 'b'



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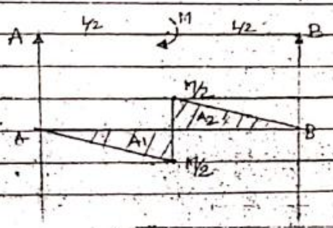
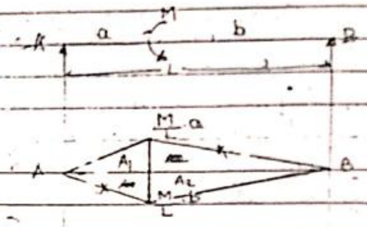
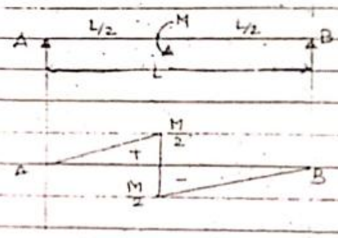


clockwise moment at centre of a beam

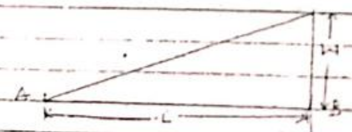
6) A beam is subjected to an anticlockwise moment 'M' acting at 'a' & 'b'

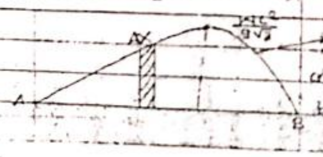




anticlockwise moment at centre of a beam

6) A beam subjected to gradually varying loads.

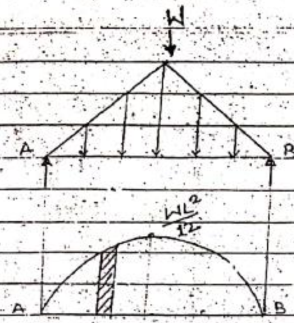




2nd deg. parabola

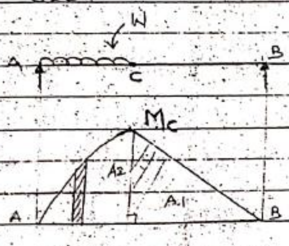
area & C.G. calculated by integration

7)

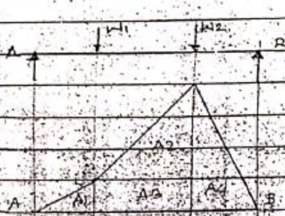


uniform triangular loading

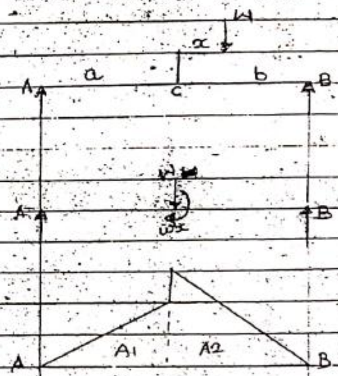
8) Partial UDL



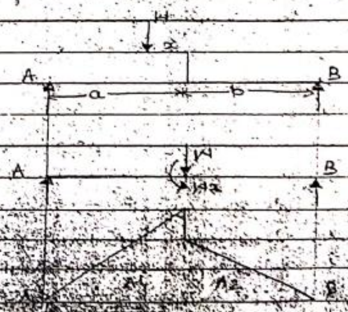
9) Beam subjected to multiple loading



10) Beam subjected to clockwise couple moment



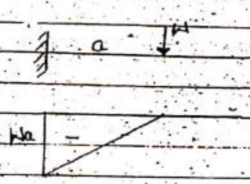
11) Beam subjected to anticlockwise couple moment



Standard BMD for cantilevers

1)

2)



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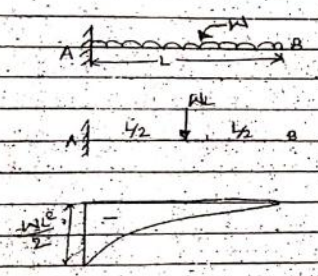

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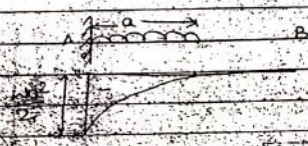

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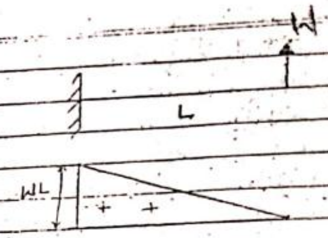
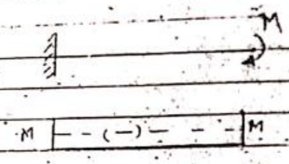
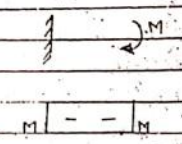


6) Imp



7)



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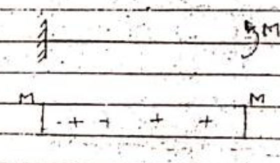

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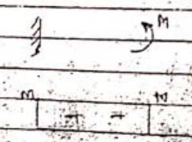
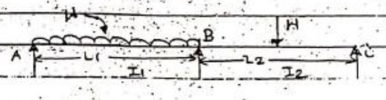
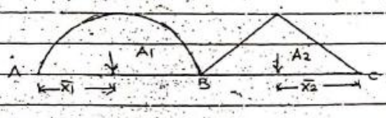


Clapeyron's TMT.





As per Clapeyrons theorem of three moment for a continuous beam ABC supported ABC subjected to any type of loading, the moments developed at supports A, B, C are calculated by equation:-

$$\boxed{M_A\left(\frac{L_1}{I_1}\right)+2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_C\left(\frac{L_2}{I_2}\right)=-\frac{6A_1\bar{x}_1}{L_1I_1}-\frac{6A_2\bar{x}_2}{L_2I_2}}$$

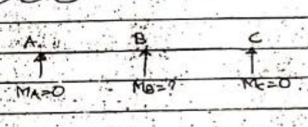
where M_A, M_B & M_C are moments developed at supports A, B, C.

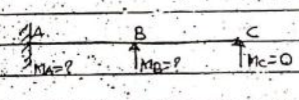
L_1 & I_1 corresponds to length & moment of inertia of span AB.

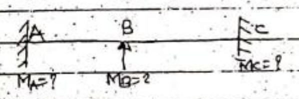
L_2 & I_2 corresponds to length & moment of inertia of span BC.

A_1 is the area of simply supported BMD over span AB with $\bar{x}_1$ as a C.G measured from 'A'

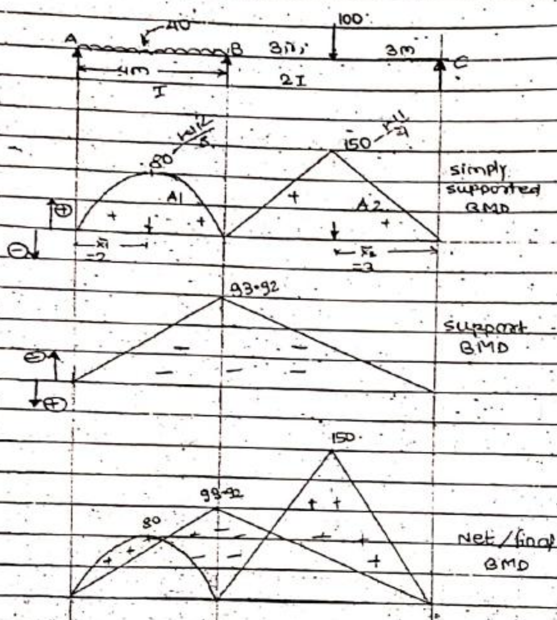
A_2 is the area of simply supported BMD over span BC with $\bar{x}_2$ as a C.G measured from 'C'







1) For a continuous beam ABC as shown in figure. Determine the support moments & plot BMD



applying TMT betn ABC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$

$A_1 = \frac{2}{3}BH \Rightarrow \frac{2}{3} \times 4 \times 80$ $\qquad \bar{x}_1 = 2m$

$A_1 = 213.33\ kN\,m^2$.

$A_2 = \frac{1}{2}BH \Rightarrow \frac{1}{2} \times 6 \times 150$ $\qquad \bar{x}_2 = 3m$

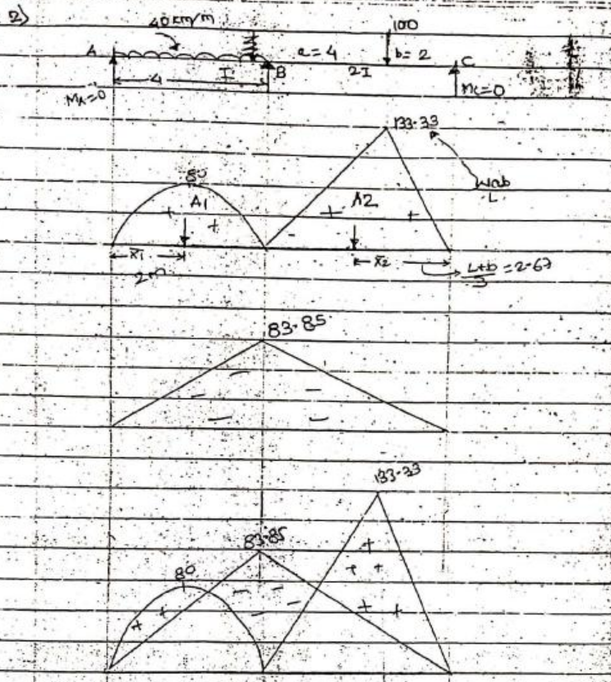
$A_2 = 450$.

$$0 + 2M_B\left(\frac{4}{I}+\frac{6}{2I}\right) + 0 = -\frac{6 \times 213.33 \times 2}{4 \times I} - \frac{6 \times 450 \times 3}{6 \times 2I}$$

$2M_B \times 7 = -639.99 - 270$

$\boxed{M_B = -93.92\ kNm}$

2)



by TMT at ABC

$$M_A\left(\frac{L_1}{I_1}\right)+2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_C\left(\frac{L_2}{I_2}\right) = \frac{-6A_1\bar{x}_1}{L_1I_1} - \frac{6A_2\bar{x}_2}{L_2I_2}$$

$A_1 = \frac{2}{3}BH \Rightarrow \frac{2}{3}\times 4\times 80$

$A_1 = 213.33$ $\qquad \bar{x}_1 = 2m$

$A_2 = \frac{1}{2}BH$

$= \frac{1}{2}\times 6\times 133.33$

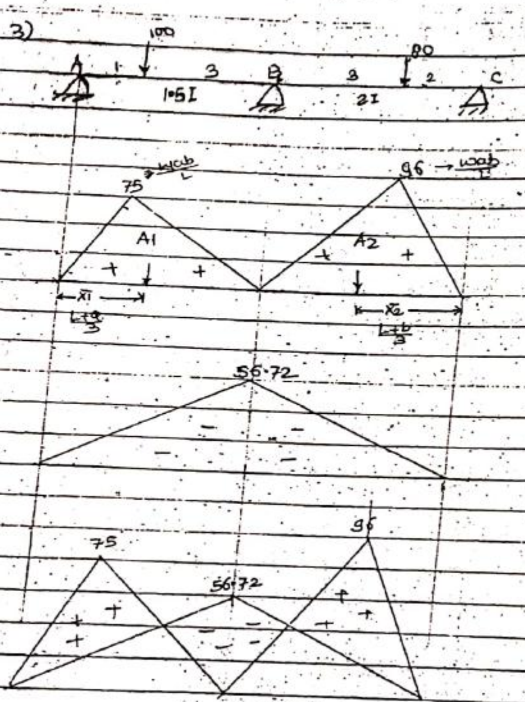
$A_2 = 400$ $\qquad \bar{x}_2 = \frac{L+b}{3} = \frac{6+2}{3} = 2.67$

$$0+2M_B\left(\frac{4}{I}+\frac{6}{2I}\right)+0 = \frac{-6\times 213.33\times 2}{4\times I} - \frac{6\times 400\times 2.67}{6\times 2I}$$

$14M_B = -639.99-534$

$\boxed{M_B = -83.85\ KN.m}$

3)



by TMT of ABC.

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$

$A_1 = \frac{1}{2} \times B \cdot H$ $\qquad \bar{x}_1 = \frac{L+a}{3} = \frac{4+1}{3}$

$= \frac{1}{2} \times 4 \times 75$

$A_1 = 150$ $\qquad \bar{x}_1 = 1.67\,m$

$A_2 = \frac{1}{2} \times B \cdot H$ $\qquad \bar{x}_2 = \frac{L+b}{3} = \frac{5+2}{3}$

$= \frac{1}{2} \times 5 \times 96$

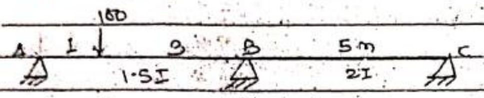
$A_2 = 240$ $\qquad \bar{x}_2 = 2.33\,m$

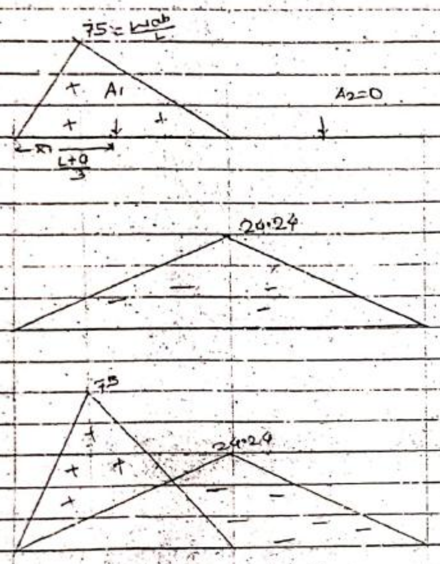
$$0 + 2M_B\left(\frac{4}{1.5I} + \frac{5}{2I}\right) + 0 = -\frac{6 \times 150 \times 1.67}{4 \times 1.5I} - \frac{6 \times 240 \times 2.33}{5 \times 2I}$$

$$10.33 M_B = -250.5 - 335.52$$

$$\boxed{M_B = -56.72}$$

4)





by TMT at ABC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) =$$

$$\frac{-6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$

$A_1 = \frac{1}{2} BH$ $\qquad \bar{x}_1 = \frac{L+a}{3} = \frac{4+1}{3}$

$= \frac{1}{2} \times 4 \times 75$

$A_1 = 150$ $\qquad \bar{x}_1 = 1.67\,m$

$A_2 = 0$.

$$0 + 2M_B\left(\frac{4}{1.5I} + \frac{5}{2I}\right) + 0 =$$

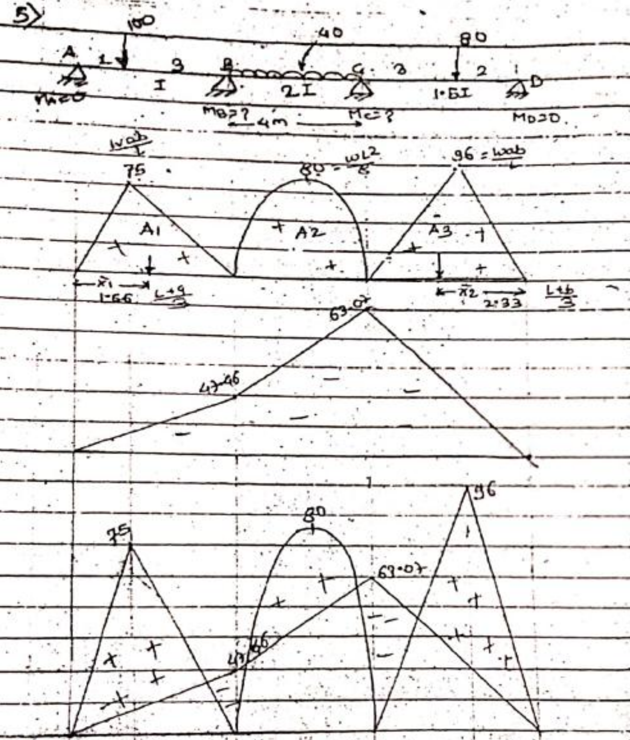
$$\frac{-6 \times 150 \times 1.67}{4 \times 1.5I} - 0$$

$$2M_B\left(\frac{8I + 7.5I}{3I}\right) = -250.5I$$

$$10.33I\,M_B = -250.5I$$

$$\boxed{M_B = -24.24\ KNm}$$

5)



$A_1 = 150$ $\qquad \bar{x}_1 = 1.67$

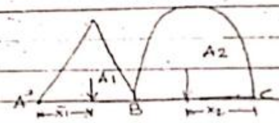
$A_2 = \frac{2}{3} BH = \frac{2}{3} \times 4 \times 80$ $\qquad \bar{x}_2 = 2$

$A_2 = 213.33$ $\qquad \bar{x}_3 = 2.33$

$A_3 = 240$

Applying TMT between ABC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = \frac{-6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$

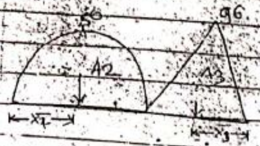


$$0 + 2M_B\left(\frac{4}{I} + \frac{4}{2I}\right) + M_C\left(\frac{4}{2I}\right) = \frac{-6 \times 150 \times 1.67}{4 \times I} - \frac{6 \times 213.33 \times 2}{4 \times 2I}$$

$$12 M_B + 2 M_C = -695.75 \quad \text{---(1)}$$

Applying TMT betn BCD

$$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + M_D\left(\frac{L_3}{I_3}\right) = \frac{-6A_2\bar{x}_2}{L_2 I_2} - \frac{6A_3\bar{x}_3}{L_3 I_3}$$



$A_2 = 213.33$, $\bar{x}_2 = 2$

$A_3 = 240$, $\bar{x}_3 = 2.33 = \frac{L+b}{3} = \frac{5+2}{3}$

$$M_B\left(\frac{4}{2I}\right) + 2M_C\left(\frac{4}{2I} + \frac{5}{1.5I}\right) + 0$$

$$= -\frac{6 \times 213.33 \times 2}{4 \times 2I} - \frac{6 \times 240 \times 2.33}{5 \times 1.5I}$$

$2M_B + 10.66M_C = -767.35$ — (2)

from (1) & (2)

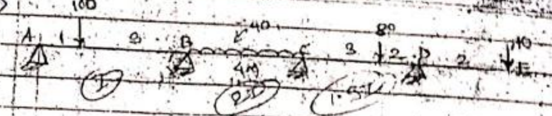
$M_B = -47.90$

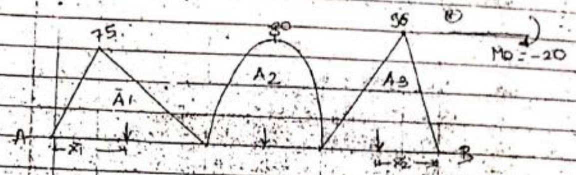
$M_C = -63.07$

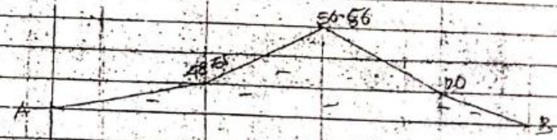
joint moment.

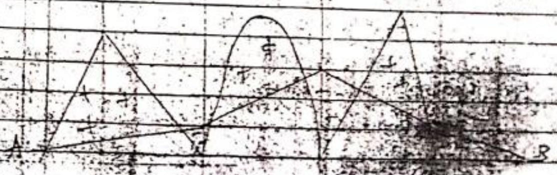
Overhang is not a member, it is determinant part of structure

6)









Applying TMT b/w ABC.

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right)$$

$$= -\frac{6A_1\bar{x}_1}{L_1I_1} - \frac{6A_2\bar{x}_2}{L_2I_2}$$

$A_1 = 150$ $\quad \bar{x}_1 = 1.67$

$A_2 = 213.33$, $\bar{x}_2 = 2$

$$0 + 2M_B\left(\frac{4}{I}+\frac{4}{2I}\right) + M_C\left(\frac{4}{2I}\right) =$$

$$-\frac{6\times150\times1.67}{4\times I} - \frac{6\times213.33\times2}{4\times2I}$$

$$12M_B + 2M_C = -695.75 \quad \text{—} (1)$$

Applying TMT b/w BCD

$$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left(\frac{L_2}{I_2}+\frac{L_3}{I_3}\right) + M_D\left(\frac{L_3}{I_3}\right)$$

$$= -\frac{6A_2\bar{x}_2}{L_2I_2} - \frac{6A_3\bar{x}_3}{L_3I_3}$$

$$M_B\left(\frac{4}{2I}\right) + 2M_C\left(\frac{4}{2I}+\frac{5}{1.5I}\right) - 20\left(\frac{5}{1.5I}\right) = -767.35$$

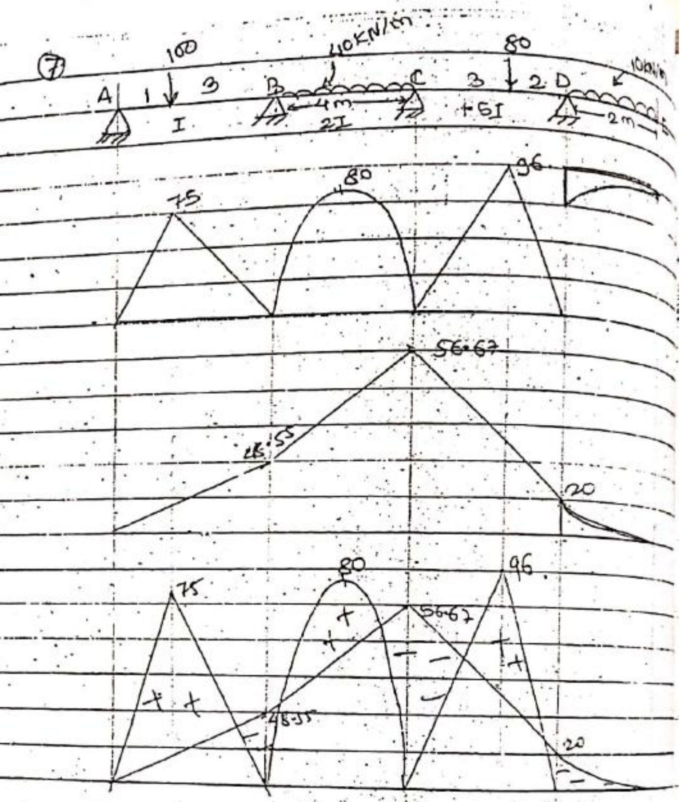
$$2M_B + 10.67M_C = -700.68 \quad \text{—} (2)$$

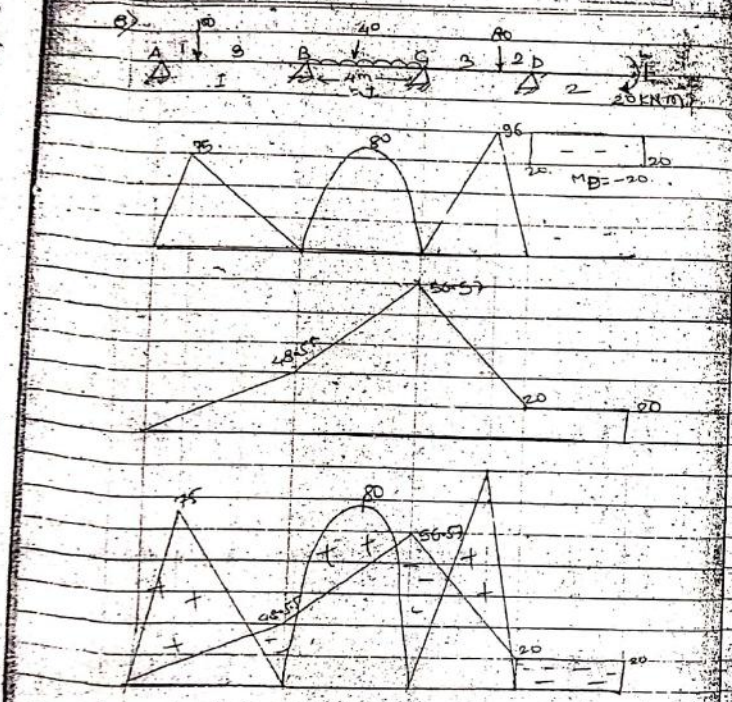
from (1) & (2)

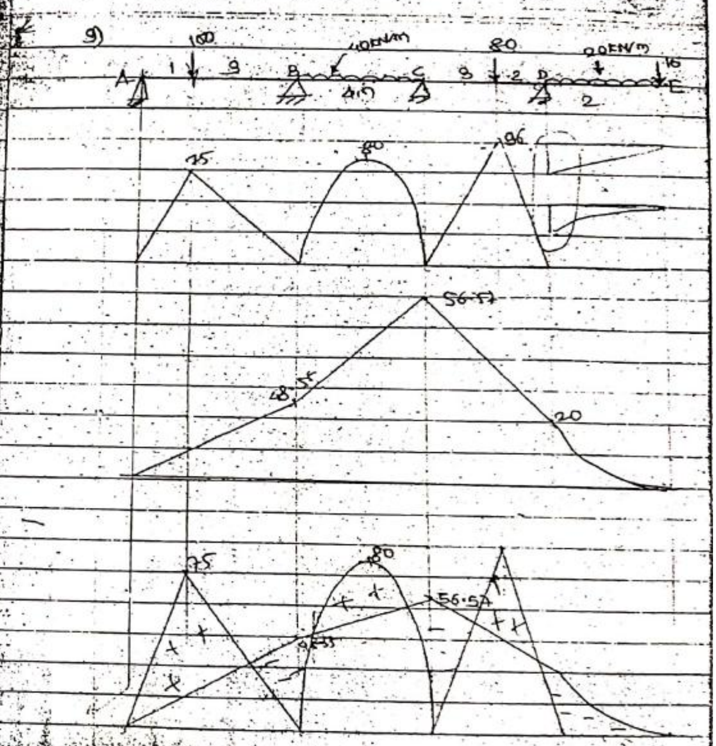
$M_B = -48.55$ kNm

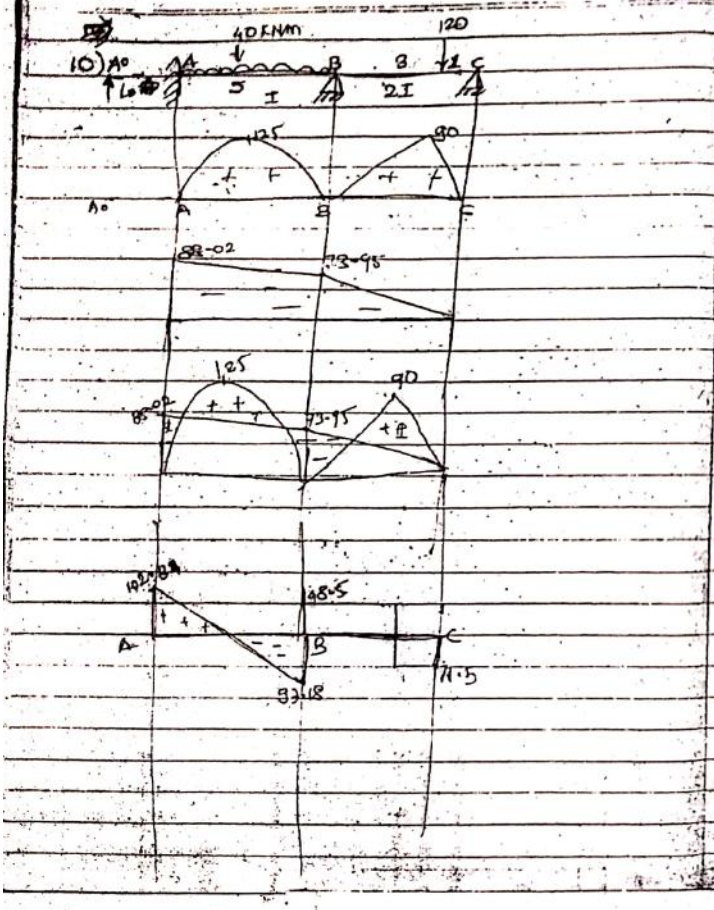
$M_C = -56.56$ kN.m

(7)



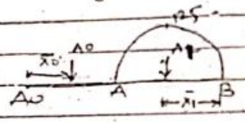

8)
A
B
C
D
E
100
40
80
3
2
4m
20KN/m
75
80
96
20
MB = -20
56.57
48.57
20


9)
A
B
C
D
E
100
40KN/m
80
20KN/m
4m
2
75
80
96
56.57
48.57
20



Applying TMT betn A_0AB

$$M_{A_0}\left(\frac{L_0}{I_0}\right) + 2M_A\left(\frac{L_0}{I_0}+\frac{L_1}{I_1}\right) + M_B\left(\frac{L_1}{I_1}\right) = -\frac{6A_0\bar{x}_0}{L_0 I_0} - \frac{6A_1\bar{x}_1}{L_1 I_1}$$



$A_1 = \frac{2}{3}BH = 416.67$

$\bar{x}_1 = 2.5$

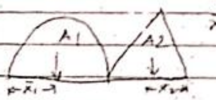
$$0 + 2M_A\left(0+\frac{5}{I}\right) + M_B\left(\frac{5}{I}\right) = 0 - \frac{6\times416.67\times2.5}{5\times I}$$

$$10M_A + 5M_B = -1250 \quad \text{—— (1)}$$

Applying TMT betn ABC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) = -\frac{6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$

zero



$A_1 = 416.67$ $A_2 = 180$

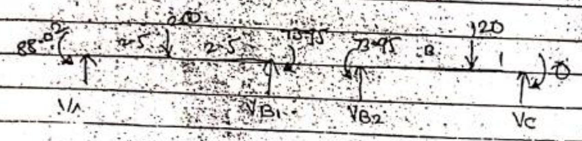
$\bar{x}_1 = 2.5$ $x_2 = 1.66$

$$M_A\left(\frac{5}{I}\right) + 2M_B\left(\frac{5}{I}+\frac{4}{2I}\right) = -\frac{6\times416.67\times2.5}{5\times I} - \frac{6\times180\times1.66}{4\times2I} \Rightarrow 5M_A + 14M_B = -1475.46 \quad \text{—— (2)}$$

(1) & (2) $\boxed{M_A = -88.02}$ $\boxed{M_B = 78.95}$

Hogging sagging

SFD calculation:



$V_A + V_{B1} = 200$ —— ①

$\Sigma M_A = 0;\ 200 \times 2.5 - 88.02 + 73.95 - V_{B1} \times 5 = 0$

$\boxed{V_{B1} = 97.186}$

$V_A = 200 - 97.18$

$\boxed{V_A = 102.81}$

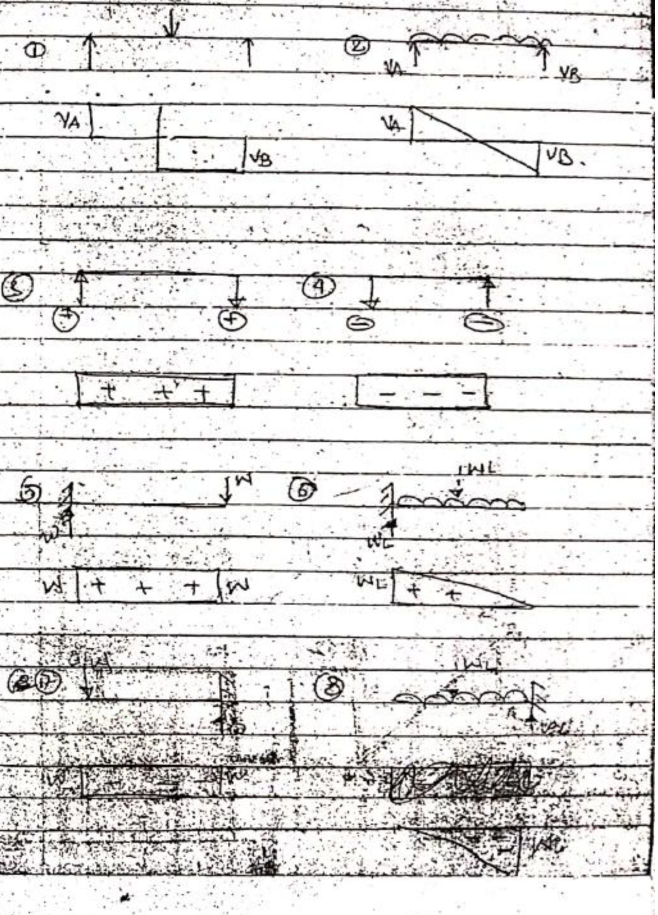
$V_{B2} + V_C = 120$

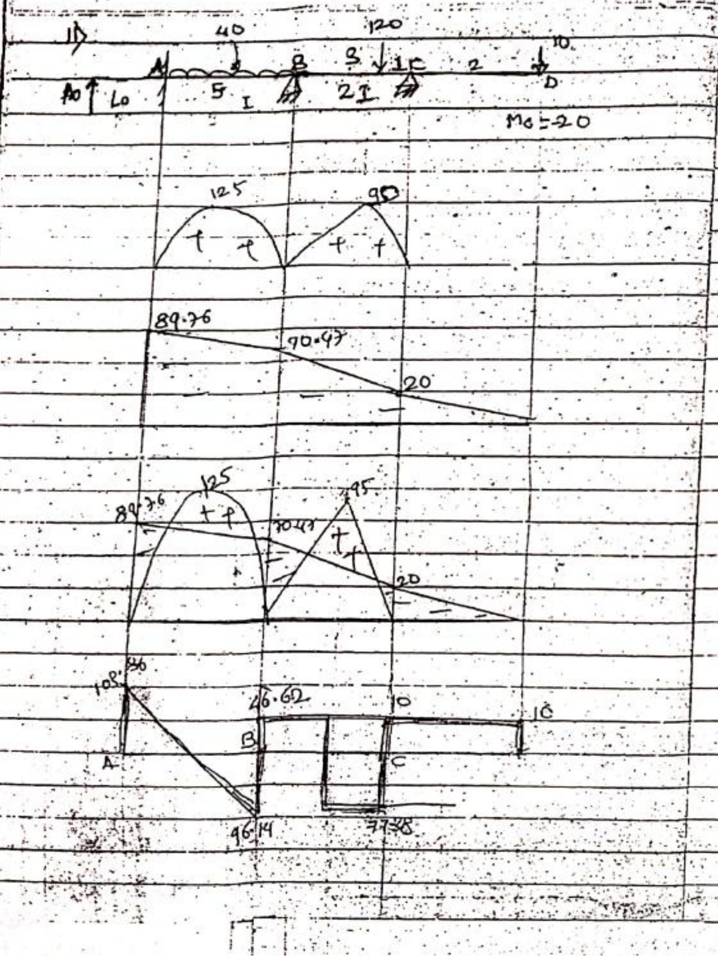
$\Sigma M_{B2} = 0;\ 120 \times 8 - 73.95 + 0 + V_C \times 4 = 0$

$\boxed{V_C = 71.5}$

$\boxed{V_{B2} = 48.98}$

Standard S.F.D





Applying TMT b/w A_0AB

$$M_{A0}\left(\frac{L_0}{I_0}\right)+2M_A\left(\frac{L_0}{I_0}+\frac{L_1}{I_1}\right)+M_B\left(\frac{L_1}{I_1}\right)=$$

$$\frac{-6A_0\bar{x}_0}{L_0I_0}-\frac{6A_1\bar{x}_1}{L_1I_1}$$

$$10M_A+5M_B=-1250 \quad \text{—(1)}$$

Applying TMT b/w^n ABC

$$M_A\left(\frac{L_1}{I_1}\right)+2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_C\left(\frac{L_2}{I_2}\right)=$$

$$\frac{-6A_1\bar{x}_1}{L_1I_1}-\frac{6A_2\bar{x}_2}{L_2I_2}$$

$$M_A\left(\frac{5}{I}\right)+2M_B\left(\frac{5}{I}+\frac{4}{2I}\right)-20\left(\frac{4}{2I}\right)=-1475.5$$

$$5M_A+14M_B-40=-1475.5$$

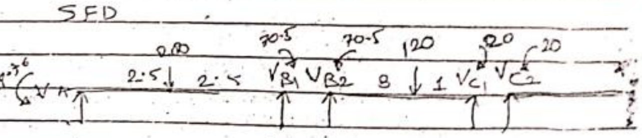
$$5M_A+14M_B=-1435.5 \quad \text{—(2)}$$

from (1) & (2)

$M_A = 89.76$

$M_B = 70.47$

SFD



$V_A + V_{B1} = 200$

$-89.76 + 200 \times 2.5 - V_{B1} \times 5 = 0$

~~$V_{B1} = 102.86$~~ $V_{B1} = \cancel{96.14}\ 82.04$

~~V_B~~ $V_A =$ $V_A = \cancel{103.86}\ 282.04$

$V_{B2} + V_{C1} = 120$

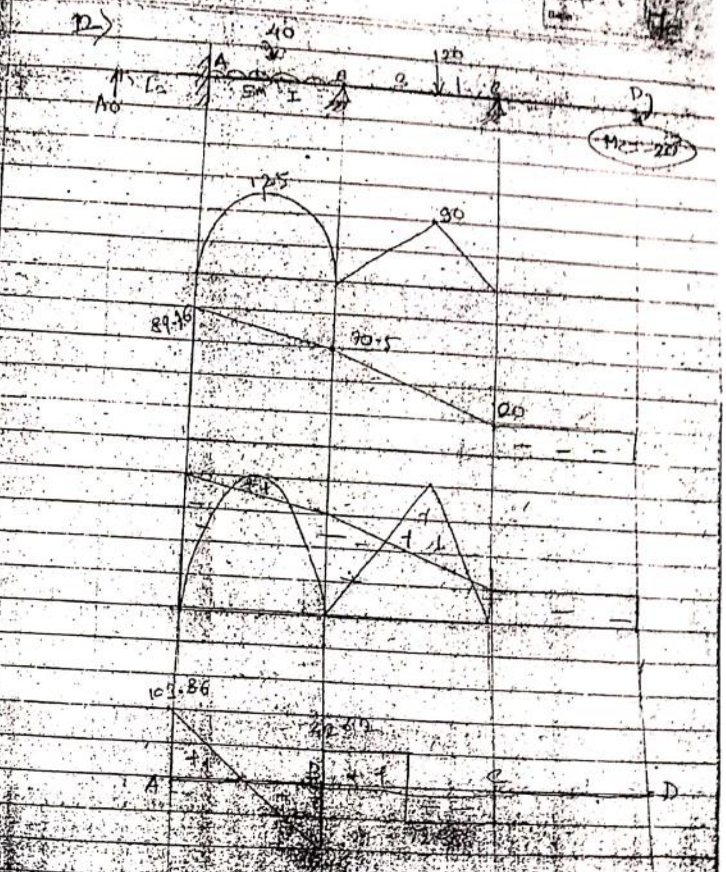
$-70.5 + 120 \times 3 - V_{C1} \times 4 = 0$

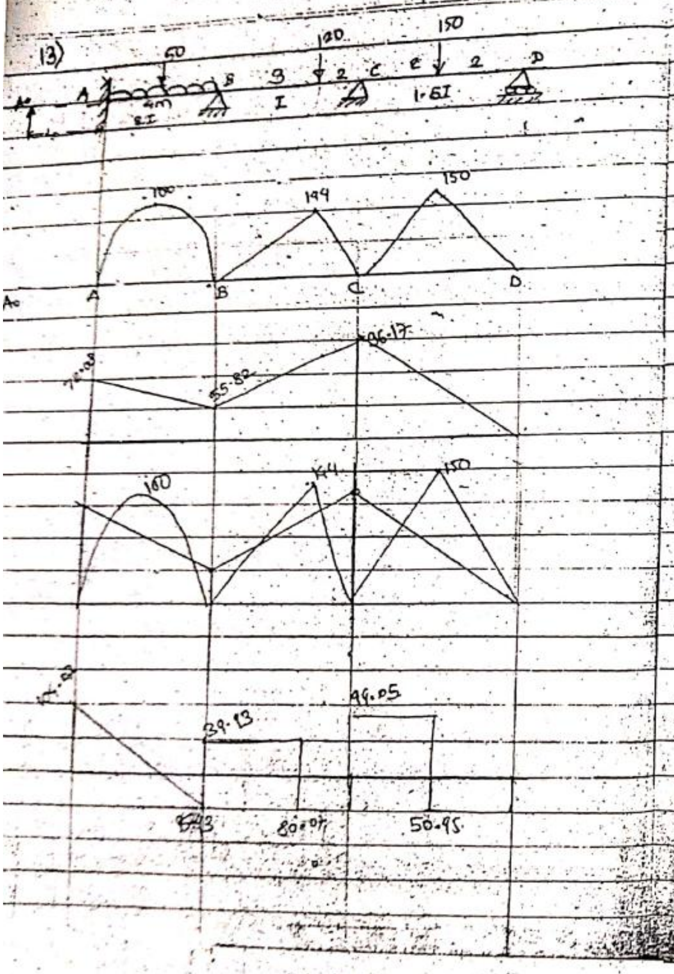
$V_{C1} = 77.38$

$V_{B2} = 42.62$

$V_{C2} = 10$

overhang pe जितना लोड है इतना upward reaction develop होगा।

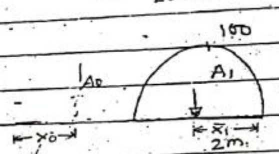




Applying TMT betn A_0AB.

$$M_{A_0}\left(\frac{L_0}{I_0}\right)+2M_A\left(\frac{L_0}{I_0}+\frac{L_1}{I_1}\right)+M_B\left(\frac{L_1}{I_1}\right)=$$

$$-\frac{6A_0\bar{x}_0}{L_0I_0}-\frac{6A_1\bar{x}_1}{L_1I_1}$$



$A_1 = 2/3\,BH$

$A_1 = 266.67$

$$0+2M_A\left(0+\frac{4}{2I}\right)+M_B\left(\frac{4}{2I}\right)=$$

$$-0-\frac{6\times 266.67\times 2}{4\times 2I}$$

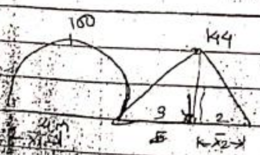
$8M_A + 4M_B + 0M_C = -800.$

$8M_A + 4M_B = -800$ —— (1)

Applying TMT betn ABC.

$$M_A\left(\frac{L_1}{I_1}\right)+2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_C\left(\frac{L_2}{I_2}\right)=$$

$$-\frac{6A_1\bar{x}_1}{L_1I_1}-\frac{6A_2\bar{x}_2}{L_2I_2}$$



$A_1 = 266.67$, $\bar{x}_1 = 2m$

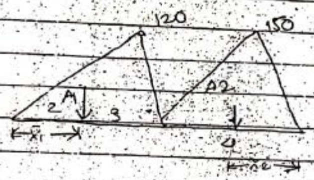
$A_2 = 360$, $\bar{x}_2 = 2.33m$

$$M_A\left(\frac{4}{2I}\right) + 2M_B\left(\frac{4}{2I} + \frac{5}{I}\right) + M_C\left(\frac{5}{I}\right) = \frac{-6 \times 266.67 \times 2}{4 \times 2I} - \frac{6 \times 360 \times 2.33}{5 \times I}$$

$$2M_A + \cancel{4} \; 14M_B + 5M_C = -1406.5 \quad \text{--- (2)}$$

consider span BCD

· applying TMT to BCD.



$A_1 = 360 \quad \bar{x}_1 = 2.33$

$A_2 = 300 \quad \bar{x}_2 = 2$

$$M_B\left(\frac{L_1}{I_1}\right) + 2M_C\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_D\left(\frac{L_2}{I_2}\right) = \frac{-6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2}$$

$$= M_B\left(\frac{5}{I} + \frac{4}{1.5I}\right) + 2MC$$

$$= M_B\left(\frac{5}{I}\right) + 2M_C\left(\frac{5}{I} + \frac{4}{1.5I}\right) + \overset{0}{M_D}\left(\frac{4}{1.5I}\right)$$

$$= \frac{-6 \times 360 \times 2.67}{5 \times I} - \frac{6 \times 300 \times 2}{4 \times 1.5I}$$

$$5M_B + 15.33M_C + \cancel{2.667M_D} = \cancel{-1608.56} \; -1753.44 \quad \text{--- (3)}$$

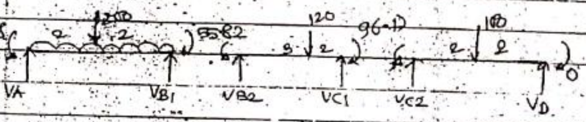
from (1) (2) & (3).

$M_A = -72.08$ KNM

$M_B = -55.82$ KNM

$M_C = -96.17$ KNM

<u>SFD calcul^n</u>



$-72.08 + 200 \times 2 + 55.82 - V_{B1} \times 4 = 0$

$V_A + V_{B1} = 200$

$V_A = 104.07$

$V_{B1} = 95.93$

$V_{B2} + V_{C1} = 120$

$-55.82 + 120 \times 3 + 96.17 - V_{C1} \times 5 = 0$

$V_{B2} = 39.93$

$V_{C1} = 80.07$

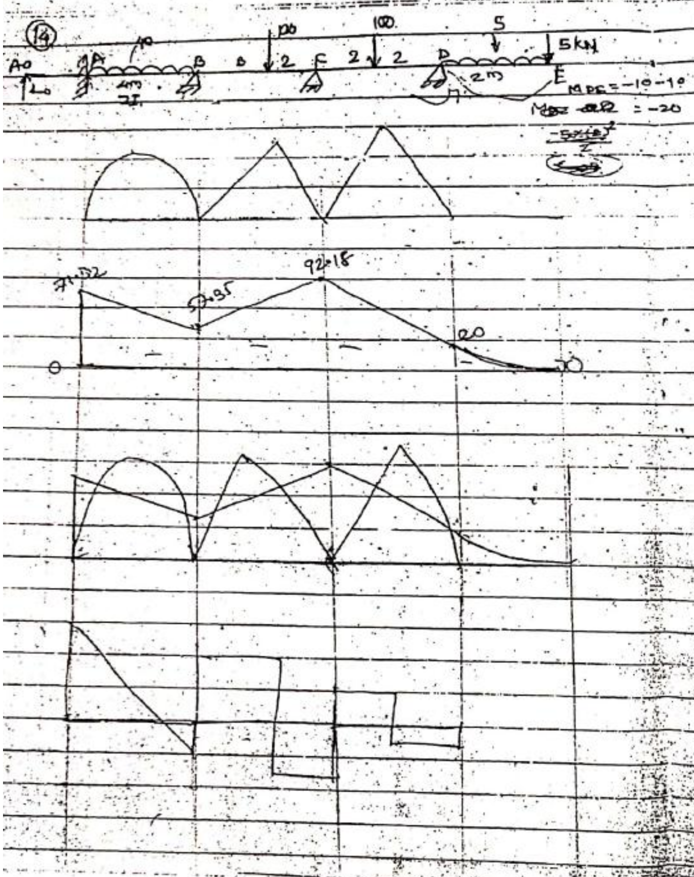
$V_{C2} + V_D = 100$

$-96.17 \times 100 \times 2 + V_D \times 4 = 0$

$V_{C2} = 98.05$

$V_D = 50.85$

(14)



$8M_A + 4M_B + 0M_C = -800$ — (1)

$2M_A + 14M_B + 5M_C = -1406.6$ — (2)

$5M_B + 15.83M_C + (-20)\left(\frac{4}{1.5I}\right) = -1753.35$

$5M_B + 15.33M_C - 53.33 = -1753.34$

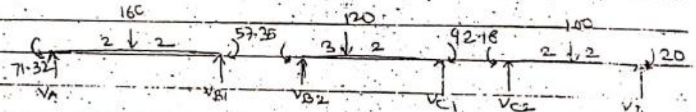
$\therefore 0M_A + 5M_B + 15.33M_C = -1700$ — (3)

from (1) (2) & (3)

$M_A = -71.32$

$M_B = -57.35$

$M_C = -92.18$



$V_A + V_{B_1} = 160$

$-71.32 + 160 \times 2 + 57.35 - V_{B1} \times 4 = 0$

$\boxed{V_{B1} = -76.50}$

$\boxed{V_A = 236.5}$

$V_{B2} + V_{C1} = 120$

$-57.35 + 120 \times 3 + 92.18 - V_{C1} \times 5 = 0$

$\boxed{V_{C1} = -78.96}$

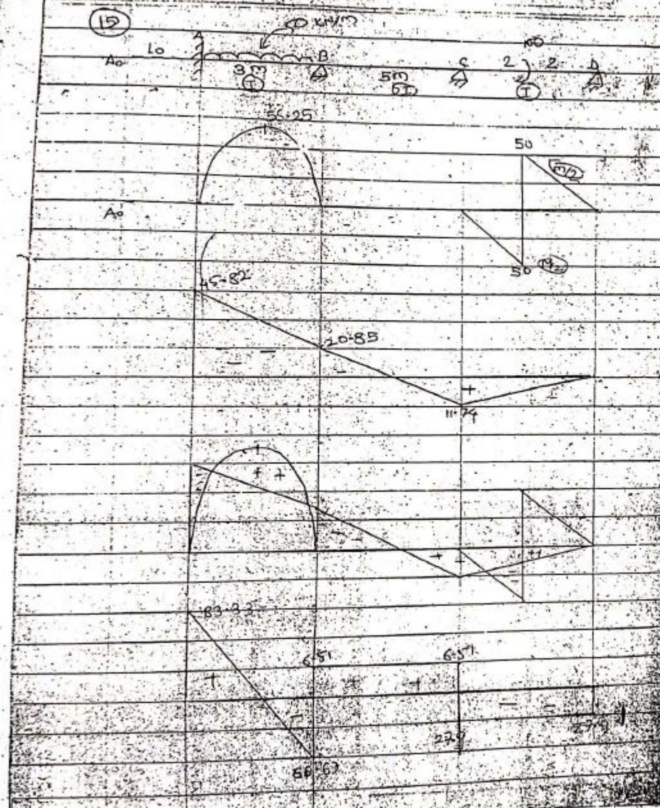
$\boxed{V_{B2} = 198.96}$

V_{C2} $-92.18 + 100 \times 2 + 20 - V_D \times 4 = 0$ $V_{C2} + V_D = 100$

$\boxed{V_D = -31.95}$

$\boxed{V_{C2} = 131.95}$

Scanned by CamScanner



Apply 3M TMT b/w A_0AB

$$M_{A_0}\left(\frac{L_0}{I_0}\right) + 2M_A\left(\frac{L_0}{I_0} + \frac{L_1}{I_1}\right) + M_B\left(\frac{L_1}{I_1}\right) = $$

$$-\frac{6A_0\bar{x}_0}{L_0I_0} - \frac{6A_1\bar{x}_1}{L_1I_1}$$

$x_1 = 1.5$

$A_1 = \frac{2}{3} \times 56.25 \times 3 = 112.5\ m^2$

$$0 + 2M_A\left(\frac{3}{I}\right) + M_B\left(\frac{3}{I}\right) = -\frac{6 \times 112.5 \times 1.5}{I \times 3}$$

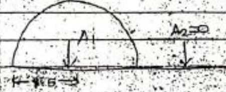
~~$6M_A + 3M_B = -168.76$~~

$6M_A + 3M_B = -337.5 \longrightarrow (1)$

consider ABC

$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) =$$

$$-\frac{6A_1\bar{x}_1}{L_1I_1} - \frac{6A_2\bar{x}_2}{L_2I_2}$$



$A_2 = 2/3 \times 56.25 \times 3 =$

$A_2 = 112.5\ m^2$

$x_2 = 1.5$

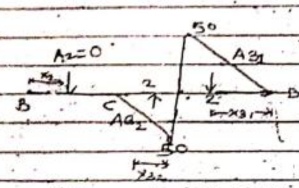
$$M_A\left(\frac{3}{I}\right) + 2M_B\left(\frac{3}{I} + \frac{5}{2I}\right) + M_C\left(\frac{5}{2I}\right) = -\frac{6 \times 112.5 \times 1.5}{I \times 3}$$

$3M_A + 11M_B + 2.5M_C = -337.5 \longrightarrow (2)$

Apply TMT betn BCD

$$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + M_D\left(\frac{L_3}{I_3}\right) =$$

$$-\frac{6A_2x_2}{L_2I_2} - \frac{6A_3x_3}{L_3I_3}$$



$A_{3_1} = \frac{1}{2} \times 2 \times \cancel{100} = 50$

$x_{3_1} = \left(\frac{2}{3}\right) \times 2 = 1.33$

$A_{3_2} = -\frac{1}{2} \times 2 \times 50 = -50$

$x_{3_2} = 2 + \frac{1}{3} \times 2 = 2.67$

$$A_3x_3 = A_{3_1}x_{3_1} + A_{3_2}x_{3_2} = 50 \times 1.33 - 50 \times 2.67$$

$$A_3x_3 = -67\ m^2$$

$$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + M_D\left(\frac{L_3}{I_3}\right) =$$

$$-0 - \frac{6A_3x_3}{I_3 x_3}$$

$$M_B\left(\frac{5}{2I}\right) + 2M_C\left(\frac{5}{2I} + \frac{4}{I}\right) + 0 = 0 - \frac{6 \times (-67)}{4 \times I}$$

$2.5M_B - 13M_C = 100.5$ —(3)

from ①, ② & ③.

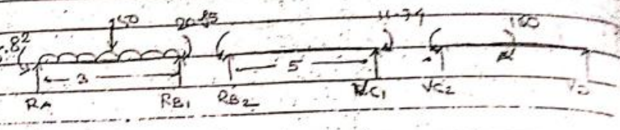
$M_A = -45.82$

$M_B = -20.85$

$M_C = 11.74$

$M_D = 0$

SFD calculation.



$RA + RB_1 = 150$ — (A)

$-45.82 + 50 \times 1.5 + 20.85 = 3RB_1$

$RB_1 = 66.67$ KN; $RA = 83.32$ KN

$RB_2 + VC_1 = 0$ — (B)

$-20.85 - 11.74 = 5VC_1$

$VC_1 = -6.51$

$VB_2 = +6.51$

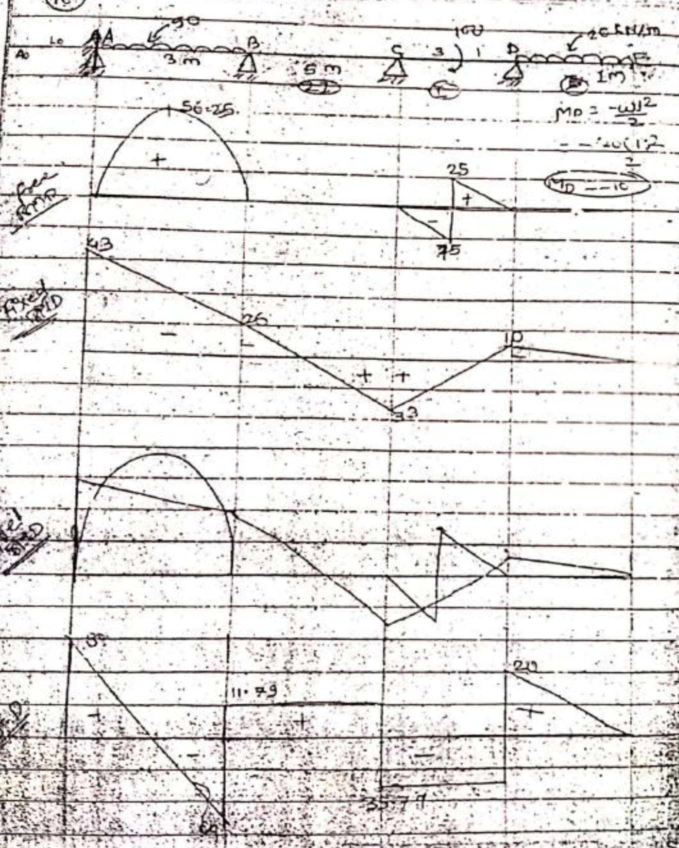
$VC_2 + V_D = 0$ — (C)

$11.74 + 100 = V_D \times 4$

$V_D = \cancel{24}\ 27.93$ KN.

$V_{C_2} = -27.93$.

//



consider A_0AB

Apply TMT betn A_0AB

$M_{A_0}\left(\frac{L_0}{I_0}\right) + 2M_A\left(\frac{L_0}{I_0} + \frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1}\right) =$

$$\frac{-6A_0\bar{x}_0}{L_0I_0} - \frac{6A_1\bar{x}_1}{L_1I_1}$$

$6M_A + 3M_B = -337.5$ —— ①

consider ABC

$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) =$

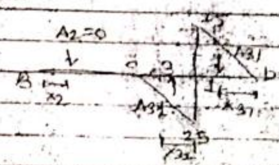
$$\frac{-6A_1\bar{x}_1}{L_1I_1} - \frac{6A_2\bar{x}_2}{L_2I_2}$$

$3M_A + 11M_B + 2.5M_C = -337.5$ —— ②

Apply TMT betn BCD

$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + M_D\left(\frac{L_3}{I_3}\right) =$

$$\frac{-6A_2\bar{x}_2}{L_2I_2} - \frac{6A_3\bar{x}_3}{L_3I_3}$$



$A_{31} = \frac{1}{2} \times 1 \times 25$
$= 12.5$

$x_{31} = \frac{2}{3} \times 1 = 0.67$

$A_{32} = \frac{1}{2} \times 3 \times 75$

$x_{32} = \frac{2}{3} \times 3$

$A_B \times 3 = 12.5 \times 0.67 - 112.6 \times 2 = -216.63 \; m^3$

$$M_B\left(\frac{5}{2I}\right) + 2M_C\left(\frac{5}{2I} + \frac{4}{I}\right) + (-10)\left(\frac{4}{I}\right) =$$

$$0 - \frac{6 \times (-216.63)}{4 \times I} = 324.945 + 40$$

$2.5 M_B + 13 M_C = 964.945$ —— (3)

~~M_A~~ from (1), (2) & (3)

$M_A = -43.00$

$M_B = -26.49$

$M_C = +33.167$

SFD calc^n

$V_A + V_{B_1} = 150$	$V_{B2} + V_{C1} = 0$
$-43 + 150 \times 1.5 + 26.49 = 3V_{B_1}$	$-26.49 - 33.16 = 5V_{C1}$
$V_{B1} = 69.49$	$V_{C1} = -11.93$
$V_A = 80.50$	$V_{B2} = +11.93$

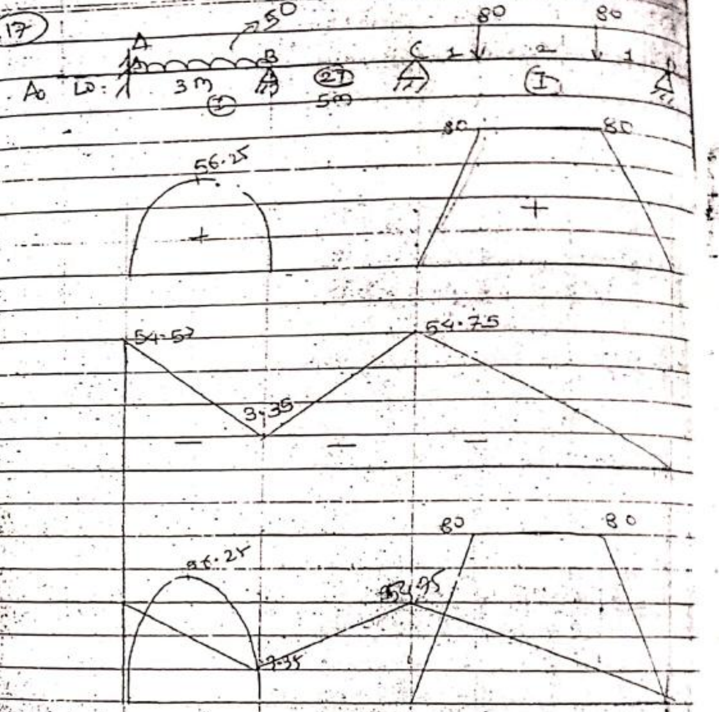
$V_{C2} + V_{D1} = 0$

$100 + 33.16 + 10 = 4V_D$

$V_D = 35.79$

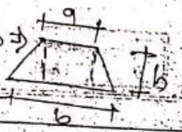
$V_{C2} = -35.79$

(17)



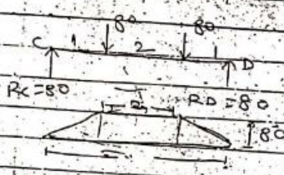
12

$A = \frac{a+b}{2} \times h \Rightarrow$



~~Apply TMT bet^n~~

Simply Support CD



$A = \left(\frac{a+b}{2}\right) \times h$

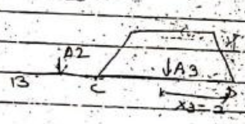
$6M_A + 3M_B = -337.5$ — (1)

$3M_A + 11M_B + 2.5M_C = -337.5$ — (2)

Apply TMT bet^n BCD

$M_B\left(\frac{L_2}{I_2}\right) + 2M_C\left(\frac{L_2}{I_2} + \frac{L_3}{I_3}\right) + M_D\left(\frac{L_3}{I_3}\right) =$

$-\frac{6A_2\bar{x}_2}{I_2L_2} - \frac{6A_3\bar{x}_3}{L_3I_3}$



$A_3 = \left(\frac{2+4}{2}\right) \times 80$

$= 240\ m^2$

$M_B\,(5/2I) + 2M_C\left(5/2I + 4/I\right) + 0 =$

$-0 - \frac{6 \times 240 \times 2}{I \times 4} = -72$

$0M_A + 2.5M_B + 13M_C = -720$ — (3)

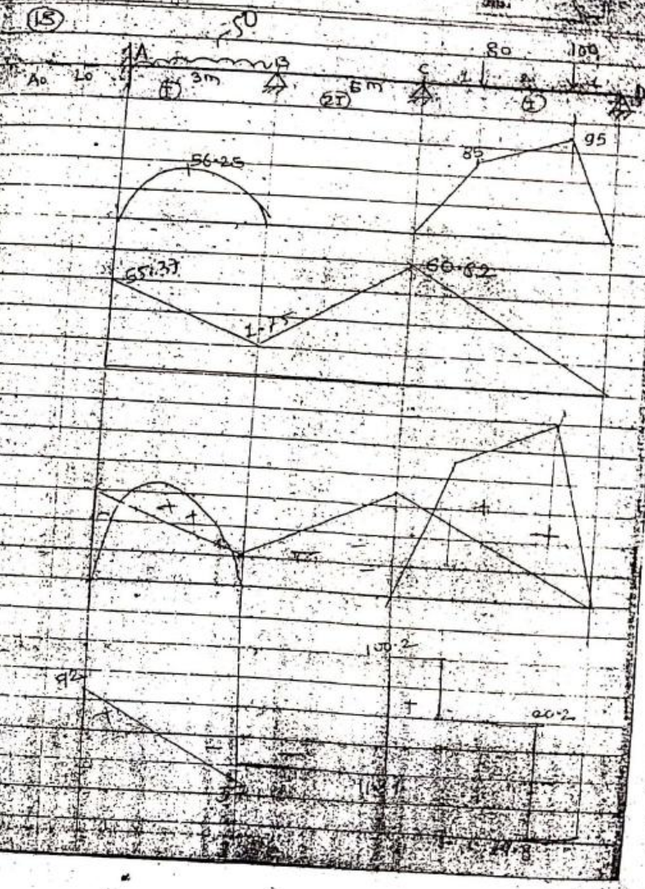
from (1) (2) & (3)

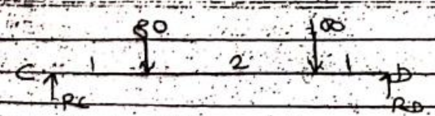
$M_A = -54.57$

$M_B = -3.35$

$M_C = -54.75$

(15)



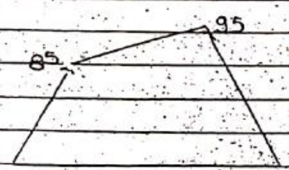


$R_C + R_D = 180$

$80 \times 1 + 100 \times 3 = 4R_D$

$R_D = 95\,KN$

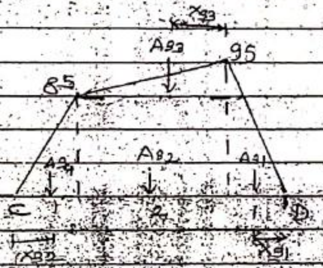
$R_C = 85\,KN$



$6M_A + 3M_B + 0M_C = -337.5 \quad \text{—(1)}$

$3M_A + 11M_B + 25M_C = -337.5 \quad \text{—(2)}$

Apply TMT betn B C D



$A_{31} = \frac{1}{2} \times 95 \times 1 = 47.5\,m^2$

$x_{31} = 2/3 \times b = 2/3 \times 1 = 0.67\,m$

$A_{32} = b \times d = 85 \times 2 = 170\,m^2$

$x_{32} = 1 + 1 = 2\,m$

$A_{33} = 1/2 \times 2 \times 10 = 10\,m^2$

$x_{33} = 1 + 1/3 \times b = 1 + 1/3 \times 2 = 1.67\,m$

$A_{34} = 1/2 \times 1 \times 3.5 = 42.5\,m^2$

$x_{34} = 1 + 2 + 1/3 \times b = 1 + 2 + 1/3 \times 1 = 3.33$

$A_3 x_3 = A_{31} x_{31} + A_{32} x_{32} + A_{33} x_{33} + A_{34} x_{34}$

$= 47.5 \times 0.67 + 170 \times 2 + 10 \times 1.67 + 42.5 \times 3.33$

$A_3 x_3 = 530.05$

$M_B\left(\frac{5}{2I}\right) + 2M_C\left(\frac{5}{2I} + \frac{4}{I}\right) + 0 = 0 - \frac{6 \times A_3 x_3}{I_3 L_3}$

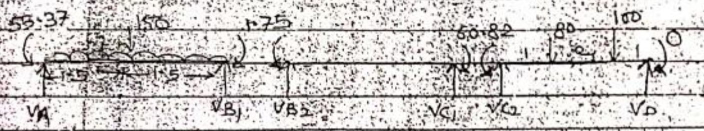
$= \frac{-6 \times 530.05}{4 \times 1}$

$0 M_A + 2.5 M_B + 13 M_C = -795$ —— (3)

from (1) (2) & (3)

$M_A = -55.37$ KNm
$M_B = -1.75$ KNm
$M_C = -60.82$ KNm

<u>SFD</u>



~~$V_{C1} + V_{B1} = 150$~~

$V_A + V_{B1} = 150$

$-55.37 + 150 \times 1.5 + 1.75 = 3 V_{B1}$

$V_{B1} = 57.18$ KN

$V_A = 92.82$ KN

$-1.75 + 60.82 = 5 V_{C1}$

$V_{C1} = +11.81$ KN ↑

$V_{B2} = -11.81$ KN ↓

$V_{C2} + V_D = 180$

$-60.82 + 80 \times 1 + 100 \times 3 = 4 V_D$

$V_D = 79.8$ KN

$V_{C2} = 100.20$ KN.

SF = 100.2

$(SF)_L = 100.2$

~~$(SF)_R = 100.2 - 80 = 20.2$~~

$(SF)_L = 20.2$

$(SF)_R = 20.2 - 100$

$= -79.8$

$(SF)_D = 0$

TMT with sinking of support

The modified TMT after accounting the support sinking is

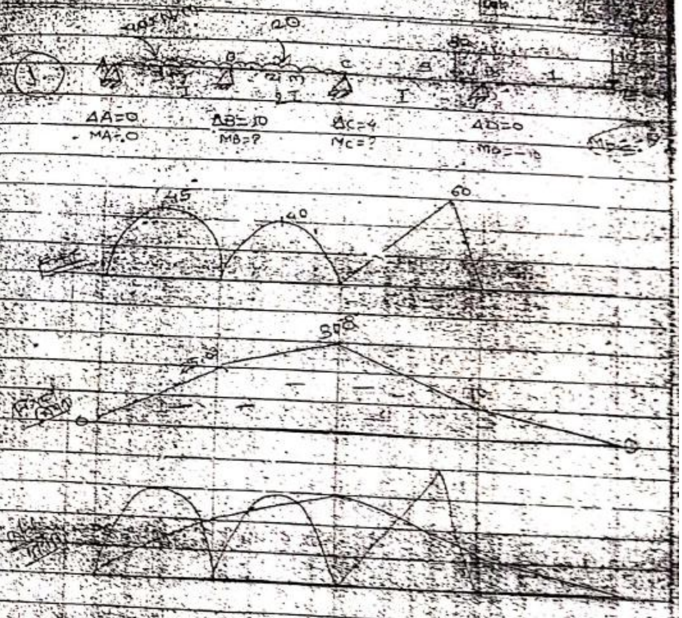
$$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right)$$

$$= \frac{-6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2} + \frac{6E\Delta_1}{L_1} + \frac{6E\Delta_2}{L_2}$$

Δ_1 :- relative displacement of central support w.r.t left support

Δ_2 :- relative displacement of central support w.r.t right support

Q 1) For a continuous beam ABCD as shown in fig. determine the support moments & plot SFD, BMD if support B sinks by 10 mm & C sinks by 4 mm. Take EI = 4000 kNm²

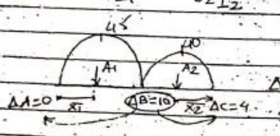


m c d M

Applying TMT betn ABC

$$M_A\left(\frac{L_1}{I_1}\right)+2M_B\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_C\left(\frac{L_2}{I_2}\right)=$$

$$\frac{-6A_1\bar{x}_1}{L_1I_1}-\frac{6A_2\bar{x}_2}{L_2I_2}+\frac{6E\Delta_1}{L_1}+\frac{6E\Delta_2}{L_2}$$



$\Delta_1 = 10-0 = 10 = 0.01$

$\Delta_2 = 10-4 = 6 = 0.006\,mm$

$\bar{x}_1 = 1.5 \quad \bar{x}_2 = 2$

$A_1 = \frac{2}{3}BH = \frac{2}{3}\times 3\times 45 = 90$

$A_2 = \frac{2}{3}BH = \frac{2}{3}\times 4\times 40 = 106.67$

$$0+2M_B\left(\frac{3}{I}+\frac{4}{2I}\right)+M_C\left(\frac{4}{2I}\right)=$$

$$\frac{-6\times 90\times 1.5}{3\times I}\;\frac{-6\times 106.67\times 2}{4\times 2I}+\frac{6E(0.01)}{3}+\frac{6E(0.006)}{4}$$

$$\frac{10M_B}{I}+\frac{2M_C}{I}=-\frac{270}{I}-\frac{160}{I}+0.029E$$

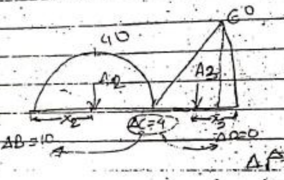
$10M_B + 2M_C = -270 - 160 + 0.029\,EI$

$10M_B + 2M_C = -314$ —— 1

Applying TMT betn BCD

$$M_B\left(\frac{L_1}{I_1}\right)+2M_C\left(\frac{L_1}{I_1}+\frac{L_2}{I_2}\right)+M_D\left(\frac{L_2}{I_2}\right)=$$

$$\frac{-6A_2\bar{x}_2}{L_1I_1}-\frac{6A_3\bar{x}_3}{L_2I}+\frac{6E\Delta_1}{L_2}+\frac{6E\Delta_2}{L_3}$$



$A_2 = 106.67, \quad \bar{x}_2 = 2$

$A_3 = \frac{1}{2}bh = \frac{1}{2}\times 4\times 60 = 120$

$\bar{x}_3 = \frac{L+b}{3} = 1.67$

$\Delta_1 = 4-10 = -6 = -0.006\,mm$

$\Delta_2 = 4-0 = 4 = 0.004\,mm$

$$M_B\left(\frac{4}{2I}\right)+2M_C\left(\frac{4}{2I}+\frac{4}{I}\right)-10\left(\frac{4}{I}\right)=$$

$$\frac{-6\times 106.67\times 2}{4\times 2I}-\frac{6\times 120\times 1.67}{4\times I}+\frac{6E(-0.006)}{4}$$

$$+\frac{6E(0.004)}{4}$$

$$\frac{2M_B}{I}+\frac{12M_C}{I}-\frac{40}{I}=-\frac{160}{I}-\frac{300}{I}-0.009E$$

$+0.006E$

$$\frac{2M_B}{I}+\frac{12M_C}{I}-\frac{40}{I}=-\frac{460}{I}-0.003E$$

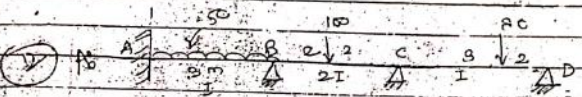
$2M_B + 12M_C - 40 = -460 - 12$

$2M_B + 12M_C = -432$ —— 2

from (1) (2)

$M_B = -25.03$ KNm

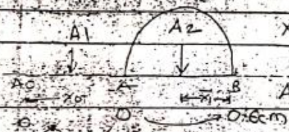
$M_C = -31.822$ KNm



Analyse the given continuous beam if supports B sinks by 0.6cm & C sinks by 1cm. Take EI = 8000 KNm^2

⇒ Applying TMT betn A0AB.

$$M_{A0}\left(\frac{L_0}{I_0}\right) + 2M_A\left(\frac{L_0}{I_0}+\frac{L_1}{I_1}\right) + M_B\left(\frac{L_1}{I_1}\right) = -\frac{6A_0\bar{x}_0}{L_0 I_0} - \frac{6A_1\bar{x}_1}{L_1 I_1} + \frac{6E\Delta_1}{L_0} + \frac{6E\Delta_2}{L_1}$$



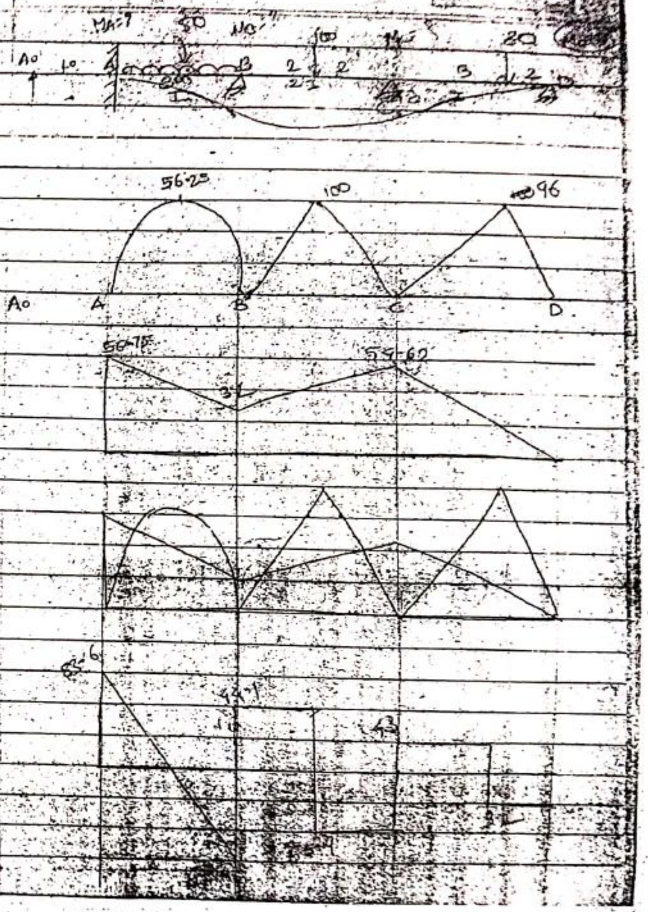
$x_0 = 0$, $A_0 = 0$

$\bar{x}_1 = 1.5$

$A_1 = 112.5$

$\Delta_1 = 0$

$\Delta_2 = 0 - 0.6 = -0.6 = -0.006$ m



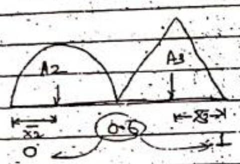
$2M_A\left(\frac{3}{I}\right) + M_B\left(\frac{3}{I}\right) = \frac{-6 \times 112.5 \times 1.5}{3 \times I} + 6E\frac{(-0.006)}{3}$

$\frac{6M_A}{I} + \frac{3M_B}{I} = \frac{-337.5}{I} - 0.012E$

~~$6M_A + M_B$~~

$6M_A + 3M_B = -433.5$ —— (1)

applying TMT betn ABC



$A_1 = 112.5$

$\bar{x}_1 = 1.5$ m

$A_2 = 200$

$\bar{x}_2 = 2$ m

0.6

$\Delta_1 = 0.6 - 0 = 0.6 = 0.006$ m

$\Delta_2 = 0.6 - 1 = -0.4 = -0.004$ m

$M_A\left(\frac{L_1}{I_1}\right) + 2M_B\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_C\left(\frac{L_2}{I_2}\right) =$

$\frac{-6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2} + \frac{6E\Delta_1}{L_1} + \frac{6E\Delta_2}{L_2}$

$M_A\left(\frac{3}{I}\right) + 2M_B\left(\frac{3}{I} + \frac{4}{2I}\right) + M_C\left(\frac{4}{2I}\right) =$

$\frac{-6 \times 112.5 \times 1.5}{3 \times I} - \frac{6 \times 200 \times 2}{4 \times 2I} + \frac{6E \times 0.006}{3}$

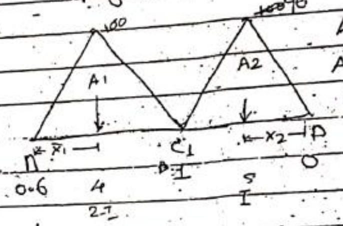
$+ \frac{6E \times (-0.004)}{4}$

$\frac{3M_A}{I} + \frac{10M_B}{I} + \frac{2M_C}{I} = \frac{-337.5}{I} - \frac{300}{I} + 0.006E$

$3M_A + 10M_B + 2M_C = -337.5 - 637.5 + 0.006E$

$3M_A + 10M_B + 2M_C = -589.5$ —— (2)

applying TMT betn BCD



$A_1 = 200$

$A_2 = 240$

$X_1 = 2$, $X_2 = 2.33$

$\Delta_1 = (0 - 0.6) = 0.4$

$= 0.004$ m

$\Delta_2 = 1 - 0 = \frac{1}{0.01}$ m

$M_B\left(\frac{L_1}{I_1}\right) + 2M_C\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_D\left(\frac{L_2}{I_2}\right) =$

$\frac{-6A_1\bar{x}_1}{L_1 I_1} - \frac{6A_2\bar{x}_2}{L_2 I_2} + \frac{6E\Delta_1}{L_1} + \frac{6E\Delta_2}{L_2}$

$M_B\left(\frac{4}{2I}\right) + 2M_C\left(\frac{4}{2I} + \frac{5}{I}\right) + M_D\left(\frac{5}{I}\right) \to 0 =$

$\frac{-6 \times 200 \times 2}{4 \times 2I} - \frac{6 \times 240 \times 2.33}{5 \times I} + \frac{6 \times E(0.004)}{4} + \frac{6E(0.01)}{5}$

$\frac{2M_B}{I} + \frac{14M_C}{I} + \cancel{\ldots} = \frac{-300}{I} - \frac{671.90}{I} +$

$\frac{2M_B}{I} + \frac{14M_C}{I}$ 40 = $\frac{-300}{I}$ $\frac{63.49}{I}$ -1200·E

2MB - $0M_A + 2M_B + 14M_C = -827.04$ — (3)

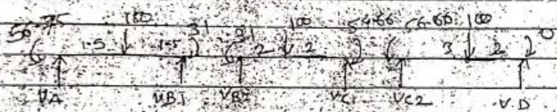
from (1) (2) & (3)

$M_A = 56.75$

$M_B = 31$

$M_C = 54.61$

SFD calc^n



$-56.75 + 150 \times 1.5 + 31 - V_{B1} \times 3 = 0$

$\boxed{V_{B1} = 66.4}$

$\boxed{V_A = 83.6}$

$-31 + 100 \times 2 + 54.36 - V_{C1} \times 4 = 0$

$\boxed{V_{C1} = 55.9}$

$\boxed{V_{B2} = 44.1}$

-56.66 + 100×3 + 20 - ... = 0

[illegible]

(13) Analyse the given cont beam if support C sinks down by 1cm.