Department of Civil Engineering

Name of Subject: - Strength of Materials	**Subject Code:- BECVE302T**
Unit-I : Introduction	**Semester: - III**
Basic Introduction: Basic concept of the subject , Structural laws , Basic fundamentals of mechanics , Law of machines Basic Assumption in mechanics, torque , spring camp and cranks , Application of mechanics in industry	

Course Outcome (CO):- The students would be able to understand the behavior of materials under different stress and strain conditions.

Learning Outcomes (LOs) :- (4 to 5 are expected and as per the COs)

- To make students learn and apply basic theories and concepts of equilibrium.
- To understand the basic fundamentals of mechanics
- To Remember the basic concept of structures in industry
- To utilize the basic lows for problems solving
- To learn about compression and tension behavior of materials

Department of Civil Engineering

1. INTRODUCTION
STRENGTH OF MECHANICS

The subject strength of material is a foundation stone for any engineering course. Today the Engineering application are mostly interdisciplinary, involving the basics of various fundamental subject .One of the such subject of vital importance is “Strength of Materials “

Department of Civil Engineering

About the Subject

Strength of material is an interdisciplinary subject. is used by the undergraduate and post graduate student of Civil, Mechanical, Chemical Engineering, Electrical engineering and architecture . Architecture and Civil Engineer like to see that the trusses slabs beams and columns of building and bridge are safe. The subject knowledge is also useful for the design of machinery and pressure vessels. . A metallurgist must understand this subject well so that he can think about the further improvement in material properties.

A computer electronics and electrical engineer should have the basic knowledge of subject to design some mechanical components they need in their product This book has been presented in various parts dealing with basic mechanics, simple stresses and strain. Shear force and bending moment, stresses in beam. Thick and thin cylinder, torsion, fracture mechanics, column and struts. .The basic course on the strength of material and related numerical required for all Engineer in covered in this book

Many structural elements like tube , beam truss cylinder sphere shaft coupling are used for the benefit of man kind . all the material is made up of timber , steel iron , aluminum , copper concrete . The application of laws of mechanics to find the support reaction due to the applied forces is normally covered under this subject engineering mechanics . In transferring this force from their point of application to support the material of the structures develops resting force and undergoes the deformation. The effect of these resisting force on the structural elements is treated under the subject strength of materials

Civil engineering, as consists of designing and constructing different parts necessary in day to day life, whether it is constructing a building, laying down sewage pipes, constructing dam, making piles, drift. In all of these, the main aim is to make the structure operational and safe for a certain amount of time, which is called its design period. It is necessary that the material and methods being used are adequate to meet the standards of safety set by the country/region.

Strength of materials is a rudimentary subject which encompasses everything from behavior in axial forces to bending to shear to buckling under huge loads, it prepares you to estimate material behavior when it is subjected to a force or combination of different forces. Without it's knowledge you cannot understand and appreciate the beauty behind other subjects (how cracking moment in reinforced bar is generated, what happens to a steel frame after it is loaded beyond it's capacity, how load varies in soil or asphalt which makes up the road).It is essential that you make yourself acquainted with basic subjects without which you cannot progress any further,

BRIEF HISTRORICAL REVIEW

The many material used for the benefit of the mankind. In early days of evolution of mankind itself . the great designer of the seventeenth century Galileo may be given the credit of presenting rational approach in the strength of materials . He studied the design of hull in Italian ships on the rational basis . However , the major development in the strength of materials took places in early part of nineteenth century by the contribution of French Significant *contributions to* the development of *strength of materials* were made by the Russian *scientists* M. V. Ostrogradskii, whose studies in *strength of* strength of materials .

INTRODUCTION TO STRUCTURAL MECHANICS :

Mechanics of structures is the computation of deformations, deflections, and internal forces or stresses(*stress equivalents*) within structures, either for design or for performance evaluation of existing structures. It is one subset of structural analysis. Structural mechanics analysis needs input data such as structural loads, the structure's geometric representation and support conditions, and the materials' properties. Output quantities may include support reactions, stresses and displacements. Advanced structural mechanics may include the effects of stability and non-linear behaviors. Mechanics of structures is a field of study within applied mechanics that investigates the behavior of structures under mechanical loads, such as bending of a beam, buckling of a column, torsion of a shaft, deflection of a thin shell, and vibration of a bridge.

Engineers engaged in designing buildings, bridges, transmission towers, machines, motor vehicles, aero planes, aerospace, mines, etc. Need to ascertain that their designs meet the following requirements.

1. Structural stability requirement, i.e., overturning or sliding should be avoided.
2. Each and every part of the structure should be strong enough to resist the force to which it is subjected during its life.
3. The deformation of any of its part should not be so llarge as to make it non-functional.
4. Every part is tough enough to sustain the vibrations likely to arise in the normal life of the structure.

It is not enough for the engineer to provide a safe design. His design has to be economical too. It is may no require very high skill to provide safe design but to produce an economically safe design the engineer should:

1. Consider the types of loads and their combinations.
2. Idealize the structure into simple elements like bar, beam, slab, shell, etc.
3. Analyse the idealized structure to find the forces each component has to receive.
4. Find internal forces developed in the components in transferring the forces acting on them to the supports and ultimately to the ground.
5. Proportion the members to resist the internal forces economically.

6. Check the deformations of structural members so that functioning of the member or other members is not adversely affected. If necessary, redesign the member.

Idealizing a structure and finding forces acting on its component is the subject matter of structural analysis. Treating a mender as rigid body and finding the reactions to its support is treated under engineering mechanics. Considering a member as a deformable body and finding internal forces developed in the body in mechanics of solids. Proportioning the member size to resist the forces is Structural Design. However, the lines of demarcation between engineering mechanics, mechanics of solids, structural analysis and structural design are not distinct. Hence, criss-crossing of many topics in the above subjects is very common which may be seen in many books and the university syllabi.

Structural Mechanics can be briefly described as the study of the behavior of structures using the knowledge of mechanics and strength of materials. Such a description needs some understanding of the terms “structure” and “mechanics”. Structures include a wide variety of systems, such as buildings, bridges, dams, aircrafts, etc., that are built to serve some specific human needs (for example, habitation, transportation, storage, etc.). Students of Structural Mechanics should already have some basic knowledge of mechanics through the prerequisite courses of Engineering Mechanics (or Rigid-body Mechanics or Vector Mechanics) and Solid Mechanics (Mechanics of Deformable Solids or Mechanics of Materials). In Structural Mechanics, we apply our knowledge of the mechanics of rigid bodies and of deformable solids to the understanding of the behavior of engineered structures.

In Structural Mechanics, we mostly deal with mechanics of solids (i.e. deformable bodies). However, here we move on from studying the behavior of structural members/materials (as in a course of Solid Mechanics) to studying the behavior of real structures, or parts thereof. For example, instead of dealing with a beam or a column, we study how a building frames composed of several beams and columns, behaves. In a similar way, we first learn about the loads that are applied to the whole structure, and not to individual members. Our knowledge of Structural Mechanics enables us to find the forces that act on individual members based on the loads that are acting on the whole structure. Stresses, strains, internal forces and deformations in members, then, can be obtained by using what we have already learned about the behavior of deformable solids.

Structures and its element

Strength of Materials

Knowledge of strength of materials is important for analysis and design of different structural element

Such structures and its element are classified into various categories depending on the system/mode of classification: On the basis of its intended function/usage: Buildings, bridges, dams, industrial sheds, cable ways, chimneys, etc.

BASIC ASSUMPTIONS

The Procedure developed in this book has made the following assumptions:

1. Materials are continuous and remain continuous under the action of loads considered, hence, The Equations developed may be differentiated or integrated .
2. All Material are Homogeneous i.e.. a material has identical properties at all the points.
3. Material are isotropic i.e.. a material identical property an all direction .
4. Materials are free form inertial force prior to Loading considered , i.e. residual stress due to manufacturing defect are ignored .

FUNDAMENTAL LAW

The Following Fundamental Laws of mechanics are use in this text through .

1. Elastic Behaviour : It is assume that a material regains its shape and size, when a load is released and stress-strain relation is linear in the range of loads expected on the structure.
2. Law of Superposition : According to this law the total effect of a set of loads is same as the sum of the effects of individual loads.
3. St. Venant Principle : According to this, except near the point of application of loads and geometric discontinuities, the stress distribution is independent of actual mode of application of the load. The distance at which stress distribution is in dependent of actual mode of application is equal to or grater than the width of the member. In other words stress concentrations at the points of load and geometric discontinuities are not considered in the analysis.

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Laws of Mechanics

The study of elementary mechanics rests on six fundamental principles based on experimental evidence.

Newton’s Three Fundamental Laws: - Formulated by Sir Isaac Newton in the latter part of the seventeenth century, these laws can be stated as follows:

Strength of Materials

1First Law: - If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

2 Second Law: - If the resultant force acting on a particle is not zero, the particle will have acceleration proportional to the magnitude of the resultant and in the direction of this resultant force. This law can be stated as

$F = ma$ Where F, m, and a represent, respectively, the resultant force acting on the particle, the mass of the particle, and the acceleration of the particle, expressed in a consistent system of units.

3 Third Law: - The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense.

4 Newton's Law of Gravitation: - This states that two particles of mass M and m are mutually attracted with equal and opposite forces of magnitude F given by the formula

$F=G\ Mm/d^2$ Where d is the distance between the two particles, G is universal constant called the constant of gravitation.

5 Parallelogram Law: - This states that two forces acting on a particle may be replaced by a single force, called their resultant, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces.

6Law of Transmissibility of forces: - This law states that the conditions of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action.

BASIC MECHANICS

The branch of physics that deals with the action of forces on matter is referred to as mechanics. All considerations of motion are addressed by mechanics, as well as the transmission of forces through the use of simple machines. In our class, the goal is a mechanical goal (placing blocks into a bin) and electronics are used to control the mechanics.

Strength of Materials

While it is not necessary to sit down and draw free body diagrams or figure out the static coefficient of friction between the LEGO tires and the game board, it is helpful to keep certain mechanical concepts in mind when constructing a robot. If a robot's tires are spinning because they do not grip the floor, then something must be done to increase the friction between the tires and the floor. One solution is to glue a rubber band around the circumference of the tire. That problem/solution did not require an in-depth study of physics. Simply considering the different possibilities can lead to more mechanically creative robots. Describing motion involves more than just saying that an object moved three feet to the right. The magnitude and direction of the displacement are important, but so are the characteristics of the object's velocity and acceleration.

1.Torque

A torque is a force applied at a distance from a pivot. When describing torques, one must include magnitude, direction, and perpendicular distance from the pivot. For torques the line of action is a circle centered on the pivot. As torque is a product of force and distance, one may be "traded" for the other. By applying more force closer to the pivot, one may produce the same torque. This concept of "trading" distance applied for force experience applied key to many simple machine

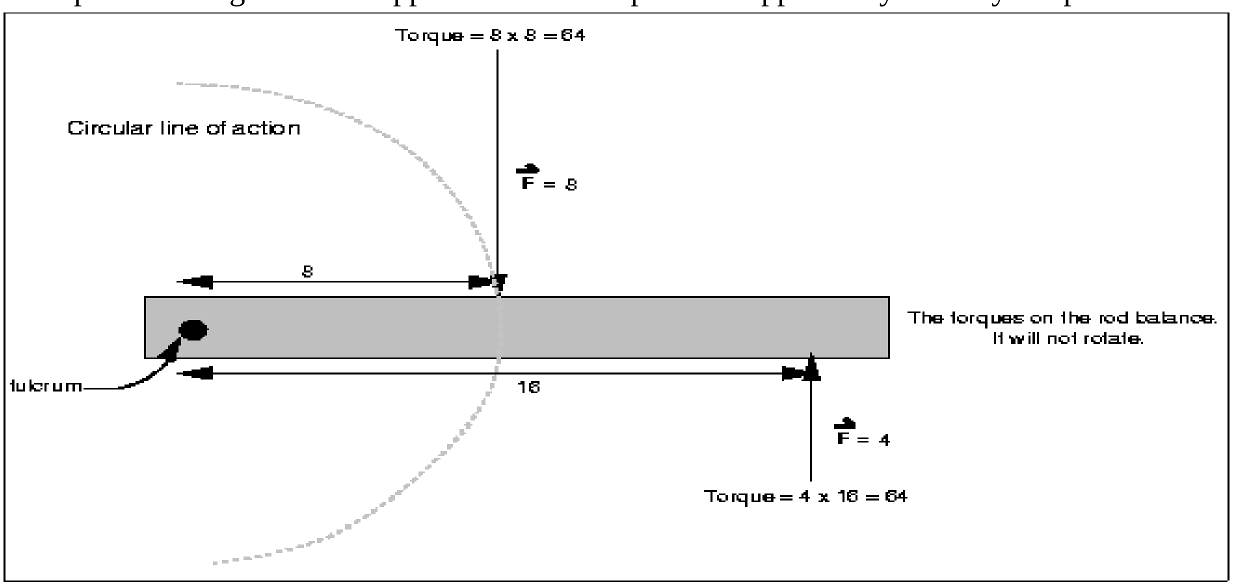

Fig : 1.2 Applied torque

2 Cams and Cranks

Both cams and cranks are useful when a repetitive motion is desired. Cams make rotary motion a little more interesting by essentially moving the axle off-center. Cams are often used in conjunction with a rod. One end of the rod is held flush against the cam by a spring. As the cam rotates the rod remains stationary until the "bump" of the cam pushes the rod away from the cam's axle. Cranks convert rotary motion into a piston-like linear motion. The best examples of cranks in action are the drive mechanism for a steam locomotive and the automobile engine crankshaft. In a crank, the wheel rotates about a centered axle, while an arm is attached to the wheel with an off-centered peg. This arm is attached to a rod fixed in a linear path. A crank will cause the rod to move back and forth, and if the rod is pushed back and forth, it will cause the crank to turn. On the other hand, cams can move their rods, but rods cannot move the cams.

Cams can be used to create linear repetitive motion or a repetitive rotational motion

3. Springs

A favorite device for storing potential energy is the spring. Everything from clocks to catapults makes use of springs. There are two distinctive forms of springs: the familiar coil and the bending bar. A common use for springs is to return something to its original position. A more interesting application is to use them to measure force -- springs in scales. The third use is to store energy. All springs perform all three functions all of the time, but specific devices are built to exploit certain functions of the spring.

A coil spring works for more or less the same reason as a bar spring, it's just in a different shape. To understand a spring, one must zoom in to the microscopic level where molecules interact. Molecules are held together in rigid bodies because of electromagnetic forces. Some of these forces are repulsive, and some of them are attractive. Normally they balance out so that the molecules are evenly spaced within an object; however, by bending a bar, some molecules are forced farther apart and others are shoved closer together. Where the molecules have been spread out, the attractive forces strive to return the original spacing. Where molecules have been forced together, the repulsive forces work to return the object to the original shape.

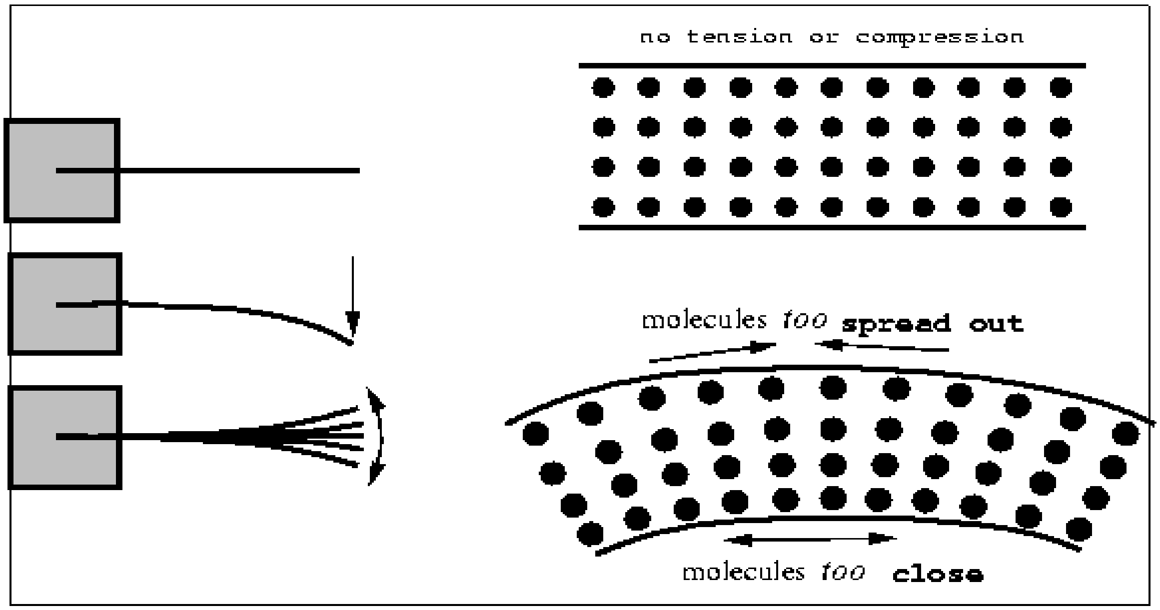

Fig : 1.4 Tension and compression Module

4 Counterweights

Counter weighting is a necessary evil in constructing even a simple robot. Examples of common counterweights are shown in below fig . If a robot that has been traveling along at high speed suddenly comes to a halt, there is danger of the robot overturning if the location of the robot's center of mass has not been well placed. The ELEC 201 robots carry around a fairly massive battery, and its placement within the robot's structure is important. When an arm extends, the robot should remain stable. This is accomplished through the use of counterweights.

Counter weighting might also prove useful to raise a bin carrying blocks. Rather than committing an entire motor to raising a bin, a set of counterweights known to be heavier than the bin plus contents could be suspended until the time when the bin should rise. Of course if a motor was used to take care of the counterweights then no motors have been saved. A motor could be used for more than one task if a mechanical transmission was employed. Another solution would be to use the high current LED outputs to operate a solenoid.

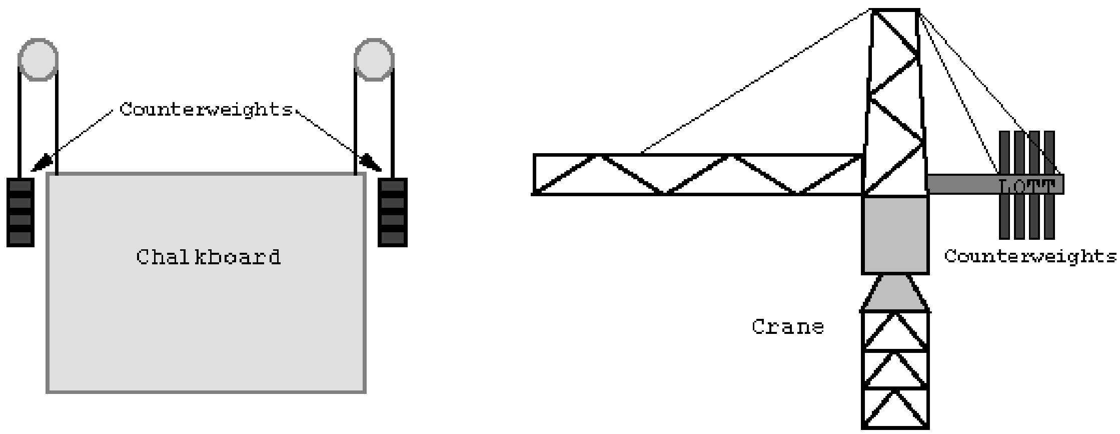

Fig : 1.5 Counterweight

Summary of Unit No 1 (Basic Introduction of Mechanics)

Mechanics may be defined as the physical science which describes and predicts the conditions of rest or motion of bodies under the action of force systems. In other words, where there is motion or force, there is mechanics. In engineering, mechanics is generally based on Newton's Laws and is often called Newtonian (or Classical) Mechanics after the English scientist Sir Isaac Newton (1642-1727).

Conditions involving speed of bodies close to the speed of light (about 300 x 106 m/s) and conditions requiring consideration of bodies with extremely small mass and size (such as subatomic particles with sizes in the order of 10-12 m and smaller) cannot be adequately described by Newton's Laws.

These extreme conditions are treated in Relativistic Mechanics and Quantum Mechanics. However, for a vast range of problems between these extremes, Newton's Laws give accurate results and are far simpler to apply. Although the fundamental principles of Newtonian Mechanics are surprisingly few in numbers, they have exceedingly wide range of applications. Modern research and development in the fields of vibrations, stability and strength of machines and structures, rocket and spacecraft design, robots, automatic control, engine performance, fluid flow, electrical

Strength of Materials

machines and apparatus are highly dependent upon the basic principles of mechanics. Mechanics is divided into two parts statics and dynamics

Exercise :

Q1. What are the various fundamentals laws of mechanics?

Q2. Explain the torque with example

Q3. What are the various application of spring

Q4. Explain the Newton's law of motion

Q5 Draw a neat sketch of Cram and Cranks

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Department of Civil Engineering

Name of Subject: - Strength of Materials	**Subject Code:- BECVE302T**
Unit-II : Shear Force and Bending Moment	**Semester: - III**

SFD and BMD

Axial force, shear force and bending moment diagram Concepts of free body diagrams, types of loads, Determination of axial forces, shear forces and bending moment at a section, axial force, shear force and bending moment in beams and simple frames,

Differential relations between shear force and bending moment, Relation between load and shear force.

Course Outcome (CO):- The students would be able to understand the concept of Shear force and Bending Moment

Learning Outcomes (LOs) :- (4 to 5 are expected and as per the COs)

- To Understand then basic concept of shear force and bending moment
- To remember the basic concept of axial force at the section
- To analysis the basic concept of bending moment in beam and simple frame
- To utilize the basic lows for problems solving
- To learn about compression and tension behavior of materials

INTRODUCTION

Shear force and bending moment calculation in structural design is very important Beam is a structural member, which carries lateral, or transverse forces i.e. forces at right angles to the axis of the member. The study of these transverse loads, however, is complicated by the fact that the loading effects vary from section to section of the beam. As a preliminary to study the stresses in beams, we shall study the variation of shear and bending moment at various cross section of the beam. Beams may be straight or curved, but we shall study only straight plane. All the beams disused will be statically determinate i.e. unknown reactions can be determined by applying equations of static equilibrium.

The shear force at a section of a beam is the force that shears off the section and is obtained as the algebraic sum of all forces including the reactions acting normal to the axis of the beam either to the left or right of the beam. Below, a force of 10N is exerted at point A on a beam. This is an external force. However, because the beam is a rigid structure, the force will be transferred internally all along the beam. This internal force is known as a shear force. The shear force between point A and B is usually plotted on a shear force diagram. As the shear force is 10 N all along the beam, the plot is just a straight line in this example.

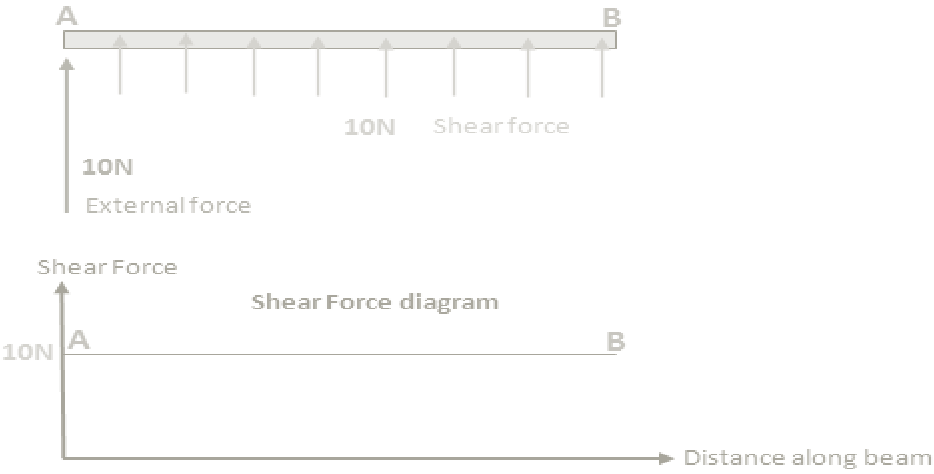


Fig 9.1 a shear force diagram

A bending moment is the reaction induced in a structural element when an external force or moment is applied to the element causing the element to bend.

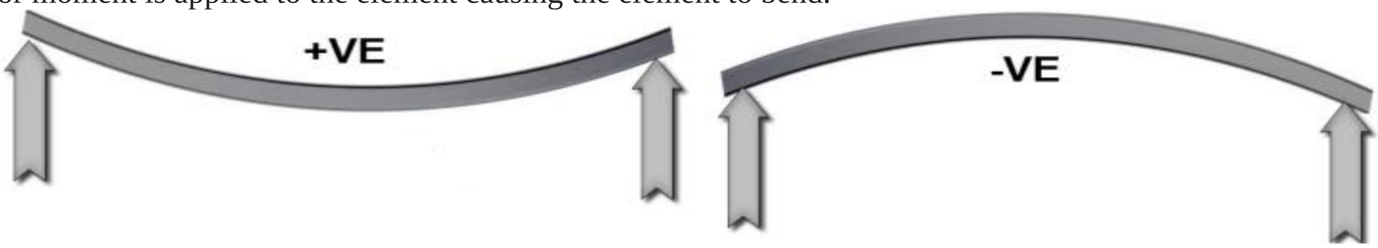


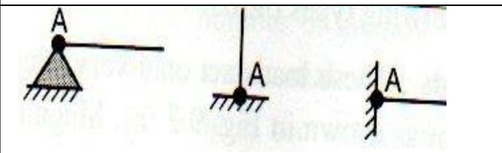
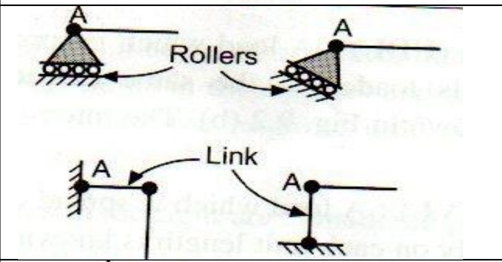
Fig 9.1 b Bending Moment diagram

9.2 TYPES OF SUPPORTS

In making sketches of structures or structual members it will be convenient to make use of symbols to show the manner in which it is supported. The various types of supports commonly employed in structural arrangements and their description are given in Table 9.1

Table 9.1 supports and Descriptions

Sr.	Support type	Symbol	Description

No.			
1	Hinged or pinned		Horizontal or vertical movement of the point of suport. A, is prevented, but free rotation about the point of support is possible.
2	Roller / Link		Movement normal to the plane on which rollers are supported is prevented, but movement parallel to the surface of support and rotation about the point of support is possible.
3	Fixed		Movement or rotation in any direction is absolutely prevented.

From the above discussion of various types of supports, it must be clear that, a hinged or a pinned support is capable of resisting a force acting in any direction of the plane. Hence , in general, the reaction at such a support may have two components, one in the horizontal and one in the vertical direction. The roller or link support is capable of resisting a force only in the direction normal to the plane on which it is supported. The fixed support is capable of resisting a force in any direction and is also capable of resisting a couple or a moment. Thus, two reaction components are possible at hinged end, one reaction component is possible at roller end and three reaction components are possible at the fixed end as shown in Fig.9.1.

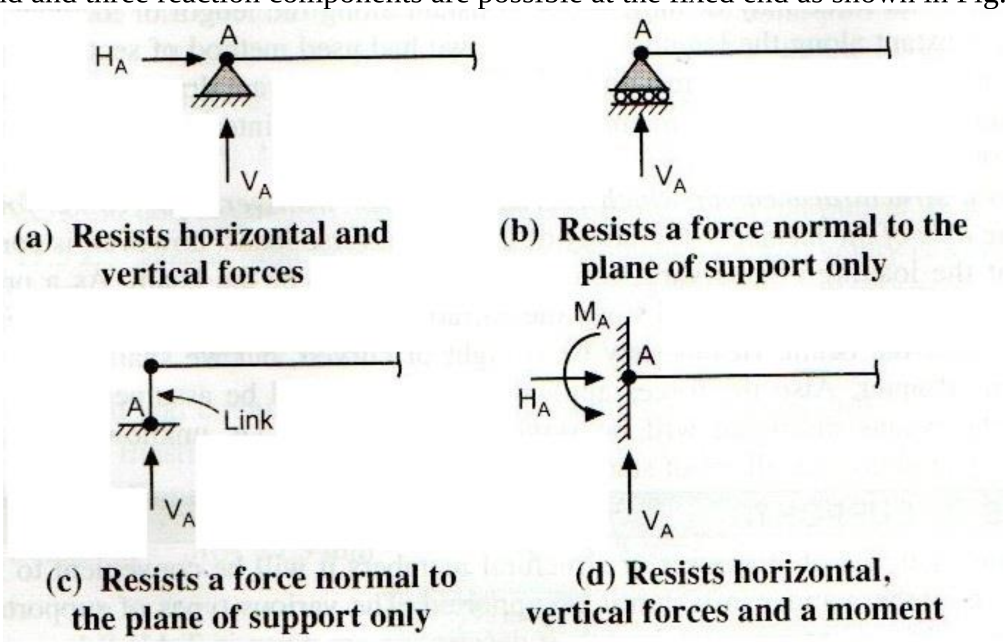


Fig. 9.1

9.3 TYPES OF LOADS

A beam may be subjected to the following types of loads:

Concentrated or Point Loads: These loads act on a very small area and hence it can be assumed to act at a point as shown in Fig. 9.2(a). Magnitude of the point load is expressed in N or kN.

(i) Uniformly Distributed Load (UDL) : A load which is spread over a length of beam such that, each unit length is loaded to the same extent is known as uniformly distributed load (UDL) , AS SHOWNB IN Fig. 9.2(b). The intensity of UDL is expressed in N/m or kN/m.

(ii) Uniformly Varying Load (UVL) : A load which is spread over a length of beam such that, its extent varies uniformly on each unit length is known as uniformly varying load (UVL): as shown in Fig. 9.2(c), (d). The intensity of UVL is expressed in N/m or kN/m.

(iii) Couple or Moment: A beam may be subjected to couple or moment at a point as shown in Fig. 9.2(d). The magnitude of couple or moment is expressed in Nm or kNm.

(iv) Bracket Loads: A beam may be subjected to a bracket load as shown in Fig. 9.2(g) which ultimately result in point load and couple at the point of attachment of bracket with the beam.

The reactions from secondary beams, columns, brackets etc. are generally idealized as concentrated or point loads. The self weight of beam, live load etc. are generally uniformly distributed over the length of beam. Loading on side walls of water tanks, lintels, retaining walls etc. is uniformly varying load. Point loads on brackets or point loads eccentric to the axis of the member result in couples.There may also be various combinations of these loadings. All these loads are assumed to act in one plane, called as plane of loading. To get the reactions set up by various distributed loads, one needs only to consider the load replaced by an equivalent concentrated load acting at its centre of gravity.

9.4 REACTIONS

If a body, subjected to forces acting in a plane, is at rest, there are three conditions of equilibrium which must be satisfied.

$$\sum F_x = 0; \quad \sum F_y = 0 \text{ and ; } \quad \sum M_z = 0$$

It follows that, in determining the reactions acting on the body, the loads being known, not more than three unknown quantities can be found. There are three things which must be known about each force in order to have it fully determined: its magnitude , its direction, and its point of application or line of action. If knowledge of all these concerning one reaction is lacking, then the magnitude, direction and point of application or line of action of all other forces acting on the structure must be known. This reaction would then hold body at equilibrium. It must be remembered that, the number of unknowns which can be determined is fixed by the number of equations available, and it makes no difference whether they pertain to one reaction or to several reactions as long as the unknown quantities do not exceed in number of independent equations.

To find the reactions of given beam or structure, first of all we remove the supports and show the respective reaction components in their places and draw complete free body diagram (FBD). Then apply laws of statics to get the required unknown reactions. Sign convention used for applying laws of statics is as under

(i) All horizontal forces to the right are considered positive.

(ii) All vertical forces acting upwards are considered positive.

(iii) All anticlockwise moments are considered positive.

Forces, moments, opposite of above are considered negative.

9.5 TYPES OF BEAMS

Beams are basically classified as statically determinate and statically indeterminate beams. When unknown reactions can be obtained by use of equations of static equilibrium alone, it is called as statically determinate. The various types of statically determinate beams are (i) Simply supported beams, (ii) Simple beam, (iii) Cantilever beam, (iv) Overhanging beam etc. While the various tyupes of statically indeterminate beams are (i) Fixed beams, (ii) Continuous beams (iii) Propped cantilevers, etc. These various types of beams are shown in Fig. 9.2.

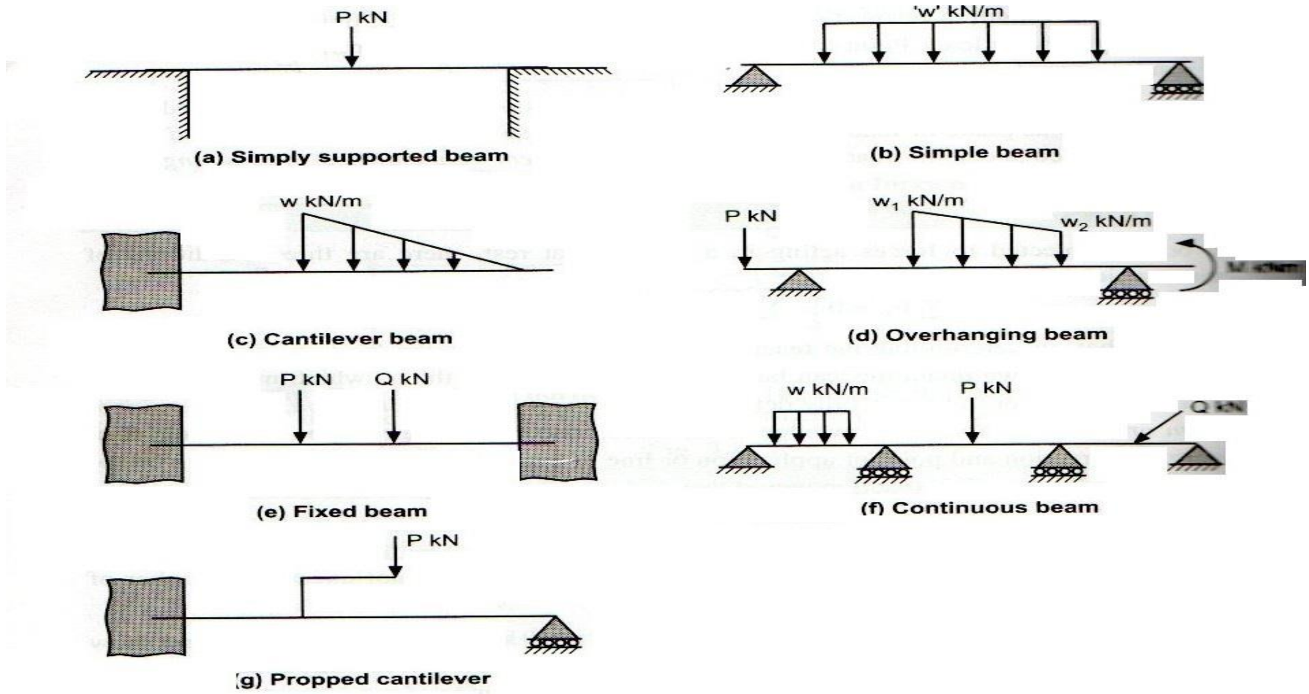


Fig. 9.2 : Various types of beams

9.6 SHEAR FORCE AND BENDING MOMENT

Consider a FBD of beam as shown in Fig. 9.3(a). The unknown reactions are determined by equations of laws of statics and having obtained the unknown reactions, method of section is used to find shear force and bending moment at a cross section.

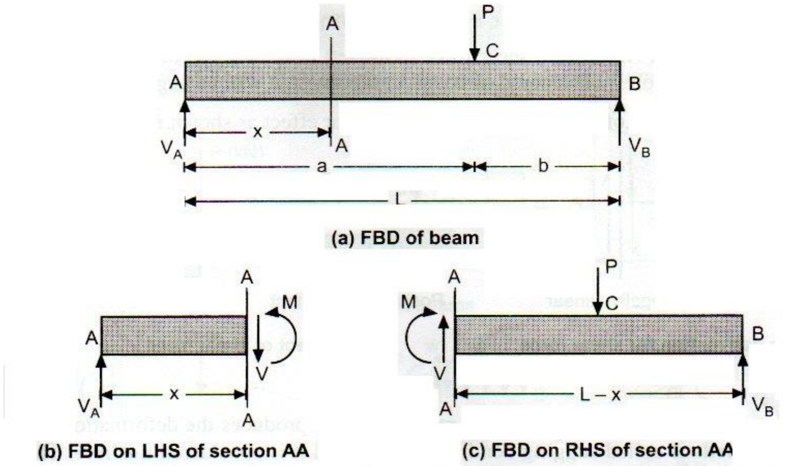


Fig. 9.3

Fig. 9.3(9b) shows the FBD of part of the beam on left hand side of the section considered at a distance x from A. since the beam and all its parts are in equilibrium, this section of the beam must be in equilibrium. First consider the equilibrium of the vertical forces i.e. This equation shows that, there must be a downward vertical force equal in magnitude to the reaction at left support. The only surface on which such a force can act is that on the right end of the free body. This downward force is called as the sher force and is designated as V or SF. Since, there are no horizontal forces, a moment equation is all that remains to be satisfied for equilibrium of the element shown. We can, of course, write a moment equation about any point in the plane of the diagram, but it is convenient to write it with respect to the right hand end of the section. Assuming unknown couple at this section of magnitude M, we have

$M - V_A(x) = 0$

$$M - V_A(x)$$

This quantity is known as the bending moment and is designated as M or BM.

Fig. 9.3(c) shows FBD of part of the beam on right hand side of the section considered. It should be noted that magnitude and nature of the shear force and bending moment at a section whether computed from left or right hand side of the section remains the same. The side to be chosen for calculation is that side on which fewer external forces are acting and the section is to be considered normal to the axis of the beam.

Thus, shear force and bending moment at a section can be defined as follows :

Shear force : It is the algebraic sum of all the external forces acting parallel to the section, on any one side of the section.

Bending moment: It is the algebraic sum of the moments of all the external forces acting on any one side of the section taken about the centre of gravity of section.

9.6.1 Sign Conventions for Shear Force and Bending Moment

(a) Shear Force : An upward shear force to the left of the section or downward shear force to the right of the section is considered as positive; otherwise it will be negative. See Fig.9.4(a).

This sign convention of shear force leads to the shear effect as shown in Fig. 9.4(b).

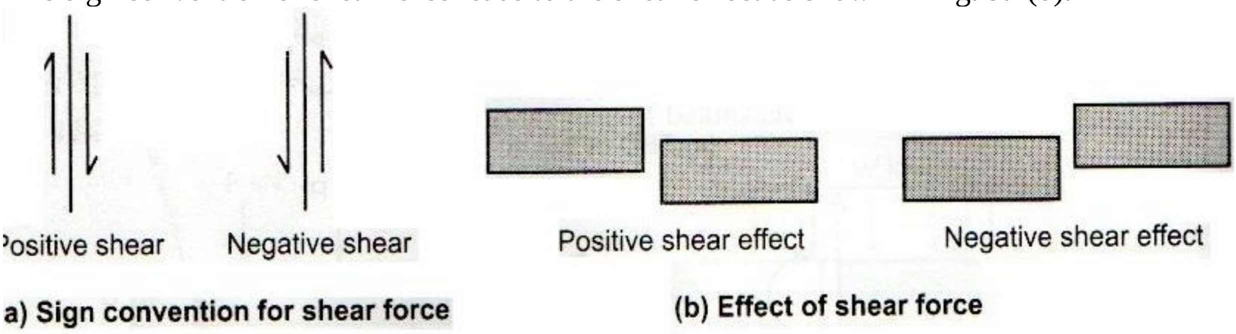


Fig. 9.4

(b) Bending Moment : the bending moment which produces the deformation of the beam concave upwards is called sagging bending moment and it is considered positive. The bending moment which produces the deformation of the beam convex upwards is called hogging bending moment and it is considered negative. See Fig. 9.5(a). This sign convention of bending moment leads to the bending effect as shown in Fig. 9.5(b).

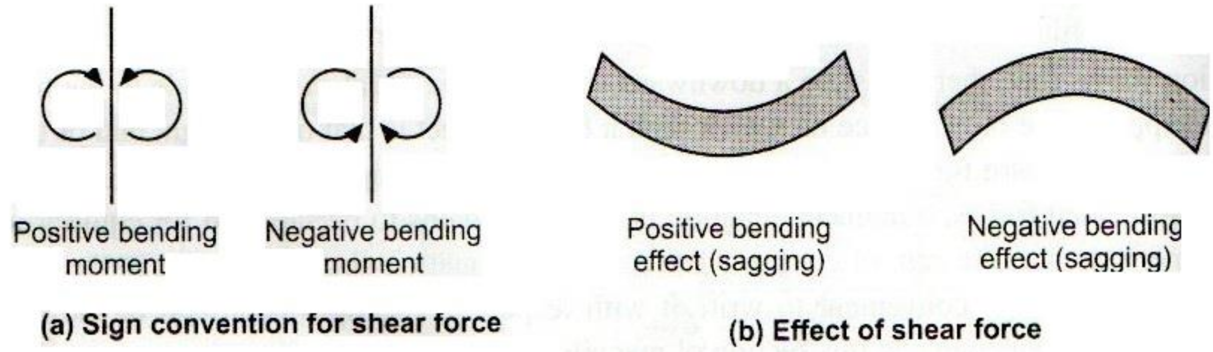


Fig. 9.5

It should be noted that, for a horizontal beam, upward external forces cause positive bending moments with respect to any section, while the downward external forces cause negative bending moments.

9.7 SHEAR FORCE AND BENDING MOMENT DIAGRMS (SFD AND BMD)

The shear force and bending moment can be calculated numerically at any particular section. But from the design point of view, we are interested in knowing the manner in which these values vary, along the length of beam. This can be done by plotting the shear force or the bending moment as ordinate and the position of the cross section as abscissa to give shear force diagram (SFD) and bending moment diagram (BMD) respectively. While plotting these diagrams, positive values are plotted above the reference line and negative values below it. The shear force and bending moment diagrams can be plotted from shear force and bending moment equations written for respective zones. The nature of these curves depends on the type of loading the beam is subjected to. This is illustrated in following simple examples.

SOLVED EXAMPLES

Example 9.1: Drew SFD and BMD for the following cantilever

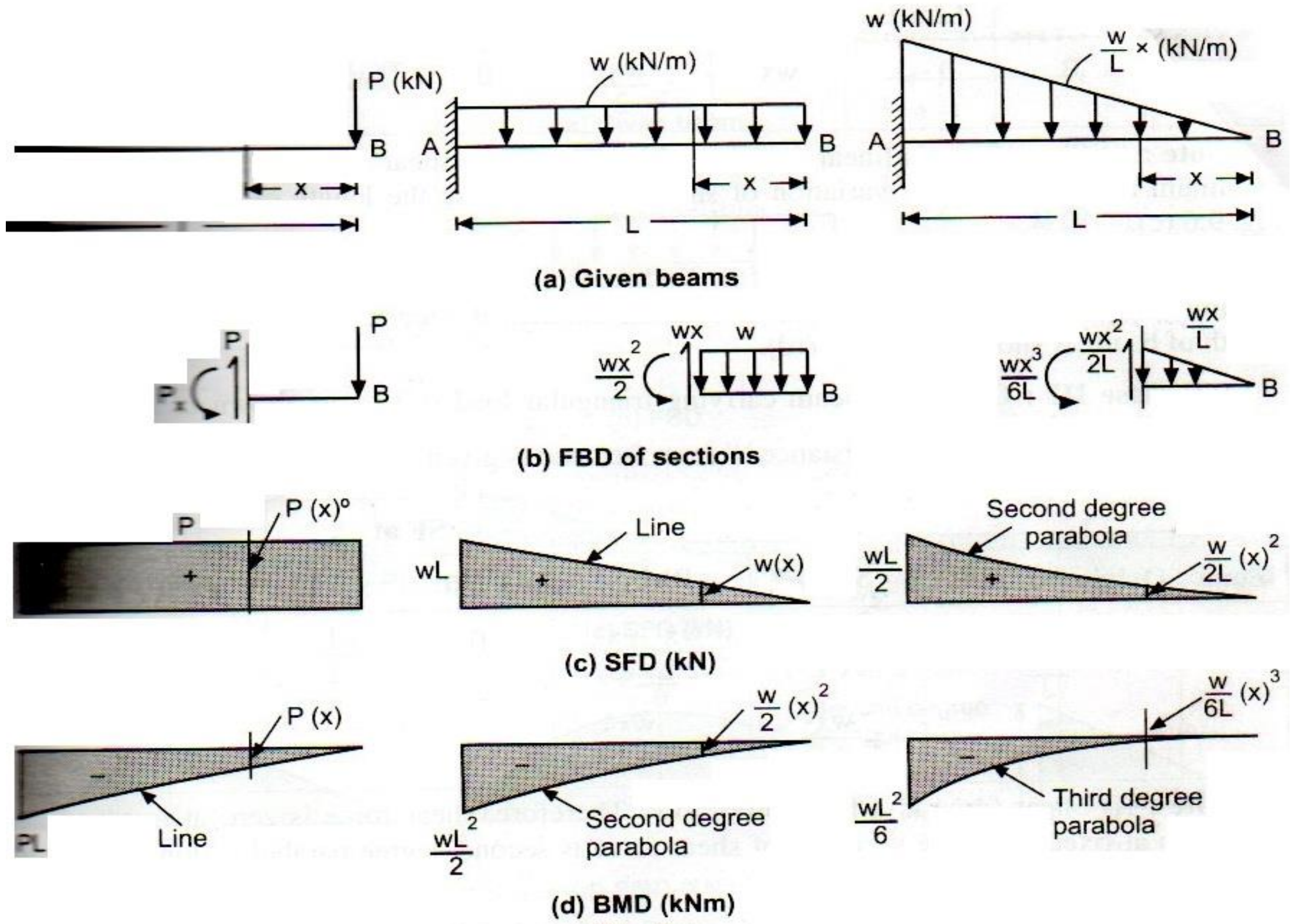


Fig. 9.6

Data : Give beams as shown in Fig. 9.6 (a)

Required : SFD and BMD

Concept : Consider a section at a distance ‘x’ from free end B as shown in Fig. 9.6(a) and write the equations of SF and BM for the respective beams.

Solution : (i) Case I : Cantilever beam with point load at free end.

					SF at		BM at	
Zone	Origin	Limits	SF (x)	BM (X)	X=0	X=L	X=0	X=L
BA	B	0-L	P	-PX	P	P	0	-PL

Note : (i) Shear force is independent of position of section defined by distance ‘x’. Therefore, shear force is constant throughout the beam; and SFD is as shown in Fig. 9.6(c).
(ii) Bending moment is linear function of ‘x’. Therefore, bending moment is zero at free end and maximum at fixed end. The variation of BM is linear along the length of beam as shown in Fig. 9.6(d)

(ii) Case II : Cantilever beam with UDL throughout the span.

					SF at		BM at	
Zone	Origin	Limits	SF (x)	BM (X)	X=0	X=L	X=0	X=L
BA	B	0 - L	Wx	$-\frac{wx^2}{2}$	0	WL	0	$-\frac{wL^2}{2}$

Note : (i) Shear force is linear function of ‘x’. Therefore, shear force is zero at free end and maximum at fixed end. The variation of shear is linear along the length of beam as shown in fig. 9.6(c) .

(ii) Bending moment is function of $(x)^2$. Therefore, bending moment is zero at free end and maximum at fixed end. The variation of bending moment is second degree parabolic along the length of beam as shown in Fig. 9.6(d).

(iii)Case III : Cantilever beam carrying triangular load (UVL) as shown inFig.9.6(a). Intensity of triangular load at a distance 'x' from free end is given by ------------------- from similar triangles.

					SF at		BM at	
Zone	Origin	Limits	SF $_{(x)}$	BM $_{(X)}$	X=0	X=L	X=0	X=L
BA	B	0 - L	$\frac{1}{2}\times\frac{wx}{L} = \frac{wx^2}{2L}$	$-\frac{wx^2}{L2}\frac{x}{3} = \frac{wx^2}{6L}$	0	$\frac{wL}{2}$	0	$-\frac{wL^2}{6}$

Note : (i) Shear force is a function of $(x)^2$. Therefore, shear force is zero at free end and maximum at fixed end. The variation of shear force is second degree parabolic along the length of beam as shown in Fig. 9.6(c).

(ii) Bending moment is a function of $(x)^3$. Therefore, bending moment is zero at free end and maximum at fixed end. The variation of bending moment is third degree parabolic along the length of beam as shownn Fig. 9.6(d) .**Example 9.2 :** Draw SF and BM diagrams for following simple beams

Data : As shown in Fig. 9.7(a).

Required : SFD and BMD.

Solution : (i) Case I : Simple beam with single point load.

$\sum M_A = 0$: $V_B \times -Pa = 0$ $\therefore$ $V_B = \frac{Pa}{L}$ (↑)

$\sum Fy = 0$; $V_A + V_B - P = 0$

$\therefore V_A = P - \frac{P}{L} = \frac{Pb}{L}$ (↑)

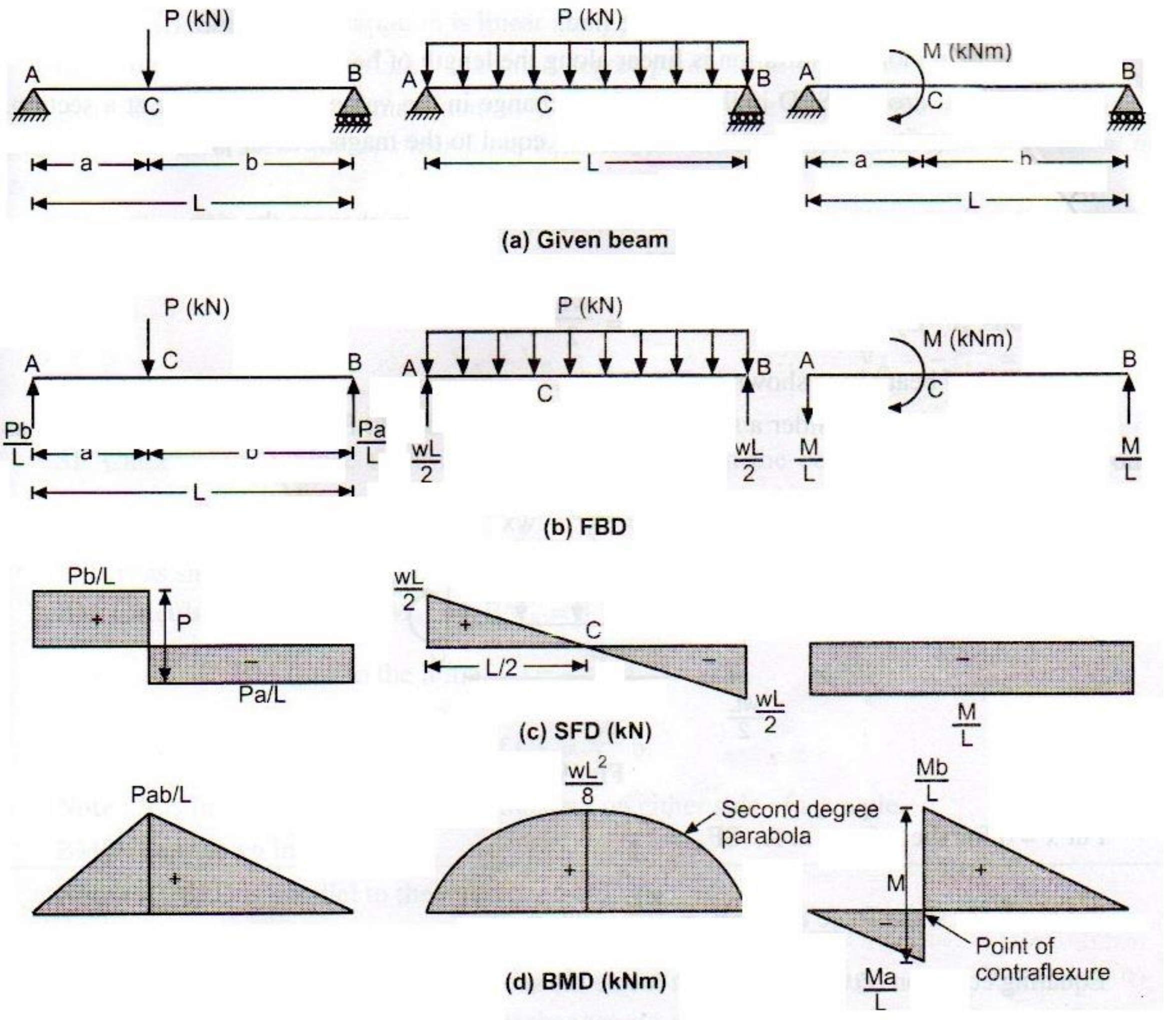


Fig. 9.7

SF Calculation :-

SF (Just to the left of C) $= \frac{Pb}{L}$

SF (Just to the Right of C) = $\frac{Pb}{L} - p = -\frac{Pb}{L}$

$SF_B = \frac{Pb}{L}$

Note : It is important to find shear force on either side of the concentrated force. SFD is as shown in Fig. 9.7(c).

BM Calculation :- $BM_A = BM_B = 0$

$BM_C = \frac{Pab}{L}$

BMD is as shown in Fig. 9.7(d)

Note : (i) SFD is parallel to the reference line.

(ii) Bending moment variation is linear along the length of beam.

(iii)Vertical drop in SFD indicates sudden change in the value of shear force at a section. The magnitude of drop in SF diagram is equal to the magnitude of point load acting at a section.

(iv)Bending moment is maximum at 'C' where, shear force changes the sign.

(ii) Case II : Simple beam with UDL.

Reaction : $V_A = V_B = \frac{wL}{2}$ (↑)

FBD of beam is as shown in fig 9.6(b)

SF Calculations : Consider a section at a distance 'x' from A; FBD of part of the beam is as shown in Fig 9.8.

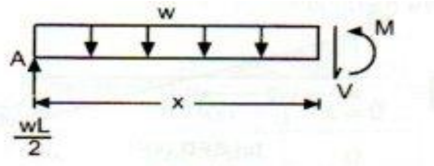


Fig. 9.8

Type equation here.

Put x =0 for shear force at A, $SF_A = \frac{wL}{2}$

Put x =L for shear force at B, $SF_B = \frac{wL}{2} - wL = \frac{-wL}{2}$

Equating equation (i) zero we can locate the section of shear force as,

$$\frac{wL}{2} - wx = 0 \quad \therefore \quad x = \frac{L}{2}$$

SFD is as a shown in fig 9.7(c)

BM Calculations : From Fig.9.8, BM at a distance x

$$= BM_x = M = \frac{wL}{2}(x) - \frac{wx^2}{2}$$

…(ii)

Put x = 0 for bending moment at A, $BM_A = 0$

Put x = L for bending moment at B, $BM_B = \frac{wL^2}{2} - \frac{wL^2}{2} = 0$

Put $x = \frac{L}{2}$ for BM at point of zero SF i.e. at C, $BM_C = \frac{wL^2}{2}\left(\frac{L}{2}\right) - \frac{w}{2}$

$$BM_c = \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{wL^2}{8}$$

BMD is as shown in Fig. 9.7(d).

Note : (i) Shear force variation is linear along the length of beam.

(ii) Bending moment variation is second degree parabolic along the length of beam.

(iii)Bending moment is maximum at C where shear force changes the sign. For locating the section of zero shear force, shear force equation for the respective zone shall be equated to zero.

(iii)Case III : Simple beam carrying a couple.

Reaction :

FBD of beam is as shown in Fig. 9.7(b).

SF Calculations : Since there is no vertical loading on the beam, shear force is constant throughout and SF = -

SFD is as shown in Fig. 9.7(c).

BM Calculations : $BM_A = BM_B = 0$

BM (just to the left of C) = -

BM (just to the right of C) =

Note : It is important to find bending moment on either side of a couple.

BMD is as shown in Fig. 9.7(d)

Note: (i) SFD is parallel to the reference line.

(ii) Bending moment variation is linear along the length of beam.

(iii)At a section where couple is acting, shear force does not change.

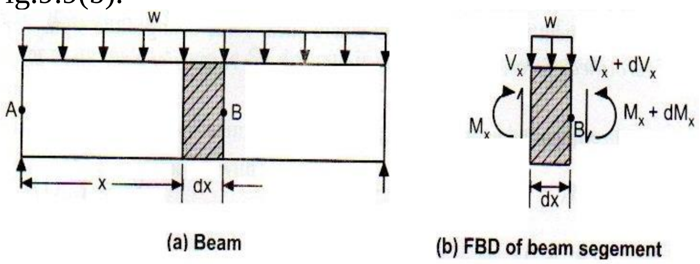
(iv)Section where bending moment changes its sign is called as point of contra flexure. Since change of sign is possible, only when bending moment at point of contra flexure is zero, it is also called as point of zero bending moment or point of inflection. It should be understood that a point of contra flexure is necessarily a point of inflection, but point of inflection need not be a point of contra flexure. For locating point of contraflexure, bending moment equation in the respective zone shall be equated to zero.

(v)Vertical drop in the BMD indicates sudden change in the value of bending moment at a section. The magnitude of drop in the BM diagram is equal to the magnitude of couple acting at a section.

9.8 RELATIONSHIP BETWEEN LOAD, SHEAR AND BENDING MOMENT

The relationship between load, shear and bending moment can be derived as under. Consider a beam supported and loaded as shown in Fig.9.9(a).

Let Vx and Mx be the shear and bending moment on a section at a distance 'x' from A. likewise Vx + dVx and Mx + dMx are the shear and bending moment on a section at a distance x + dx from A. the free body diagram of the segment of the beam which is dx in length is shown in Fig.9.9(b).



(a) Beam (b) FBD of beam segement

Fig. 9.9

Applying equations of equilibrium to FBD of segment as under :

Fy = 0 $\quad Vx – (Vx + dVx) – w . dx = 0$

$$dVx = - w . dx$$

Thus, the rate of decrease of shear force with respect to x, on any section at a distance 'x' from left end of the beam is equal to the intensity of load at the section.

$M_B = 0 \quad Vx . dx + Mx = Mx + dMx +$

The term w being very small; can be neglected.

$$dM_x = V_x . dx$$

Thus, the rate of increase of bending moment with respect to 'x', on any section at a distance 'x' from left end of the beam, is equal to shear at the section.

Example 9.3 : The beam is supported and loaded as shown in Fig. 9.10(a). Draw SFD and BMD indicating all the important values.

Data : As shown in Fig. 9.10(a)

Required : SFD, BMD.

Solution : (i) Reactions:

$\sum F_y = 0 \rightarrow =V_X -(V_X$

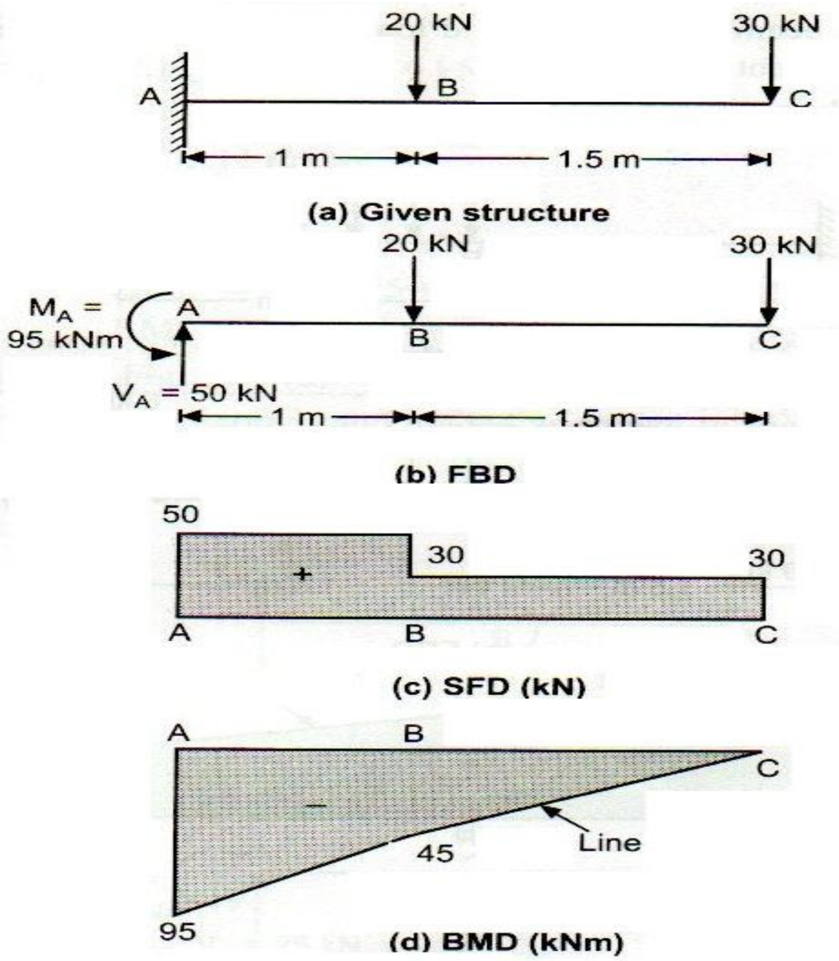


Fig. 9.10

(ii) SF calculation :

SF_A =50kN

SF_B (just to the left) =50kN

SF_B (just to the right) = 50-20=30kN

SF_c = 30kN

SFD Is as shown in fig. 9.10(c)

(iii) BM calculation

BM_A =-95kNm

BM_B = -30×1.5= - 45kNm

BM_c = 0

BMD is as shown Fig.9.10(d)

Example 9.4 : The beam is supported and loaded as shown in Fig.9.11(a). Draw SFD and BMD indicating all important values.

Data : As shown in Fig. 9.11(a).

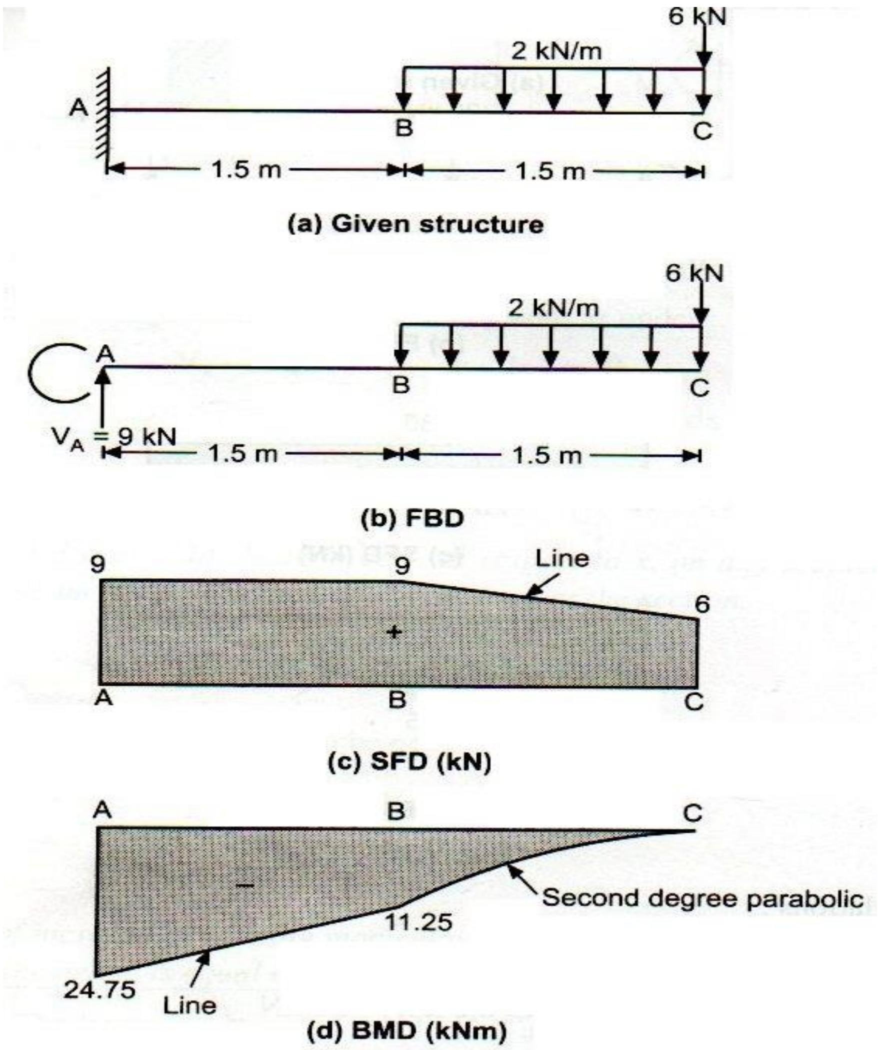


Fig. 9.11

Required : SFD, BMD.

Solution : (i) Reactions

$$V_A - 2 \times 1.5 - 6 = 0$$
$$V_A = 9\text{kN}$$
$$M_A - 2 \times 1.5 \times 2.25 - 6 \times 3 = 0$$
$$M_A = 24.75 \text{ kN.}_m$$
$$H_A = 0$$

FBD of beam is as shown in Fig.9.11(b).

(ii) SF calculations :

$$SF_A = SF_B = 9\text{kN}$$
$$SF_C = 6\text{kN}$$

SFD is as shown in Fig. 9.11(c).

(iii)BM calculations :

$$BM_A = -24.75\text{Kn.}_m$$

$BM_B = -24.75 + 9 \text{ X } 1.5 = -11.25 \text{ kN}_m$

$BM_C = 0$

BMD is as shown in Fig.9.11(d)

Example 9.5 : The beam is supported and loaded as shown in Fig. 9.12(a) . Draw SFD and

BMD indicating all the important values.

Data : As shown in Fig. 9.12(a).

$\sum F_y = 0$; $V_A - 2\times1.5 - 6 = 0$

$V_A = 9kN$ (↑)

$\sum M_A = 0$; $M_A - 2\times1.5\times2.25 - 6\times3 = 0$

$M_A = 24.75kN.m$ ( )

$\sum F_X = H_A = 0$

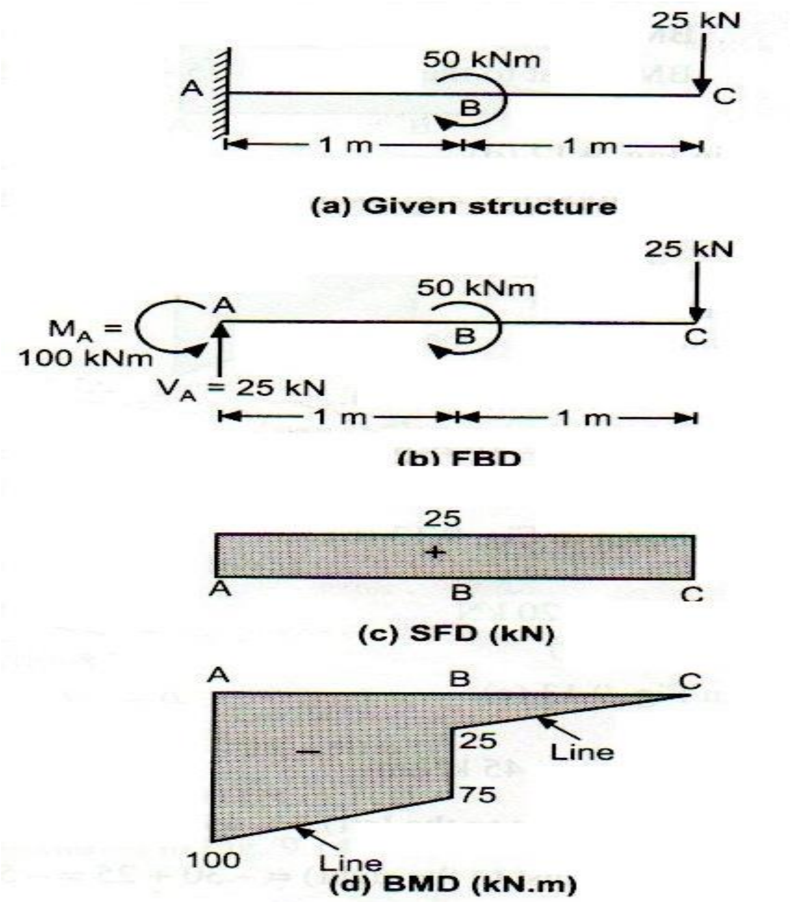


Fig. 9.12

Required : SFD, BMD.

Solution : (i) Reactions :

$F_y = 0$; $V_A - 25 = 0$ $V_A = 25kN$

$M_A = 0$; $M_A - 50 - 25x2 = 0$

$M_A = 100kN._m$

$F_x = 0$; $H_A = 0$

FBD of beam is as shown in Fig.9.12(b).

(ii) SF calculations :

$SF_A = SF_B = SF_c = 25kN$

SFD is as shown in Fig.9.12 (c).

(iii)BM calculations :

$BM_A = -100kN._m$

BM_B (just to the left) = - 100 + 25 x 1 = - 75kN.$_m$

BM_B (just to the right) = - 75 + 50 = - 25kN.$_m$

$BM_C = 0$

BMD is as shown in Fig. 9.12(d).

Example 9.6 : The beam is supported and loaded as shown in Fig. 9.13(a). Draw SFD and BMD indicating all the important values.

Data : As shown in Fig. 9.13(a).

Required : SFD, BMD.

Solution : (i) Reactions:

$F_y = 0$; V_A -10x2=0 V_A =20kN

M_A =0; M_A -25-10x =0 M_A =45kN.m

F_x =0 ; H_A =0

FBD of beam is as shown in Fig.9.13(b).

(ii) SF Calculations :

SF_A =20kN

SF_c =0

SFD is as shown in Fig.9.13(c) .

(iii)BM calculations :

BM_A =-45Kn.$_m$

BM_B (just to the left) =-45+20x1-10x = -30kN$_m$

BM_B (just to the right) = -30 + 25 = -5kN.$_m$

$BM_C = 0$

BMD is as shown in Fig. 9.13(d).

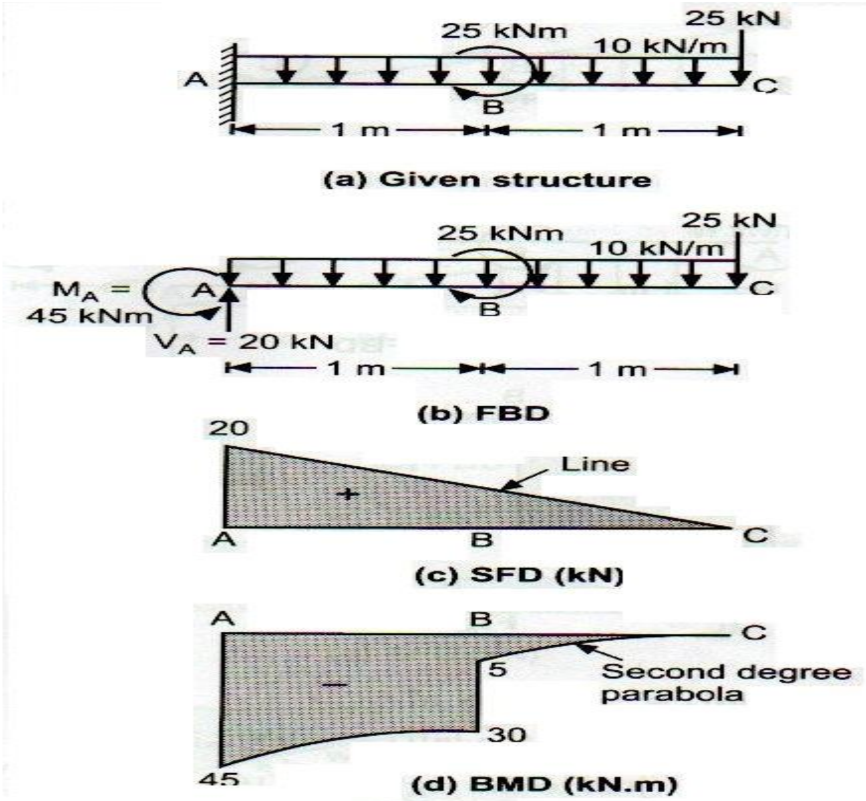


Fig. 9.13

Example 9.7 : The beam is supported and loaded as shown in Fig. 9.14 (a). Draw SFD and BMD indicating all the important values.

Data : As shown in Fig.9.14(a).

Required : SFD , BMD.

Solution : (i) Reactions :

$F_y = 0$; $V_A = 0$

$M_A = 0$; $M_A - 75 + 25 - 50 = 0$

$M_A = 100$ kN.m

FBD of beam is as shown in Fig. 9.14 (b).

(ii) SF calculations :

Beam is not subjected to any shear force.

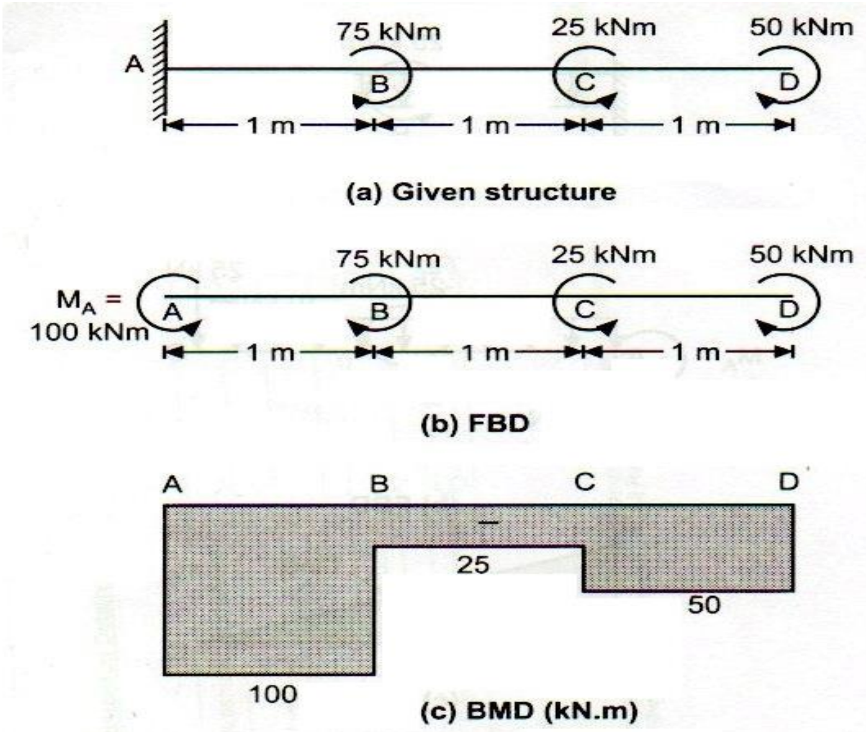


Fig. 9.14

(iii) BM calculation :

BM_A = -100kN.m

BM_B (just to the left) = 100kN.m

BM_c (just to the Right)= 100+75=-25kN.m

BM_C (just to the left)= 25kNm

BM_D (just to the Right)=-25-25=-50kN.m.

BMD is as shown in fig.9.14(c)

Example 9.8 : Draw SFD and BMD for the cantilever beam shown in Fig. 9.15(a).

Data : As shown in Fig.9.15(a).

Required : SFD , BMD.

Solution : (i) Reactions :

FBD of beam is as shown in Fig. 9.15 (b).

$\sum F_Y = 0$; $V_A - \frac{1}{2} \times w \times L = 0$ $\quad V_{A=} \frac{WL}{2}$

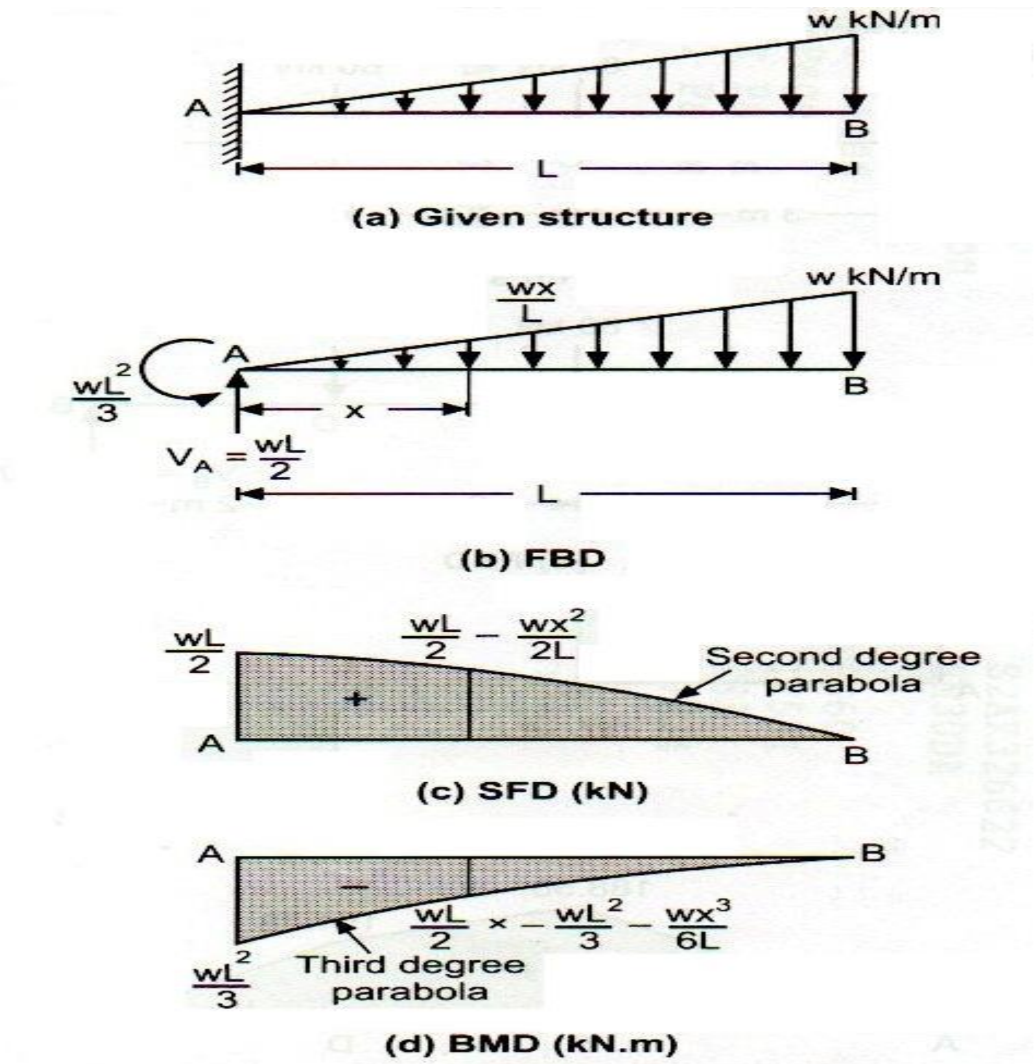


Fig. 9.15

SF and BM :

Zone	Origin	Limits	SF $_{(x)}$	BM $_{(X)}$	SF at X=0	SF at X=L	BM at X=0	BM at X=L
AB	A	0 - L	$\frac{wL}{2}-\frac{wx^2}{2L}$	$\frac{wL}{2}\times-\frac{wx^2}{2L}=\frac{wx^3}{6L}$	$\frac{wL}{2}$	0	$-\frac{wL^2}{6}$	0

SFD and BMD are as shown in Fig.9.15(c) and Fig 9.15(d) respectively.

UNIT-02 (Summary)

The algebraic sum of forces on any section is termed as shear force and algebraic sum of moment is termed as bending moment The shear force and bending moment can be calculated numerically at any particular section. But from the design point of view, we are interested in knowing the manner in which these values vary, along the length of beam. This can be done by plotting the shear force or the bending moment as ordinate and the position of the cross section as abscissa to give shear force diagram (SFD) and bending moment diagram (BMD) respectively. While plotting these diagrams, positive values are plotted above the reference line and negative values below it. The shear force and bending moment diagrams can be plotted from shear force and bending moment equations written for respective zones.

Exercise :

Q1. Define shear force and Bending moment ?

Q2. Draw the BM and SF for cantilever beam

Q3. What are the various application of SF and BM

Q4. Explain the BMD

Q5 Draw a neat sketch of SF for a concentrated load acting on Mid Span

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University, Nagpur

Department of Civil Engineering

Name of Subject: - Strength of Materials	**Subject Code:- BECVE302T**
Unit-III : Compound stresses	**Semester: - III**
Compound stresses Stress strain behaviour of ductile and brittle material in un axial state of stress, elastic, plastic and strain hardened zones stress-strain relations, Elastic constants ,relation between elastic constant, Uniaxial loading and deformation of simple cases of statically indeterminate problems under axial loading, temperature change etc.,	

Course Outcome (CO):- The students would be able to understand the ductile and brittle behavior of materials under different temperature

Learning Outcomes (LOs) :- (4 to 5 are expected and as per the COs)

- To make students learn and understand basic theories and behaviour of ductile and brittle materials
- To understand the basic relation between various elastic constant
- To Remember the basic concept of loading and deformation of simple cases
- To Analysis the basic loading condition, temperature changes
- To learn the concept of compound stresses

The elements of a force system acting at a section of a member are axial force, shear force and bending moment and the formulae for these force systems were derived based on the assumption that only a single force element is acting at the section.

Axial force Б = P/A where Б is the normal stresses

3.1 Three – dimensional stress system : Consider an element subjected to three mutually perpendicular tensile stresses σ_x, σ_y, and σ_z as shown in the figure below. 3.1

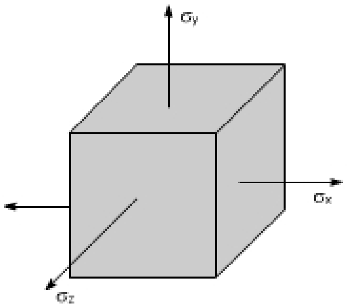


Fig 3.1 stress system

If σ_y, and σ_z were not present the strain in the x direction from the basic definition of Young's modulus of Elasticity E would be equal to

$б_x = \sigma_x / E$

The effects of σ_y, and σ_z in x direction are given by the definition of Poisson's ratio ' m ' to be equal as $-\mu\ \sigma_y/E$ and $-\mu\ \sigma_z/E$. The negative sign indicating that if σ_y, and σ_z are positive i.e. tensile, these they tend to reduce the strain in x direction thus the total linear strain is x direction is given by

$$\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

In the absence of shear stresses on the faces of the elements let us say that σ_x, σ_y, and σ_z are in fact the principal stress. The resulting strain in the three directions would be the principal strains.

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \mu\sigma_2 - \mu\sigma_3]$$

$$\epsilon_2 = \frac{1}{E}[\sigma_2 - \mu\sigma_1 - \mu\sigma_3]$$

$$\epsilon_3 = \frac{1}{E}[\sigma_3 - \mu\sigma_1 - \mu\sigma_2]$$

i.e. We will have the following relation.

3.1.1 Two – dimensional stress system :

system, the stress in the third direction becomes zero i.e σ_z

Although we will have a strain in this direction owing to stresses σ_1 and σ_2

$$\epsilon_1 = \frac{1}{E}[\sigma_1 - \mu\,\sigma_2]$$

$$\epsilon_2 = \frac{1}{E}[\sigma_2 - \mu\,\sigma_1]$$

$$\epsilon_3 = \frac{1}{E}[-\mu\,\sigma_1 - \mu\,\sigma_2]$$

Hence the set of equation as described earlier reduces to

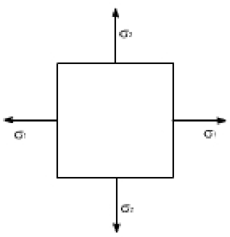


Fig: 3.2 stresses in two dimensional system

$$\text{i.e if } \sigma_3 = 0;\ \epsilon_3 = \frac{1}{E}[-\mu\sigma_1 - \mu\sigma_2]$$

Also

$$\epsilon_1 .E = \sigma_1 - \mu\sigma_2$$

$$\epsilon_2 .E = \sigma_2 - \mu\sigma_1$$

so the solution of above two equations yields

$$\boxed{\begin{aligned}\sigma_1 &= \frac{E}{(1-\mu^2)}[\epsilon_1 + \mu\epsilon_2] \\ \sigma_2 &= \frac{E}{(1-\mu^2)}[\epsilon_2 + \mu\,\epsilon_1]\end{aligned}}$$

3.2 Principal plane and principal stresses:
Analysis of Stresses:

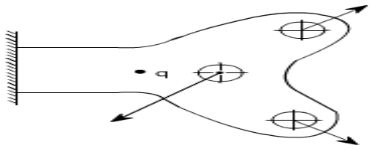


Fig : 3.3 Analysis of stresses

Consider a point ‘q' in some sort of structural member like as shown in figure below. Assuming that at point exist. ‘q' a plane state of stress exist. i.e. the state of state stress is to describe by a parameters σx, σy and τxy These stresses could be indicate a on the two dimensional diagram as shown below:

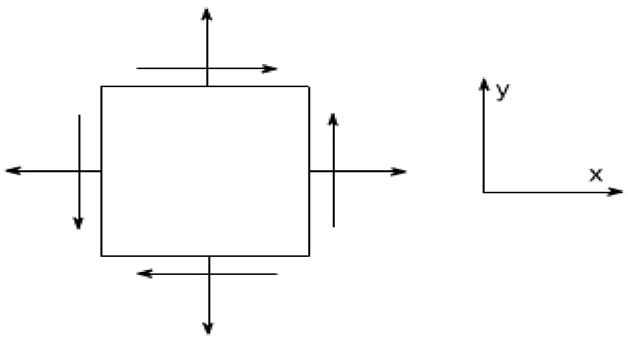


Fig : 3.4 stresses in dimensional diagram

This is a commen way of representing the stresses. It must be realize a that the material is unaware of what we have called the x and y axes. i.e. the material has to resist the loads irrespective less of how we wish to name them or whether they are horizontal, vertical or otherwise further more, the material will fail when the stresses exceed beyond a permissible value. Thus, a fundamental problem in engineering design is to determine the maximum normal stress or maximum shear stress at any particular point in a body. There is no reason to believe apriority that σx, σy and τxy are the maximum value. Rather the maximum stresses may associates themselves with some other planes located at ‘q'. Thus,
it becomes imperative to determine the values of σq and τq.

3.1.2 Stresses on oblique plane:

Till now we have dealt with either pure normal direct stressor pure shear stress. In many instances, however both direct and shear stresses acts and the resultant stress across any section will be neither normal nor tangential to the plane.

A plane stse of stress is a 2 dimensional state of stress in a sense that the stress components in one direction are all zero i.e

σ_z = τyz= τzx = 0

examples of plane state of stress includes plates and shells. Consider the general case of a bar

under direct load F giving rise to a stress σ_y
vertically

3.1.4 Material subjected to two mutually perpendicular direct stresses:

Now consider a rectangular element of unit depth, subjected to a system of two direct stresses both tensile, σ_x and σ_y acting right angles to each other.

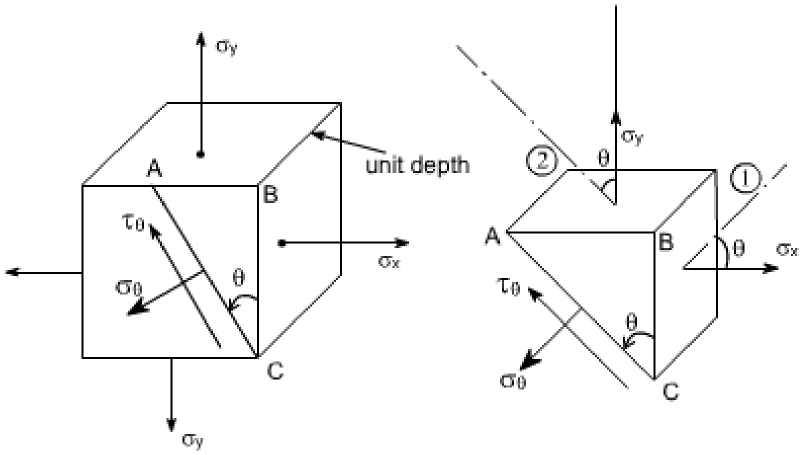


Fig : 3.8 direct stresses:

for equilibrium of the portion ABC, resolving perpendicular to AC

$s_q . AC.1 = s_y \sin q . AB.1 + s_x \cos q . BC.1$

converting AB and BC in terms of AC so that AC cancels out from the sides

$s_q = s_y \sin^2 q + s_x \cos^2 q$

Futher, recalling that $\cos^2 q - \sin^2 q = \cos 2q$ or $(1 - \cos 2q)/2 = \sin^2 q$

Similarly $(1 + \cos 2q)/2 = \cos^2 q$

Hence by these transformations the expression for s_q reduces to

$= 1/2 s_y (1 - \cos 2q) + 1/2 s_x (1 + \cos 2q)$

On rearranging the various terms we get

$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta \qquad (3)$$

Now resolving parallal to AC

s_q.AC.1= $-t_{xy}$..cosq.AB.1+ t_{xy}.BC.sinq.1

The – ve sign appears because this component is in the same direction as that of AC.

Again converting the various quantities in terms of AC so that the AC cancels out from the two sides.

$$\tau_\theta.AC.1 = [\tau_x \cos\theta \sin\theta - \sigma_y \sin\theta \cos\theta]AC$$

$$\tau_\theta = (\sigma_x - \sigma_y)\sin\theta\cos\theta$$

$$= \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta$$

$$\text{or} \quad \boxed{\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta} \qquad (4)$$

Conclusions :

The following conclusions may be drawn from equation (3) and (4)

(i) The maximum direct stress would be equal to σ_x or σ_y which ever is the greater, when q = 0^0 or 90^0

(ii) The maximum shear stress in the plane of the applied stresses occurs when q = 45^0

$$\tau_{max} = \frac{(\sigma_x - \sigma_y)}{2}$$

3.1.5 Material subjected to combined direct and shear stresses:

Now consider a complex stress system shown below, acting on an element of material.

The stresses σ_x and σ_y may be compressive or tensile and may be the result of direct forces or as a result of bending. The shear stresses may be as shown or completely reversed and occur as a result of either shear force or torsion as shown in the figure below:

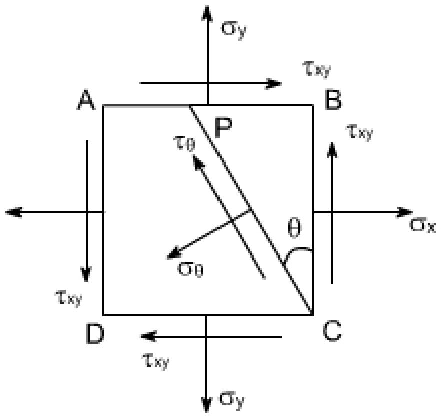


Fig : 3.9 Combined and direct stresses:

As per the double subscript notation the shear stress on the face BC should be notified as τ_{yx} , however, we have already seen that for a pair of shear stresses there is a set of complementary shear stresses generated such that $\tau_{yx} = \tau_{xy}$

By looking at this state of stress, it may be observed that this state of stress is combination of two different cases:

(i) Material subjected to pure stae of stress shear. In this case the various formulas deserved are as follows

$\sigma_q = \sigma_{yx}$ sin2 q

$\tau_q = - \tau_{yx}$ cos 2q

(ii) Material subjected to two mutually perpendicular direct stresses. In this case the various formula's derived are as follows.

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta$$
$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta$$

To get the required equations for the case under consideration, let us add the respective equations for the above two cases such tha
This eqn gives two values of 2q that differ by 1800 .Hence the planes on which maximum and minimum normal stresses occurate 900 apart.

For σ_θ to be a maximum or minimum $\frac{d\sigma_\theta}{d\theta} = 0$

Now

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\frac{d\sigma_\theta}{d\theta} = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta.2 + \tau_{xy}\cos 2\theta.2$$
$$= 0$$

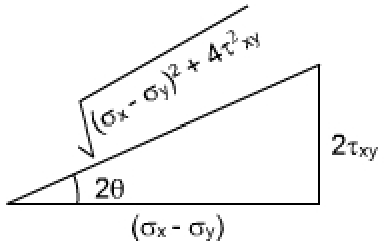
i.e. $-(\sigma_x - \sigma_y)\sin 2\theta + \tau_{xy}\cos 2\theta.2 = 0$

$\tau_{xy}\cos 2\theta.2 = (\sigma_x - \sigma_y)\sin 2\theta$

Thus, $\boxed{\tan 2\theta = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}}$

From the triangle it may be determined

$$\cos 2\theta = \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$
$$\sin 2\theta = \frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$



Substituting the values of cos2 q and sin2 q in equation (5) we get

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cdot\frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}} + \frac{\tau_{xy}.2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$= \frac{(\sigma_x + \sigma_y)}{2} + \frac{1}{2}\cdot\frac{(\sigma_x - \sigma_y)^2}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}} + \frac{1}{2}\frac{4\tau^2_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

or

$$= \frac{(\sigma_x + \sigma_y)}{2} + \frac{1}{2}\cdot\frac{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\cdot\frac{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$\sigma_\theta = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

Hence we get the two values of σ_θ, which are designated σ_1 as σ_2 and respectively,therefore

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\cdot\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

The σ_1 and σ_2 are termed as the principle stresses of the system.

Substituting the values of $\cos 2\theta$ and $\sin 2\theta$ in equation (6) we see that

$$\tau_\theta = \frac{1}{2}(\sigma_x - \sigma_y)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$= \frac{1}{2}(\sigma_x - \sigma_y)\frac{2\tau_{xy}}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}} - \frac{\tau_{xy}.(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}}$$

$$\tau_\theta = 0$$

This shows that the values oshear stress is zero on the principal planes.

Hence the maximum and minimum values of normal stresses occur on planes of zero shearing stress. The maximum and minimum normal stresses are called the principal stresses, and the planes on which they act are called principal plane the solution of equation

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

will yield two values of 2q separated by 1800 i.e. two values of q separated by 900 .Thus the two principal stresses occur on mutually perpendicular planes termed principal planes. Therefore the two – dimensional complex stress system can now be reduced to the equivalent system of principal stresses.

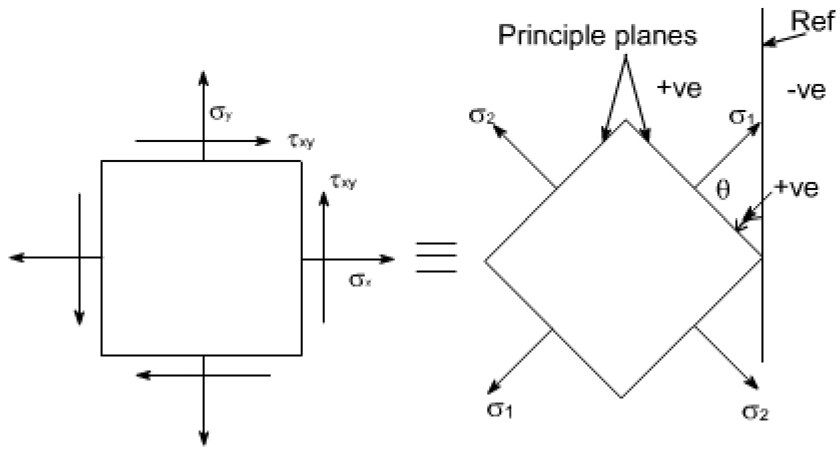
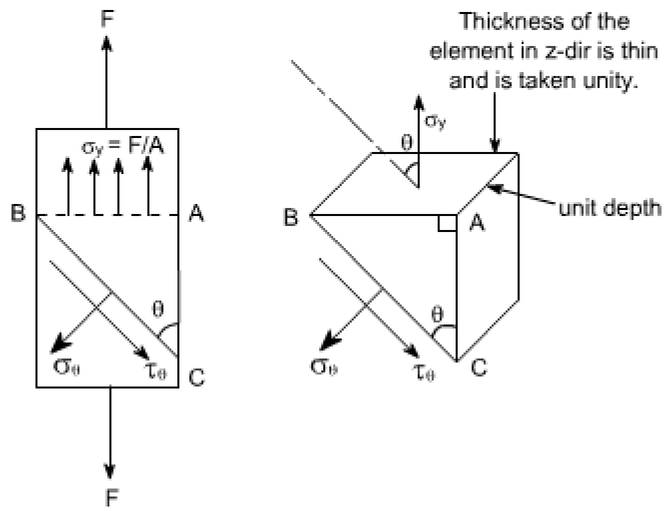


Fig : 3.10 Principal planes



ILLUSRATIVE PROBLEMS:

Example 3.1: A circular bar 40 mm diameter carries an axial tensile load of 105 kN. What is the Value of shear stress on the planes on which the normal stress has a value of 50

MN/m^2 tensile.

Solution: Tensile stress $\sigma_y = \frac{F}{A} = \frac{105 x 10^3}{\frac{\pi}{4} x 40^2}$

$$= 83.55 MN/m^2$$

Now the normal stress on an obliqe plane is given by the relation

$\sigma_q = \sigma_y \sin^2 q$

$50 \text{ x } 10^6 = 83.55 \text{ x } 10^6 \sin^2 q$

$q = 50^0 68'$

The shear stress on the oblique plane is then given by

$\tau_q = 1/2 \ \sigma_y \sin 2q$

$= 1/2 \text{ x } 83.55 \text{ x } 10^6 \text{ x } \sin 101.36$

$= 40.96 \text{ MN/m}^2$

Therefore the required shear stress is 40.96 MN/m^2

Example 3.2 : For a given loading conditions the state of stress in the wall of a cylinder is expressed as follows:

(a) 85 MN/m^2 tensile
(b) 85 MN/m^2 tensile at right angles to (a)
(c) Shear stresses of 60 MN/m^{2}on the planes on which the stresses (a) and (b) act; the shear couple acting on planes carrying the 25 MN/m^{2}stress is clockwise in effect.

Calculate the principal stresses and the planes on which they act. What would be the effect on these results if owing to a change of loading (a) becomes compressive while stresses (b) and (c) remain unchanged

Solution:
The problem may be attempted both analytically as well as graphically. Let us first obtain the analytical solution

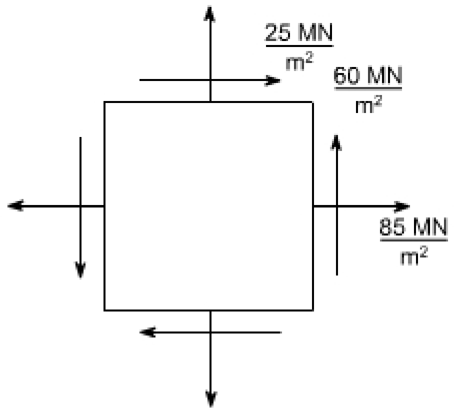


Fig : 3.12 Principal planes

The principle stresses are given by the formula

$$
\begin{aligned}
&\sigma_1 \text{ and } \sigma_2 \\
&= \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}} \\
&= \frac{1}{2}(85 + 25) \pm \frac{1}{2}\sqrt{(85 + 25)^2 + (4 \times 60^2)} \\
&= 55 \pm \frac{1}{2}.60\sqrt{5} = 55 \pm 67 \\
&\Rightarrow \sigma_1 = 122 \text{ MN/m}^2 \\
&\quad \sigma_2 = -12 \text{ MN/m}^2 (\text{compressive})
\end{aligned}
$$

For finding out the planes on which the principle stresses act us the equation

$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

The solution of this equation will yeild two values q i.e they q_1 and q_2 giving $q_1=31^0 71'$ and $q_2 = 121^0 71'$

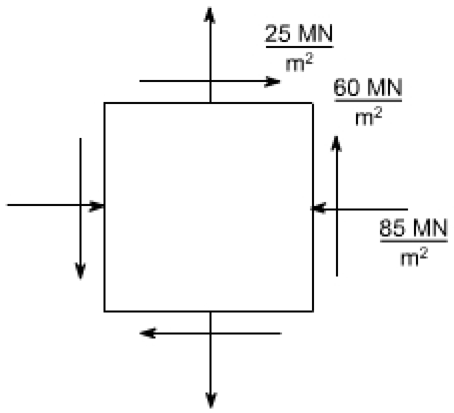


Fig : 3.13 Principal planes and stresses

Again the principal stresses would be given by the equation.

$$\sigma_1 \,\&\, \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy}}$$

$$= \frac{1}{2}(-85 + 25) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= \frac{1}{2}(-60) \pm \frac{1}{2}\sqrt{(-85 - 25)^2 + (4 \times 60^2)}$$

$$= -30 \pm \frac{1}{2}\sqrt{12100 + 14400}$$

$$= -30 \pm 81.4$$

$$\sigma_1 = 51.4 \text{ MN/m}^2;\ \sigma_2 = -111.4 \text{ MN/m}^2$$

Again for finding out the angles use the following equation.

$$\tan 2\theta = \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)$$

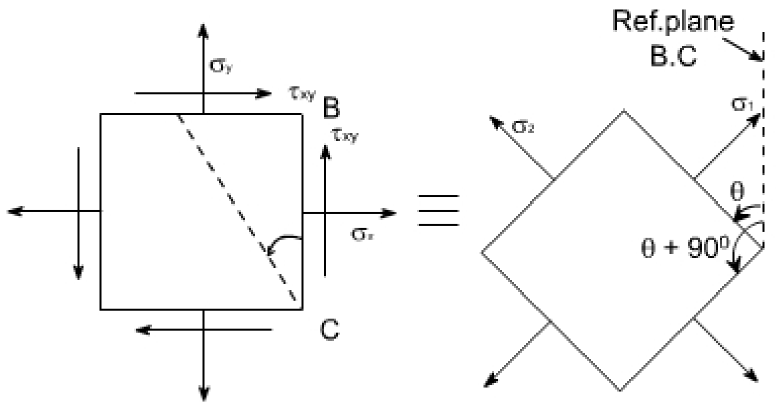
$$= \frac{2 \times 60}{-85 - 25} = +\frac{120}{-110}$$

$$= -\frac{12}{11}$$

$$2\theta = \tan\left(-\frac{12}{11}\right)$$

$$\Rightarrow \theta = -23.74^0$$

Thus, the two principle stresses acting on the two mutually perpendicular planes i.e principle planes may be depicted on the element as shown below:



3.4.3 ALITER : Let a cuboid of material having initial sides of Length x, y and z. If under some load system, the sides changes in length by dx, dy, and dz then the new volume (x+ dx)
(y +dy) (z +dz)
New volume = xyz + yzdx + xzdy + xydz
Original volume = xyz
Change in volume = yzdx +xzdy + xydz
Volumetric strain = (yzdx +xzdy + xydz) / xyz = $б_x + б_y + {}_z$
Neglecting the products of epsilon's since the strains are sufficiently small.
Volumetric strains in terms of principal stresses:

As we know that

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Futher Volumetric strain $= \epsilon_1 + \epsilon_2 + \epsilon_3$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{E} - \frac{2\mu(\sigma_1 + \sigma_2 + \sigma_3)}{E}$$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}$$

hence the

$$\boxed{\text{Volumetric strain} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}}$$

3.5 PRINCIPAL STRAIN

For the strains on an oblique plane we have an oblique we have two equations which are identical in form with the equation defining the direct stress on any inclined plane q .

$$\epsilon_\theta = \left\{\frac{\epsilon_x + \epsilon_y}{2}\right\} + \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right]$$

Since the equations for stress and strains on oblique planes are identical in form, so it isevident that Mohr's stress circle construction can be used equally well to represent strain conditions using the horizontal axis for linear strains and the vertical axis for half the shear strain. It should be noted, however that the angles given by Mohr's stress circle refer to thedirections of the planes on which the stress act and not the direction of the stresses themselves. The direction of the stresses and therefore associated strains are therefore normal (i.e. at 90^0) to the directions of the planes. Since angles are doubled in Mohr's stress circleconstruction it follows therefore that for a true similarity of working a relative rotation of
axes of 2 x $90^0 = 180^0$ must be introduced. This is achieved by plotting positive sheer strains vertically downwards on the strain circle construction.

The sign convention adopted for the strains is as follows:

Linear Strains : extension - positive
compression - negative

{ Shear of strains are taken positive, when they increase the original right angle of anunstrained element. }

Shear strains : for Mohr's strains circle sheer strain γ_{xy} - is +ve referred to x - direction the convention for the shear strains are bit difficult. The first subscript in the symbol g usually denotes the shear strains associated with direction. e.g. in γ_{xy} - represents the shear strain in x - direction and for γ_{xy} – represents the shear strain in y - direction. If under strain the line associated with first subscript moves counter clockwise with respect to the other line, the shearing strain is said to be positive, and if it moves clockwise it is said to be negative.

N.B: The positive shear strain is always to be drown on the top of σ_x .If the shear stain γ_{xy} is given .

Moh's strain circle

For the plane strain conditions can we derivate the following relations γ_{xy}

$$\epsilon_\theta = \left\{\frac{\epsilon_x + \epsilon_y}{2}\right\} + \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta \qquad (1)$$

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right] \qquad (2)$$

Re writing the equation (1) as below :

$$\left[\epsilon_\theta - \left(\frac{\epsilon_x + \epsilon_y}{2}\right)\right] = \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta \qquad (3)$$

squaring and adding equations (2) and (3)

$$\left[\epsilon_\theta - \left(\frac{\epsilon_x + \epsilon_y}{2}\right)\right]^2 + \left\{\frac{1}{2}\gamma_\theta\right\}^2 = \left[\left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta\right]^2 + \left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right]^2$$

$$\left[\epsilon_\theta - \left(\frac{\epsilon_x + \epsilon_y}{2}\right)\right]^2 + \left\{\frac{1}{2}\gamma_\theta\right\}^2 = \left(\frac{\epsilon_x + \epsilon_y}{2}\right)^2 + \frac{\gamma^2_{xy}}{4}$$

Now as we know that

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\epsilon_1 + \epsilon_2 = \epsilon_x + \epsilon_y$$

$$\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)^2 = \left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \frac{\gamma^2_{xy}}{4}$$

Therefore the equation gets transformed to

$$\left[\epsilon_\theta - \left(\frac{\epsilon_1 + \epsilon_2}{2}\right)\right]^2 + \left[\frac{\gamma_\theta}{2}\right]^2 = \left(\frac{\epsilon_1 - \epsilon_2}{2}\right)^2 \qquad (4)$$

If we plot equation (4) we obtain a circle of radius $\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)$ with center at $\left(\frac{\epsilon_1 + \epsilon_2}{2}, 0\right)$

A typical point P on the circle given the normal strain and half the sheer strain ½ γ_{xy} associated with a particular plane. We note again that an angle subtended at the centre of Mohr's circle by an arc connecting two points on the circle is twice the physical angle in the material.

Mohr strain circle :

Since the transformation equations for plane strain are similar to those for plane stress, we can employ a similar form of pictorial representation. This is known as Mohr's strain circle.

The main difference between Mohr's stress circle and stress circle is that a factor of half is attached to the shear strains.

Unit No 03 (Summary)

Simple stresses mean only tensile stress or compressive stress or only shear stress. Tensile and compressive stresses act on a plane normal to the line of action of these stresses and shear stress acts on a plane parallel to the line of action of this stress.

But when a plane in a strained body is oblique to the applied external force this plane may be subjected to tensile or compressive stress and shear stress. i.e.; Such a system or a plane in which direct or normal stresses and shear stresses act simultaneously are called compound stress or complex stress.

This is a common way of representing the stresses. It must be realize a that the material is unaware of what we have called the x and y axes. i.e. the material has to resist the loads irrespective less of how we wish to name them or whether they are horizontal, vertical or otherwise further more,
the material will fail when the stresses exceed beyond a permissible value. Thus, a fundamental problem in engineering design is to determine the maximum normal stress or maximum shear stress at any particular point in a body

Exercise :

Q1. Define simple and compound stresses ?

Q2. Explain Hooks law with example

Q3. What are the various application of Compound stresses

Q4. Define Intensity of load

Q5 Draw a neat sketch of for a concentrated load acting on Mid Span

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Department of Civil Engineering

Name of Subject: - Strength of Materials	**Subject Code:- BECVE302T**
Unit-IV : Torsion	**Semester: - III**
Torsion of Circular Shaft : Torsion of circular section, assumptions and derivation of relations between torsional moment, shear stress and angle of twist, Torsional stress in solid and circular sections, Introduction to Torsion in rectangular section, Torsion in thin walled hollow section	

Course Outcome (CO):- The students would be able to understand the torque and torsional moment of circular shaft

Learning Outcomes (LOs) :- (4 to 5 are expected and as per the COs)

- To make students learn and apply basic theories and concepts of torque
- To understand the basic fundamentals of torsional moment
- To Remember the basic concept of torsion in thin walled section
- To utilize the basic lows of circular shaft section
- To learn about shear stresses and angle of twist

7.1 Introduction

A member is said to be in torsion when it is subjected to a moment about its axis. Figure 7.1 show a shaft in torsion. The effect of torsional moment on the member is to twist it and hence a torsional moment is also called as twisting moment or torque.

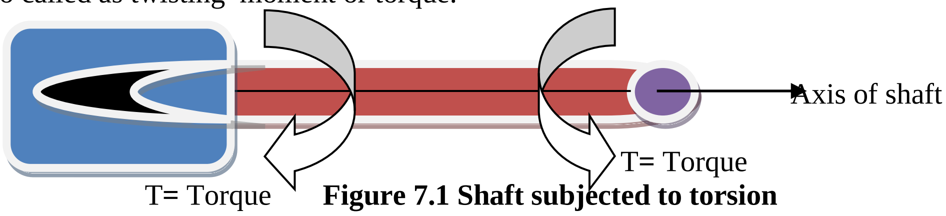
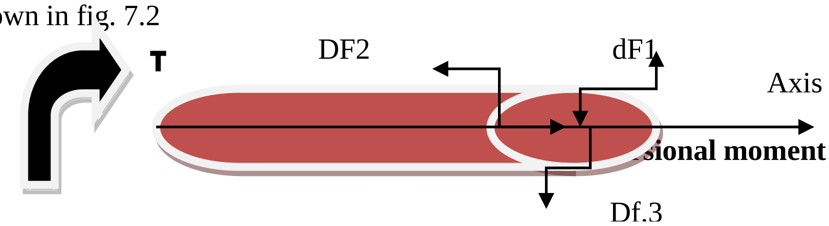


Figure 7.1 Shaft subjected to torsion

In engineering problems, many member are subjected to torsion. Shafts transmitting power from engine to the rear axle of automobile, from a motor to machine too, and from a turbine to electric motors are the common example of members in torsion. Ring beam of circular water tanks, and beam of grid flooring system are also example of member in torsion. In this chapter is we will see how this torsional moment is resisted by the members. A major part of this chapter is devoted to members with solid circular sections or a hollow circular sections. A small portion is devoted to members with non-circular section with an intention of cautioning the engineering students that the formulae developed in the earlier part cannot be applied to non-circular sections blindly.

7.2 PURE TORSION

A member is said to be in pure torsion when its cross-sections are subjected to only torsional moments and not accompanied by axial forces or bending moment. Now consider the section of a shaft under pure torsion as shown in fig. 7.2



Internal forces develop so as to counteract this torsional moment. Hence, at any element, the force *df* developed is in the direction normal to radial direction. This force is obviously shearing force and thus the elements are in pure shear. If dA is the area of the element at a distance from the axis of shaft and q is the shearing stress, then,

$dF = qdA$

$dT = dF \times r$

7.3 ASSUMPTIONS IN THE THEORY OF PURE TORSION

In the theory of pure torsion expressions will be derived for determining shear stress and the effect of torsional moment on cross-section , i.e ,in finding the angle of twist .In developing this theory the following assumptions are made.

The material is homogeneous and isotropic.

The stresses area within the elastic limit *i.e* shear stress is proportional to shear strain.Cross-section which are plane before applying twisting moment remain plane even after application of twisting moment i.e, no warping takes place.

Radial lines remain redial even after applying torsional moment.

The twist along the shaft is uniform.

7.4 Torsion of circular shafts

Definition of Torsion: Consider a shaft rigidly clamped at one end and twisted at the other end by a torque T = F.d applied in a plane perpendicular to the axis of the bar such a shaft is said to be in torsion.

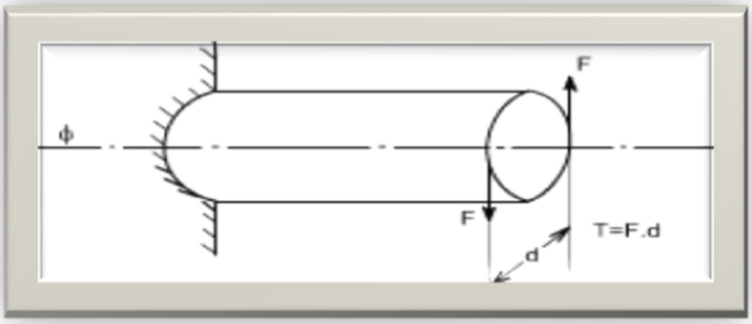


Fig 7.3 Shaft rigidly clamped

Effects of Torsion: The effects of a torsional load applied to a bar are To impart an angular displacement of one end cross – section with respect to the other end and setup shear stresses on any cross section of the bar perpendicular to its axis.

GENERATION OF SHEAR STRESSES

Fig 7.4 The physical understanding of the phenomena of setting up of shear stresses in a shaft subjected to a torsion may be understood from the figure 7.4

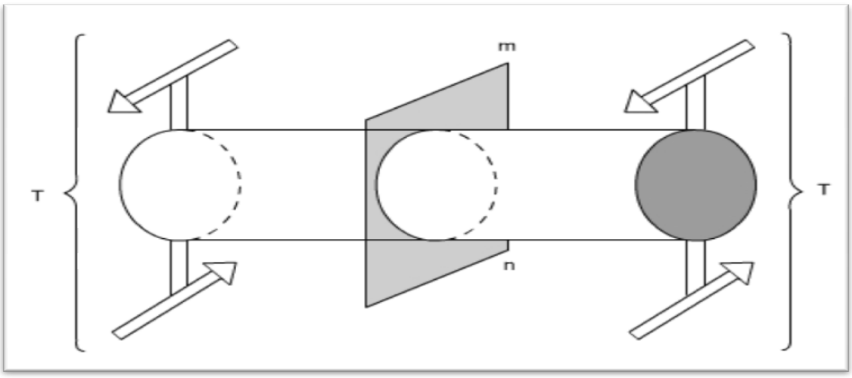


Fig 7.4 Shear stresses in shaft

Fig 7.5: Here the cylindrical member or a shaft is in static equilibrium where T is the resultant external torque acting on the member. Let the member be imagined to be cut by some imaginary plane mn.

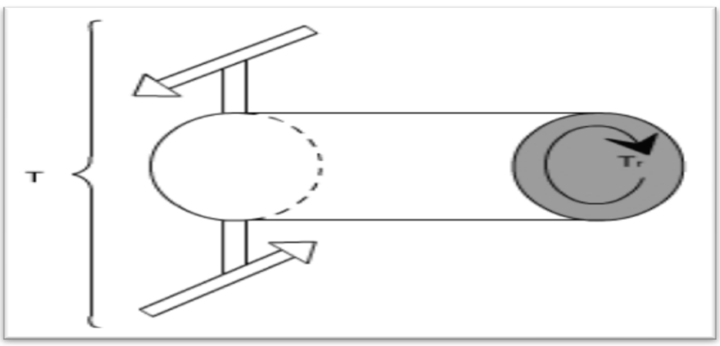


Fig 7.5 Shaft in static equilibrium

Fig 7.6: When the plane 'mn' cuts remove the portion on R.H.S. and we get a new fig. Now since the entire member is in equilibrium, therefore, each portion must be in equilibrium. Thus, the member is in equilibrium under the action of resultant external torque T and developed resisting Torque T_r .

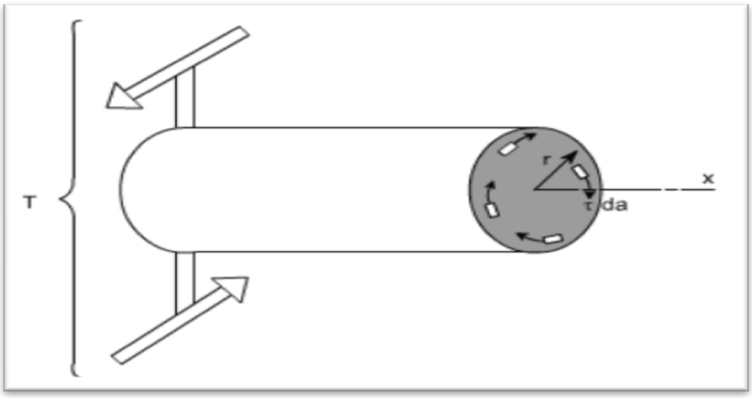


Fig 7.6: Entire member is in equilibrium,

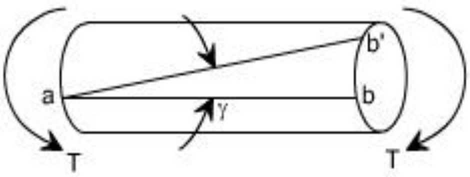
Fig 7.7: The Figure shows that how the resisting torque T_r is developed. The resisting torque T_r is produced by virtue of an infinites shear forces acting on the plane perpendicular to the axis of the shaft. Obviously such shear forces would be developed by virtue of sheer stresses.

Therefore we can say that when a particular member (say shaft in this case) is subjected to a torque, the result would be that on any element there will be shear stresses acting. While on other faces the complementary sheer forces come into picture. Thus, we can say that when a member is subjected to torque, an element of this member will be subjected to a state of pure shear.

Shaft: The shafts are the machine elements which are used to transmit power in machines.

Twisting Moment: The twisting moment for any section along the bar / shaft is defined to be the algebraic sum of the moments of the applied couples that lie to one side of the section under consideration. The choice of the side in any case is of course arbitrary.

Shearing Strain: If a generator a – b is marked on the surface of the unloaded bar, then after the twisting moment 'T' has been applied this line moves to ab'. The angle 'γ' measured in radians, between the final and original positions of the generators is defined as the shearing strain at the surface of the bar or shaft. The same definition will hold at any interior point of the bar



Modulus of Elasticity i ie shear strain is called the modulus of elasticity in shear OR Modulus of Rigidity and in represented by the symbol $G = \tau / r$

Angle of Twist: If a shaft of length L is subjected to a constant twisting moment T along its length, than the angle θ through which one end of the bar will twist relative to the other is known is the angle of twist.

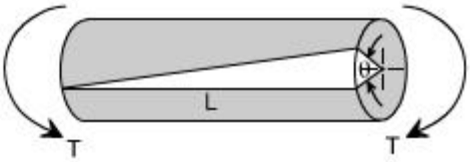


Fig 7.9 Twist angle

Despite the differences in the forms of loading, we see that there are number of similarities between bending and torsion, including for example, a linear variation of stresses and strain with position. In torsion the members are subjected to moments (couples) in planes normal to their axes. For the purpose of designing a circular shaft to withstand a given torque, we must develop an equation giving the relation between twisting moment, maximum shear stress produced, and a quantity representing the size and shape of the cross-sectional area of the shaft.

Not all torsion problems, involve rotating machinery, however, for example some types of vehicle suspension system employ torsional springs. Indeed, even coil springs are really curved members in torsion as shown in figure. 7.10

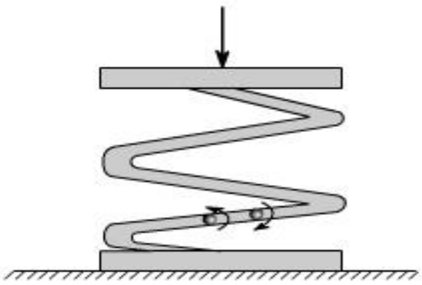


Fig 7.10 Coil Spring

- Many torque carrying engineering members are cylindrical in shape. Examples are drive shafts, bolts and screw drivers.

7.5 Simple Torsion Theory or Development of Torsion Formula : Here we are basically interested derive an equation between the relevant parameters

Relation in torsion : $\frac{T}{J} = \tau'/r = G\theta/L$

1 st Term: It refers to applied loading ad a property of section, which in the instance is the polar second moment of area.

2 nd Term: This refers to stress, and the stress increases as the distance from the axis increases.

3 rd Term: it refers to the deformation and contains the terms modulus of rigidity & combined term (τ) which is equivalent to strain for the purpose of designing a circular shaft to with stand a given torque we must develop an equation giving the relation between Twisting moments max shear stain produced and a quantity representing the size and shape of the cross – sectional area of the shaft.

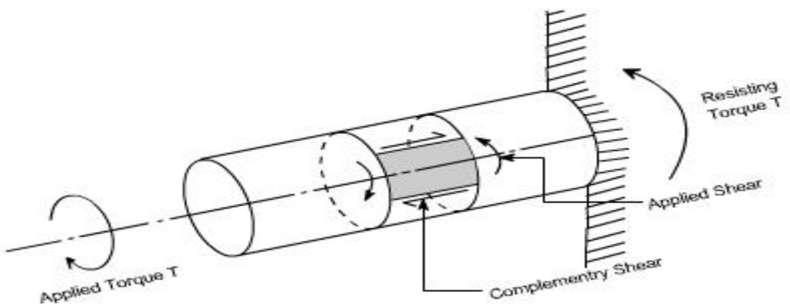


Fig 7.11 Applied Torque on shaft

Refer to the figure shown above fig 7.11 where a uniform circular shaft is subjected to a torque it can be shown that every section of the shaft is subjected to a state of pure shear, the moment of resistance developed by the shear stresses being everywhere equal to the magnitude, and opposite in sense, to the applied torque.

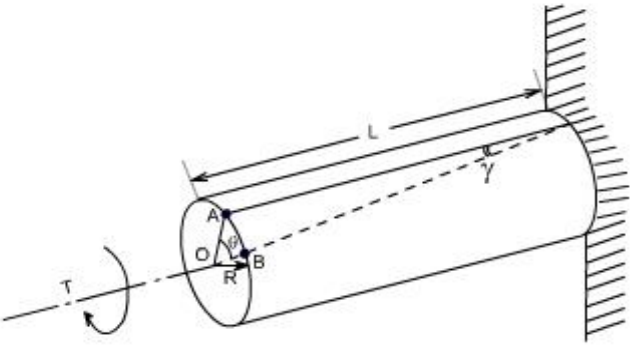


Fig 7.12 Uniform circular shaft

Consider now the solid circular shaft of radius R subjected to a torque T at one end, the other end being fixed Under the action of this torque a radial line at the free end of the shaft twists through an angle θ , point A moves to B, and AB subtends an angle ‘ γ ' at the fixed end. This is then the angle of distortion of the shaft i.e the shear strain. (Refer fig 7.12)

Since angle in radius = arc / Radius

arc AB = $R\theta$

$=L\gamma$[since L and γ also constitute the arc AB]

Thus, $\gamma = R\theta / L$ (1)

From the definition of Modulus of rigidity or Modulus of elasticity in shear

G = Shear stress (τ)/ Shear Strain (γ)

Where the (γ) is the shear stress up at radius R

Then $\tau / G = \gamma$

Equating eq 1 and 2 we get $R\theta / L = \tau / G$

$\tau / R = G\,\theta / L$ $\{ = \tau'/r \}$ where τ' is the shear stress at any radius r

Stresses: Let us consider a small strip of radius r and thickness dr which is subjected to shear stress τ'.

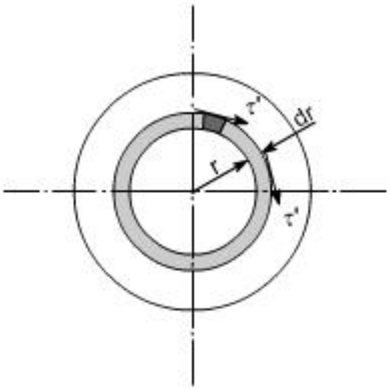


Fig 7.13 stresses

The force set up on each element

= stress x area

τ' x 2π r dr (approximately)

This force will produce a moment or torque about the center axis of the shaft.

$= \tau' . 2 \pi r dr . r$

$= 2 \pi \tau' . r^2 . dr$

$$T = \int_0^R 2\pi\tau' r^2 dr$$

The total torque T on the section, will be the sum of all the contributions.

Since τ' is a function of r, because it varies with radius so writing down τ' in terms of r from the equation (1).

.i e $\tau' = G \theta r/L$ we get $T = \int_0^R 2\pi \frac{G\theta}{L} . r3\, dr$

$T = 2\pi G\theta /L$ $[R^4 /4]$ with limit 0 to 4

0n solving we get $G\theta/L \times \pi R^4 /2$

Substitute $R = d/2$

We get $G \theta /L$ $. \pi d4/32 =$ J Polar moment of inertia or $T/J = G\theta/L$

From Eq 1 and 2 we can write $\frac{T}{J} = \tau'/r = G\theta/L$

Where

T = applied external Torque, which is constant over Length L;

J = Polar moment of Inertia

$$= \frac{\pi d^4}{32} \text{ for solid shaft}$$

$$= \frac{\pi (D^4 - d^4)}{32} \text{ for a hollow shaft.}$$

D = Outside diameter ; d = inside diameter]

G = Modules of rigidity (or Modulus of elasticity in shear)

θ = It is the angle of twist in radians on a length L.

Tensional Stiffness: The tensional stiffness k is defined as the torque per radius twist

i.e, $k = T / \theta = GJ / L$

7.6 Power Transmitted by a shaft : If T is the applied Torque and ω is the angular velocity of the shaft, then the power transmitted by the shaft is

$$P = T.\omega = \frac{2\pi NT}{60} = \frac{2\pi NT}{60.10^3} \text{kw}$$

where N= rpm

Numericals

7.1. A solid circular shaft of 100mm dia is transmitting power of 50kw at 150rpm. Determine the maximum sheer stress and the angle of twist, if the length of staft is 2.5m, G = $0.8X10^5$ N/mm the length of shaft is 2.5m.

P = 50 kw

N = 150 rpm

Fs = ?

Q = ?

L = 2.5 m, G =$8 \text{ X } 10^5$ N/mm^2.

1) $P = \frac{2\pi NT}{60,000}$

$$50 = \frac{2 \text{ X } \pi \text{ X } 150 \text{ X } T}{60,000}$$

T = 3183.09 Nm

i.e

T = $3183.09 \text{ x } 10^3$ Nmm

2) $Ip = \frac{\pi}{32} D^4$

$= \frac{\pi}{32}(100)^4$

$=9.81 \text{x} 10^6$ mm^6

$$\frac{T}{Ip} = \frac{Fs}{R} \longrightarrow \text{Strength criteria}$$

$= \frac{3183.09 \times 10^3}{9.81 \times 10^6}$

Fs = 16.22 N/mm^2

$\frac{T}{Ip} = \frac{GQ}{L}$ ⟶ Stiffness criteria

$\frac{3183.09 \times 10^3}{9.81 \times 10^6} = \frac{8 \times 10^5 \times Q}{2500}$

Q = 0.010 rad

7.2. Determine the diameter of a solid shaft which will transmit 440 kW at 280 rpm. The angle of twist should not exceed one degree per metre length and the maximum torsional shear stress is to be limited to 40 N/mm^2. Assume G=84 kN/mm^2.

Solution-

P= 440 kw

=440 x 10^3 N-m/s

= 440 x 10^6 N-m/s

N= 280 rpm

$\boldsymbol{\theta}=1^o = \frac{\pi}{180}$ red

L=1 m= 1000 mm

Q_{s} = 40 N/mm^2

G=84 x 10^3 N/mm^2

$P=\frac{2\pi \times N \times T}{60}$

NOW, 440X 10^6 = $\frac{2\pi \times 280\, T}{60}$

T= 15006037 N-mm

And J= $\frac{\pi d4}{32}$

From consideration of shear stress $\frac{T}{J} = \frac{qs}{R}$

$\frac{15006037}{\pi \frac{d4}{32}} = \frac{40}{\frac{d}{2}}$

$\frac{\pi d3}{16}$ x 40= 15006037

d =124.087 mm

from consideration of angle of twist

$\frac{T}{J} = \frac{G\theta}{L}$

$\frac{15006037}{\pi \frac{d4}{32}}$ = 84x 1000 x π/$\frac{180}{1000}$ Type equation here.

Calculate d

d = 101.048 mm

7.7 Torsional Rigidity : A torque required to introduce a unit angle of twist in a unit length is called as torsional rigidity $\frac{T}{J} = \tau'/r = G\theta/L$, GJ is called as stiffness of shaft

The angular twist θ of a shaft with given cross section is given by

$$\theta = \frac{TL}{KG} \qquad (1)$$

where T is the twisting moment (commonly measured in units of inch-pounds-force), L is the length (inches), G is the modulus of rigidity (pounds-force per square inch), and K (sometimes also denoted C) is the torsional rigidity multiplier for a given geometric cross section

Values of K are known exactly only for a small number of cross sections, and in closed form for even fewer. The following table lists approximate values for some common shapes

cross section	K/a^4 **approx**
circle	**1.570796...**
equilateral triangle	**0.021650...**
half-disk	**0.297556...**
isosceles right triangle	**0.026089...**
quarter-disk	**0.0825...**
sliced disk	**0.878055...**
square	**0.140577...**

7.3. A hollow circular shaft of 6m length and inner and outer diameter of 75 mm and 100 mm is subjected to a torque of 10 KN-m . if G=80 GPa. Determine the maximum shear produced and the total angle of twist .

L = 6m=6000 mm

.d1 = 100 mm

.d2= 75 mm

T= 10 kN-m = 10x 10^6 N-mm

G= 80 Gpa = 80 X 10^3 N-mm2

J=$\frac{\pi}{32}(d_1^4 - d_2^4) = \frac{\pi}{32}(100^4 - 75^4) = 6.711166 \times 10^6 mm^4$

Now, $\frac{T}{J} = \frac{q_S}{R}$

i.e, $\frac{10 \times 10^6}{6.711166 \times 10^6} = \frac{q}{50}$

∴ $q_s =$ 74.503 N/mm^2

and also, $\frac{T}{J} = \frac{G\theta}{L}$

$$\theta = \frac{TL}{GJ} = \frac{10 \times 10^6 \times 6000}{80 \times 10^3 \times 6.711166 \times 10^6}$$

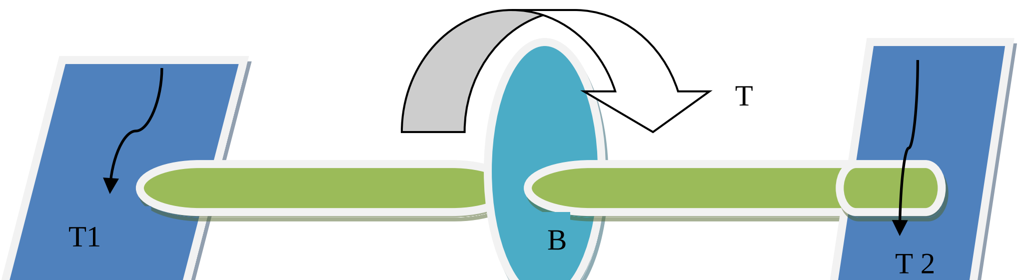
= 0.111754 rad

= 0.111754$\frac{180}{\pi}$ degrees

= 6.403 degree

7.8 STEPPED SHAFTS AND COMPOSITE SHAFTS

A Shaft may consist of a number of small shafts of different cross-section or of different materials. To analyse these shaft, first of all the torque resisted by each portion is calculated and then the individual effects are clubbed. While finding torque resisted by each portion the following points are to be noted.



L1 L2

Fig 7.14 Shaft subjected to torque

At the fixed end, torque of required magnitude develops to keep the shaft in equilibrium.

a) The torques developed at the ends of any portion are equal and opposite.

b) At the common point between two portions the angle of twist is the same.

Consider the shaft shown in fig. 7.14

The torques acting on each portion is obtained from equilibrium consideration as shown in fig

Now consider the shaft shown in fig.7.14 which is fixed at both ends and is subjected to torque T at the common point.

Let T_1 and T_2 be the torques developed at ends. Then

$$T_1 + T_2 = T$$

And $\theta_{1B} = \theta_{2B}$

i.e. $\frac{T_1 L_1}{G_1 J_1} = \frac{T_2 L_{21}}{G_2 J_2}$

from these two conditions T_1 and T_2 can de found.

7.9 Solved Numerical

7.4 The shaft shown in fig. 7.15 is securely fixed at A and is subjected to a torque of 8 kN-m . if the portion AB is solid shaft of 100mm diameter and portion BC is hollow with external diameter 100mm and internal diameter 75 mm, find the maximu ss angle of twist. Take G= 80 k

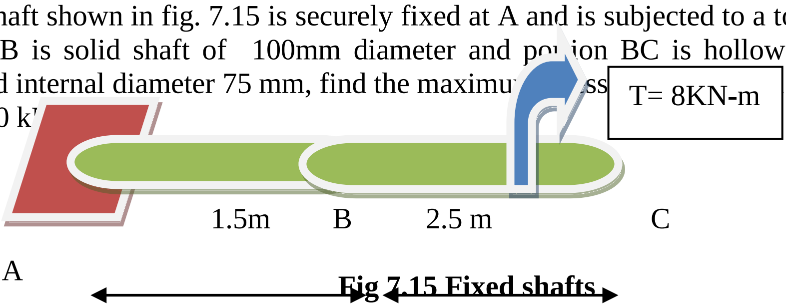


Fig 7.15 Fixed shafts

Solution:- from the free body diagram of portions AB and BC it is clear that each portion is subjected to the torque T= 8kN-m= 8×10^6 N-mm, G=80×10^3 N/mm^2.

For portion AB, $J_1 = \frac{\pi}{32} 100^4 = 9.8175 \times 10 mm^4$

For portion BC , $J_2 = \frac{\pi}{32}(100^4 - 75) = 6.71117 \times 10^6$ mm^4

∴ maximum stress occurs in portion BC , since the maximum radial distance R is the same in the two cases.

$$\frac{T}{J_2} = \frac{q_{max}}{R}$$

$q_{max} = \frac{50 \times 8 \times 10^6}{6.71117 \times 10^6} = 59.60$ N/mm^2

Let the rotation in portion AB be θ_1 and rotation in portion BC be θ_2

Total rotation at end C= $\theta_1 + \theta_2 = \frac{TL1}{GJ1} + \frac{TL2}{GJ2}$

$= \frac{8 \times 10^6}{80 \times 10^3}\left(\frac{1500}{9.8175 \times 10^6} + \frac{2500}{6.71117 \times 10^6}\right)$

= 0.05253 radians (Ans)

7.5. A composite shaft has an aluminium tube of external diameter 60 mm and internal diameter 40 mm closely fitted to a steel rod of 40 mm. If the permissible stress is 60 N/mm^2 in aluminium and 100N/mm^2 in steel, find the maximum torque the composite section can take. Given G_a= 27 kN/ mm^2 and G_s = 80 kN/ mm^2.

Sol^n :- figure 7.16 shows the cross-section of the composite shaft T_a . Ts be the torque

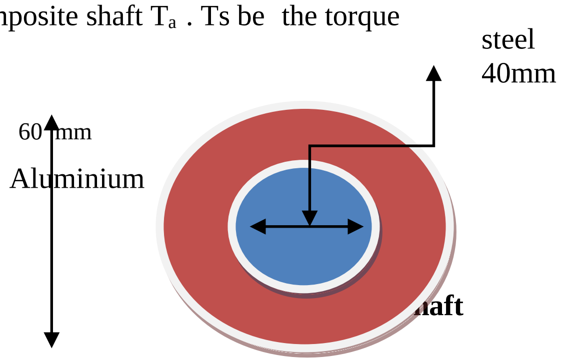


Fig 7.16

$Js = \frac{\pi}{32} \times 40^4$

$= 251327.4\ mm^4$

$J_a = \frac{\pi}{32} \times (60^4 - 40^4)$

$= 1.021018 \times 10^6 mm^4$

Now, $\theta_s = \theta_a$

$\frac{T_S L_S}{G_s J_s} = \frac{Ta\,La}{Ga\,Ja}$

Since Ls = La

$Ts = \frac{Gs}{Ga} \times \frac{Js}{Ja}\ Ta$

$= \frac{80 \times 10^3}{27 \times 10^3}\ \frac{251327.4}{1.021086 \times 10^6}\ Ta$

$= 0.7293\ Ta$

If stress in steel governs the resisting capacity :

$\frac{Ts}{Js} = \frac{q_s}{Rs}$

$Ts = \frac{251327.4 \times 100}{20}$

$= 1.2566 \times 10^6$ N-mm

$= 1.2566$ kN -m

$Ta = \frac{Ts}{0.7293}$

$= 1.723072 \times 10^6$ N-mm

$= 1.7231$ kN-m

T = Ts+Ta

= 2.9797 kN-m

If stress in aluminium governs the capacity, then

$\frac{Ta}{Ja} = \frac{q_a}{Ra}$

$T_a = \frac{1.02108 \times 10^6}{30} \times 60$

$= 2.0422 \times 10^6$N-mm = 2.0422 kN-m

$T_s = T_a \times 0.7293 = 1.4893$ kN-m

$T = T_a + T_s = 3.5315$ kN-m

we can conclude that stress in steel governs the torque-carrying capacity of the shaft and its value = 2.9797 kN-m.

7.10 SHEAR KEYS Shear keys are used to connect shafts to fly wheels/hubs in machineries as shown in fig. 7.17 the torque in shaft gets transferred to hubs/fly wheels. Let L be the length and 'b' be the width of the key. Then area of key resisting shear is Lb. If q_k is the shear stress in the keys, force resisted is q_k

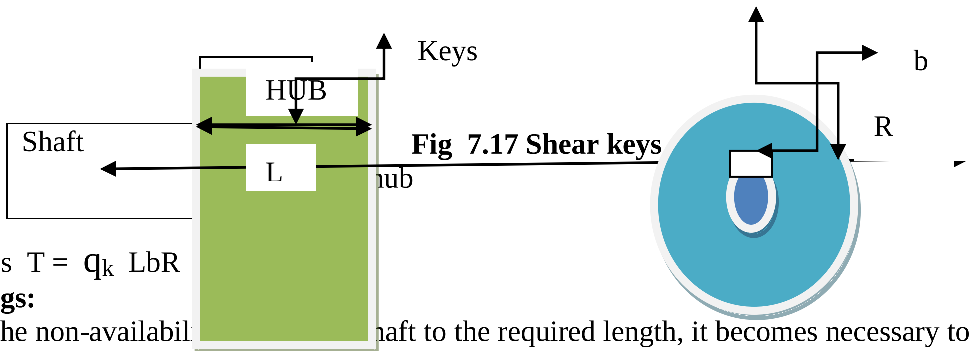


Fig 7.17 Shear keys

Its moment about

$= q_k\ LbR$

Torque transferred is $T = q_k\ LbR$

7.11 Shaft Couplings:

Sometimes, due to the non-availabil[ity of a] shaft to the required length, it becomes necessary to connect two shafts together. This is usually done by means of flanged couplings as shown in fig. 7.18 The flanges of the two shafts are joined together by bolts and nuts or rivets and the torque is then transferred from one shaft to another through the couplings. A little consideration will show that as the torque is transferred through the bolts, it is thus obvious that the bolts are subjected to shear stress. As the diameter of bolts is small, as compared to the diameter of the flanges, therefore shear stress is as assumed to be uniform in the bolts. The design of a shaft coupling means (a) design of bolts and (b) design of keys.

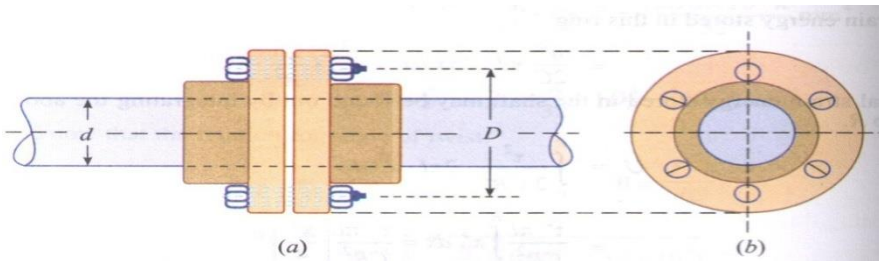


Fig 7.18 Shaft coupling

7.12 Design of Bolts:

Consider a shaft coupling, transmitting torque from one shaft to another.

Let, τ_s = Shear stress in the shaft,

d = Diameter of the shaft,

D = Diameter of the bolt pitch circle,

(i.e., the circle on which the bolts are arranged)

d_b = Diameter of the bolts,

n = No. Of bolts and

τ_b = Shear stress in the bolts.

We know that the torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau_s \times d^3 \quad \text{.................(i)}$$

And torque resisted by one bolt

= Are x Stress x Radius of bolt circle

$$= \frac{\pi}{4} \times d_b^2 \times \tau_b \times R$$

$$= \frac{\pi}{4} \times d_b^2 \times \tau_b \times \frac{D}{2}$$

$$= \frac{\pi d_b^2 \times \tau b\ D}{8}$$

Total torque resisted by the bolts

$$n \times \frac{\pi d_b^2 \times \tau b\ D}{8} \quad \text{...................(ii)}$$

Since the torque resisted by the bolts should be equal to the torque transmitted by the shaft, therefore equating (i) and (ii),

$$\frac{\pi}{16} \tau_s d^3 = \frac{n \times \pi d_b^2 \times \tau b\ D}{8}$$

This is the required equation for the number of bolts or the diameter of bolts.

Summary (Unit No 04)

Torque is a moment that twists a structure. Unlike axial loads which produce a uniform, or average, stress over the cross section of the object, a torque creates a distribution of stress over the cross section. To keep things simple, we're going to focus on structures with a circular cross section, often called rods or shafts. When a torque is applied to the structure, it will twist along the long axis of the rod, and its cross section remains circular.

To visualize what I'm talking about, imagine that the cross section of the rod is a clock with just an hour hand. When no torque is applied, the hour hand sits at 12 o'clock. As a torque is applied to the rod, it will twist, and the hour hand will rotate clockwise to a new position (say, 2 o'clock). The angle between 2 o'clock and 12 o'clock is referred to as the angle of twist, and is commonly denoted by the Greek symbol phi. This angle lets us determine the shear strain at any point along the cross section.

Exercise

1. Explain the torque and torsional moment
2. Define torsion equation
3. How the twisting of material take place
4. How bolt cane be design

Department of Civil Engineering

Name of Subject: - Strength of Materials	**Subject Code:- BECVE302T**
Unit-I : Introduction	**Semester: - III**
Slope and Deflection : Derivation of differential equation of moment curvature relation, Differential equation relating deflection and moment, shear and load, Deflection of simple beams by integration, Introduction to Deflection of linearly varying beams by integration.	

Course Outcome (CO):- The students would be able to understand the deflection of simply beams by integration

Learning Outcomes (LOs) :- (4 to 5 are expected and as per the COs)

- To make students learn and apply basic moment curvature relation
- To understand the basic fundamental of differential equation
- To Remember the relation between differential equation
- To utilize the basic concept of deflection of linearly varying beam
- To learn about the deflection, moment shear and load

There are two distinct methods of analysis for statically indeterminate structures depending on how equations of equilibrium, load displacement and compatibility conditions are satisfied: force method of analysis and displacement method of analysis. In the last module, force method of analysis was discussed. In the force method of analysis, primary unknowns are forces and compatibility of displacements is written in terms of pre-selected redundant reactions and flexibility coefficients using force displacement relations. Solving these equations,
the unknown redundant reactions are evaluated. The remaining reactions are obtained from equations of equilibrium. As the name itself suggests, in the displacement method of analysis, the primary unknowns are displacements. Once the structural model is defined for the problem, the unknowns are automatically chosen unlike the force method. Hence this method is more suitable for computer implementation. In the displacement method of analysis, first equilibrium equations are satisfied. The equilibrium of forces is written by expressing the unknown joint displacements in terms of load by using load displacement relations. These equilibrium equations are solved for unknown joint displacements. In the next step, the unknown reactions are computed from compatibility equations using force displacement relations. In displacement method, three methods which are closely related to each other will be discussed.

1) Slope-Deflection Method 2) Moment Distribution Method 3) Direct Stiffness Method

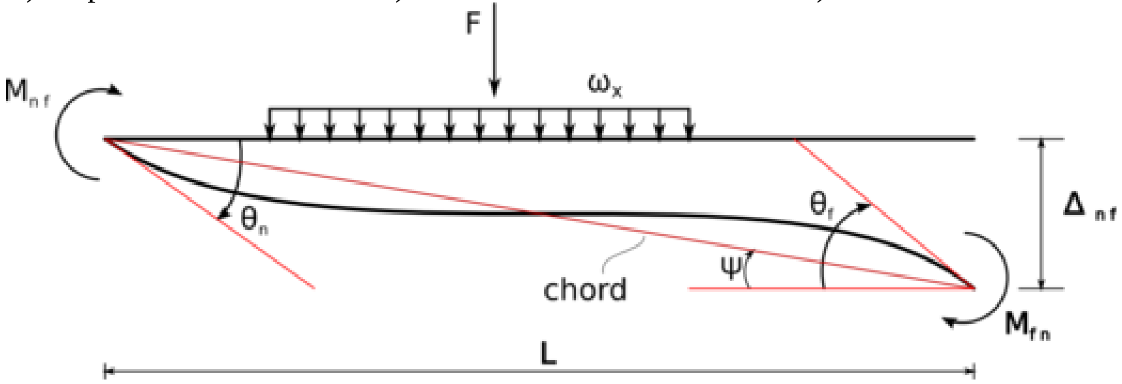


Fig 10.1 a Slope and Deflections

10.1.1 DEFLECTION OF BEAMS BY DOUBLE INTEGRATION METHOD

Materials used for beams are elastic and hence under the action of loads they deflect. A designer has to decide about beam dimensions not only based on the strength requirement but also from the consideration of deflections which should be within the prescribed limits. In mechanical components excessive deflection may cause misalignment and non-performance of the machine. In buildings, excessive deformation gives rye to psychological unrest and sometimes to breaking of the flooring, ceiling or roofing materials. Deflection calculations are also required to impose consistency conditions in the analysis of indeterminate structures. Hence, it is necessary to calculate beam deflections.

There are various methods of calculating beam deflections. This chapter discusses the double integration/direct integration/ Macaulay's method. Other methods like conjugate beam method, Castiglione's theorem and unit load method are covered in the next two chapters. The double integration method is quite simple for determinate beams. Another advantage of this method is that it gives values for all points of the structure and hence the deflected shape (elastic curve) of the beam can be drawn.

10.2 DIFFERENTIAL EQUATION FOR DEFLECTION

Consider an elements of length AB = ds as shown in Fig. 10.1. Let tangents drawn at A and B make angles 0 and 0 + d0 with x-axis and intersect it at D and E. Let M be the intersection point of these two tangents.

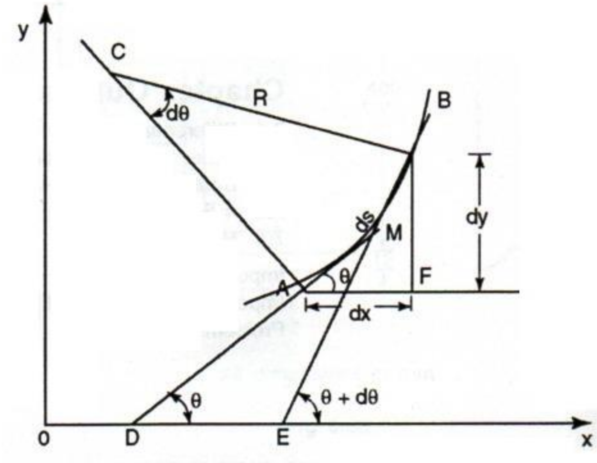


Fig 10.1 b Tangent at A and B

DME +θ= θ +dθ

DME = dθ

Also we note that

DME +AMB =180

AMB+ACB = 360-90-90

=180

AMB+ ACB = DME + AMB

ACB =DME = dθ

ds = Rdθ

Since ds is an elemental length, treating ABF as a triangle

$\frac{ds}{dx}$ = sec θ

$\frac{dy}{dx}$ = tan θ

differentiating Eqn . (10.1) w.r.t.x, we get

In beams, deflections are small and, hence, slope dy/dx is small. Therefore, in this theory, which may be called small deflection theory, $(dy/dx)^2$ is neglected compared to unity and hence,

From the bending equation of beam, we know

This equation is called differential equation for deflection.

Note that the following sign conventions are used The y-axis is upward.

(a) Curvature is concave towards the positive y-axis.

(b) This type of curvature occurs in the beam due to the sagging moment. Hence, the sagging moment is to be considered as the +ve moment.

If downward deflection is taken as positive, the moment curvature relation will be However, in any case it is not difficult to visualize the direction of deflection. In this book, the upward deflection and the sagging moments are taken as positive and, hence, the equation used is

The term El is called flexural rigidity.

10.2.1 OTHER USEFUL EQUATIONS

The differential relations relating load, shear and moments can be clubbed with Eqn. to get other useful differential equations.

Deflection = y

slope θ = $\frac{dy}{dx}$

Moment M = EI $\frac{d^2y}{dx}$

Shear force F = - $\frac{dM}{dx}$ = - EI $\frac{d^3y}{dx^3}$

Load intensity w= $\frac{df}{dx}$ = - EI $\frac{d^4y}{dx^4}$

10.3 DOUBLE INTEGRATION METHOD

In this method, moment M, at any distance x from one of the supports (usually left hand support), is written with the sagging moment as positive. Then, from we have

$$EI\frac{d^2y}{dx^2} = M$$

$$EI\frac{dy}{dx} = \int_0^x Mdx + C$$

$$EIY = \int_0^x \int_0^x Mdx + C1x + C$$

The constants C_1 and C_2 are found by making use of boundary conditions. Useful boundary conditions are listed below.

(a) At simply supported / roller ends

deflection y= 0

(b) At fixed and deflection y=0

and slop = $\frac{dy}{dx}$ = 0

(c) At point of symmetry $\frac{dy}{dx}$ = 0

* A FEW GENERAL CASES

10.3 .1 Cantilever Subjected to Moment at Free End

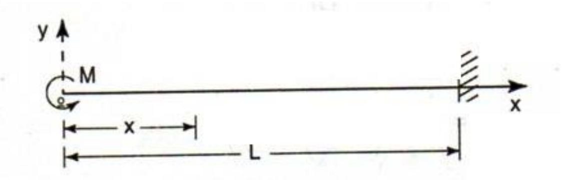


Fig 10.2 Cantilever Beam

Figure shows a cantilever beam of span L, flexural rigidity El, subjected to hogging moment M. Taking origin 'O' at the free end, moment at a distance x is given by

$$M_x = -M$$

$$EI\frac{d^2y}{dx^2} = -M$$

$$EI\frac{dy}{dx} = -M_x + C_1$$

$$EIy = \frac{Mx^2}{2} + C_{1\,x} + C_2$$

And

The boundary conditions (BC) available are

Atx = L, $\frac{dy}{dx}$ = 0

y= 0

we can write

$$0= - ML+C_1$$

$$C_{1 = ML}$$

$$0= \frac{ML^2}{2} + C_1L+ C_2$$

Substituting the value of C_1 and re - arranging

$$C_2= \frac{ML^2}{2} \quad ML^2 = -\frac{ML^2}{2}$$

$$El\frac{dy}{dx} = - M_x + M_L = M(L-x)$$

and $$Ely = \frac{-Mx^2}{2} + MLx - \frac{ML^2}{2}$$

$$= M\left[\frac{x^2}{2} + Lx - \frac{L^2}{2}\right]$$

10.3.2 A Cantilever Subjected to Concentrated Load at Free End

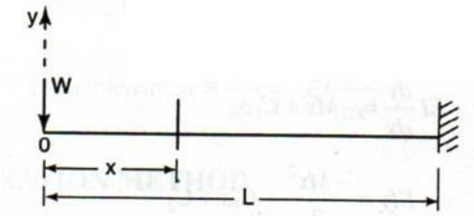


Fig 10.3 Cantilever Beam with point load

Referring to Fig. and taking hogging moment as –ve,

10.3.3 A Cantilever Subjected to Uniformly Distributed Load

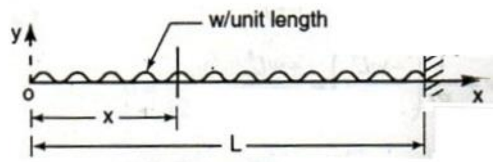


Fig 10.4 Cantilever Beam with udl

Referring to Fig. and taking hogging moment as –ve,

$$M_x = \frac{-wx^2}{2}$$

i.e $$EI\frac{d^2y}{dx^2} = - \frac{-wx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{-wx^3}{2} + C_1$$

At x = L , $$\frac{dy}{dx} = 0$$

$$0 = \frac{-wL^2}{6} + C_1$$

$$C_1 = \frac{-wL^2}{6}$$

$$EI \frac{dy}{dx} = \frac{wx^3}{6} + \frac{-wL^3}{6}$$

Integrating again we get = $$EI\frac{dy}{dx} = \frac{wx^3}{6} + \frac{-wL^3}{6}$$

At x = L, y= 0,

$$0 = \frac{-wL^3}{24} + \frac{wL^3x}{6} + C_2$$

$$C_2 = \frac{-wL^4}{6} + \frac{wL^4}{24}$$

$$= \frac{wL^4}{8}$$

$$= \frac{wL^4}{24} + \frac{wL^4}{8} + \frac{wL^3}{6}$$

At free end where x= 0 we get

$$\frac{dy}{dx} = \frac{wL^3}{6\,EI}$$

$$Y = \frac{1}{EI}\left[\frac{-wL^4}{8}\right] = \frac{-wL^4}{8EI}$$

10.3.4 A Cantilever Subjected to Load Varying Linearly from Zero at Free End to w/Unit Length at Fixed End

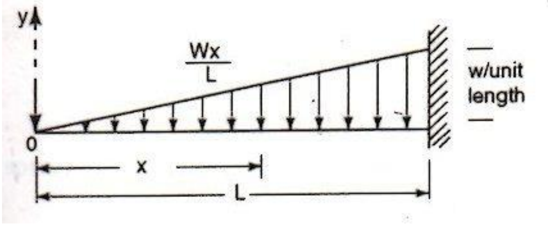


Fig 10.4 Cantilever Beam with varying udl

Consider a section at a distance x from the free end as shown in Fig. 10.4 . Here, the intensity of loading is and its CG is at from the section . Hence,

$$M_x = \frac{-1}{2} x \frac{wx}{L}\frac{x}{3} = \frac{-wx^3}{6L}$$

$$EI\frac{d^2y}{dx^2} = \frac{-wx^3}{6L}$$

$$EI\frac{dy}{dx} = \frac{-wx^4}{24L} + C_1$$

$$X = L \text{ is } \frac{dy}{dx} = 0$$

$$0 = \frac{-wx^4}{24L} + C_1$$

$$C_1 = \frac{wx^4}{24}$$

$$EI\frac{dy}{dx} = \frac{-wx^4}{120L} + \frac{wL^3 x}{24} + C_2$$

The boundary condition at x= L is y= 0

$$0 = \frac{-wx^3}{120L} + \frac{wL^3}{24} + C_2$$

$$C_2 = \frac{-wL^3}{120} - \frac{wL^4}{24} = \frac{wL^4}{120} \ (1\text{-}5)$$

$$= \frac{-wL^4}{30}$$

$$\frac{-wx^5}{120L} + \frac{wL^3}{24} + \frac{wL^4}{30}$$

At free end where x = 0 we get

$$\frac{dy}{dx} = \frac{I}{EI}\frac{wL^3}{24} = \frac{wL^4}{30EL}$$

$$y = \frac{I}{EI}\left[\frac{-wL^4}{30}\right]\frac{-wL^4}{30EL}$$

$$\frac{wL^4}{30EL} \text{ (Downward)}$$

10.3.5 A Simply Supported Beam Subjected to a Central Concentrated Load

Consider the simply sujpp;orted beam AB of span ‘L’ carrying central concentrated load W at C, which is the centre of its span (Fig.10.5).

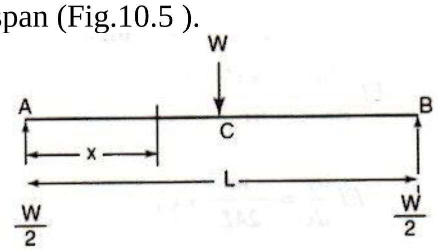


Fig 10.5 Cantilever Beam with mid point load

$$R_A = \frac{w}{2}$$

$M_x = R_{AX} = \frac{dy}{dx}$

$M_x =$

10.3.6 A Simply Suypported Beam Subjected to Uniformly Distributed Load

Let AB be the simply supported beam of span L, subjected to uniformly distributed load w/unit length throughout as shown in Fig. 7.7.

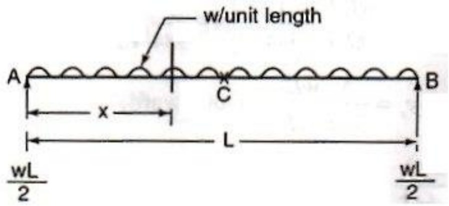


Fig 10.6 SSB with UDL

$R_A = R_a = \frac{wL}{2}$

$$M_x = \frac{wL}{2} \times - \frac{wx^2}{2}$$

$$EI \frac{d^2y}{dx^2} = \frac{wL}{2} \times - \frac{wx^2}{2}$$

$$EI \frac{dy}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1$$

Due to symmetry $\frac{dy}{dx} = 0$at $x = \frac{I}{2}$

$$0 = \frac{wL}{4} \left(\frac{L}{2}\right)^2 - \frac{w}{6} \left(\frac{L}{2}\right) + C_1$$

$$C_1 = wL^3 \left(\frac{1}{16} + \frac{1}{48}\right) = \frac{wL^3}{24}$$

$$EI \frac{dy}{dx} = \frac{wL}{4} x^2 - \frac{w}{46} x^3 - v \frac{wL^3}{24}$$

Integrating both sides w.r.t. we get

$$Ely = \frac{wL}{12} x^3 - \frac{w}{24} x^4 - \frac{wL^3}{24} x + C_2$$

at $x = 0$; $y = 0$

$0 = C_2$

Hence, $Ely = \frac{wL}{12} x^3 - \frac{w}{24} x^4 - \frac{wL^3}{24} x$

10.3.7 A Simply Supported Beam Subjected to a Load Varying Linearly from Zero at One End to w/unit Length at Other End

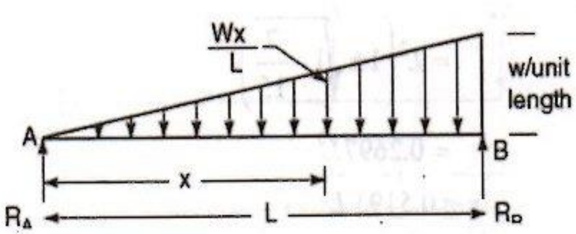


Fig 10.7 SSB with Varying UDL

$R_2L = \frac{1}{2} wL \frac{L}{3}$

$R_A = \frac{1}{2} wL$

$M_X = R_4x - \frac{1}{2} \times \frac{wx}{L} \; \frac{x}{3}$

$EI \frac{d^2y}{dx^2} = \frac{1}{6} wlx - \frac{1}{6} \frac{wL^3}{L}$

$EI \frac{dy}{dx} = C_1 + \frac{1}{12} wlx^2 - \frac{1}{24} \frac{wx^4}{L}$

$Ely = C_2 + C_1x + \frac{1}{36} wlx^2 - \frac{1}{120} \frac{wx^3}{L}$

At x = 0 y = 0

$0 = C_2$

At x = 0 y = 0

$0 = C_1L + \frac{1}{36} wl^4 - \frac{1}{120} wL^4$

Thus, the maximum deflection occurs at a distance of 0.5193 L from the end with load intensity zero and its value is 0.006523 downward.

10.4 MACAULAY'S METHOD

This is nothing but double integration method applied systematically for general loading cases in beams. It is a very useful method for the beams subjected to a set of concentrated loads and uniformly distributed loads over small lengths. The speciality of the method is in writing the expression for bending moment and retaining original forms in integration. This method is illustrated in detail with the example of a simply supported beam subjected to two concentrated loads as shown in Fig.10.8 .

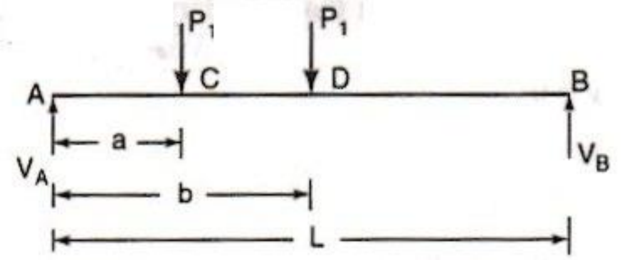


Fig 10.8 SSB with Varying UDL

Let V_A be the reaction at A. Then

And remember that if the term in brackets becomes negative for any point in the beam, that term is not applicable for that point. For example, for any point in portion CD, x-b is negative and we know the last term is not applicable for bending moment expression for this point.

It is very important to not the following :

1. Constant of integration is written first, since it is applicable to all points.
2. Integration of (x-a) is written as instead of . Both integrations are correct, the only difference is constant of integration is different which is still arbitrary. But by writing in the form , we still retain the warning message that if the term becomes negative for any point, it is not applicable for that point.

Again note the method of writing the constant of integrating and integration of the terms

The constants C_1 and C_2 are found from the boundary conditions. In this case, the boundary conditions are :

(Note: For x=0 , the quantities within the bracket in last two terms are negative and we know they are not applicable for this point which is in the portion AC)

Hence, C_2 can be found. Once C_1 and C_2 are known slope and deflection at any point can be found using Eqn.s (7.22)(and (7.23). The method is illustrated with a set of problems below.

10.5 Loading on bridges

INTRODUCTION

Reinforced concrete is increasingly used for highway and railway bridge construction due to its durability , rigidity economy , ease of construction and ease with which pleasing appearance can be made in it . reinforced concrete bridge may be of following types

1) solid slab bridge or deck slab bridge
2) Deck girder bridge or T-beam bridge
3) Balanced cantilever bridge
4) Rigid frame bridge

1.Solid Slab Bridge – This is a simplest type of construction used mostly for small bridge With a span not exceeding 8 m .Though the thickness of deck slab is considerable.Its construction is much simpler and cost of form work is also minimum

2.Deck girder bridge- Is the another type of a simple R . C. slab bridge used for the span between 10 to 20 m . The slab is built monolithic with girder so that T- beam effect is achieved

3.Balanced cantilever bridge – It is a statically determinant structure , used for the span between 25 to 50 m . It consist of alternate span with projecting cantilever the end of which are suspended to support a suspended span. A balanced cantilever bridge is used where the width of river is large .

Rigid frame bridge – It is used for only small drains, in which the vertical abutment are cast monolithic with deck slab , If the foundation condition are good , the rigid frame may be of portal type . However , if the bearing capacity is poor , a box culvert may be used

10.5.1 GENERAL TYPE- The most common type of reinforced concrete bridge deck are slab type. The reinforced concrete slab type deck is generally used for the small span

This type of super structure is economical up to the span of about 8m. Slab deck are similar for construction due to the easier fabrication of form work and reinforcement and placement of concrete. The slab deck are supported on the two opposite sides on the pier or abutment. Tee beam and slab type deck are generally adopted in the span range of 10 to 25 m . For the longer span the dead weight of girder becomes too heavy and to reduce the dead weight moment, prestressed concrete bridge deck are commonly use

10.5.2 DESIGN LOADS FOR BRIDGE

INTRODUCTION

The design of superstructure of for that matter any other component of a bridge, is based on a set of loading condition which the component must withstand. These loads may very depending on duration (permanent or temporary), direction of a action, type of deformation, and nature of structural action (shear, bending, torsion, etc.). In order to form a consistent basis for design, the Indian Road Congress (IRC) has developed a set of standard loading conditions, which are taken in to account while designing a bridge. Other nations maintain their own set of design load such as

BS 5400 loads – United Kingdom

Ontario Highway Bridge Design Code (OHBDS) – Canada

American Association of State Highway and Transportation Officials (AASHTO) – USA

10.5.3 DESIGN LOADS

1. Dead Load

The dead load on superstructure is the aggregate weight of all superstructure elements (elements above bearing) such as the deck, wearing coat, parapets, stiffeners and utilities. It will be seen in design that the first step is to calculate the dead load of all the elements. The IRC 6 provides a table where the dead load unit weights of various construction materials are listed.

2.Vehicle live Load

The term live load means a load that moves along the length along the span. By this definition a man walking on the bridge is also a live load. But a highway bridge is designed to withstand much more than just pedestrian loading. To give the designers the ability to accurately model the live load on a structure, hypothetical vehicles were evolved by IRC along ago in 1946. The loads are categorized based on their configuration and intensity. They are explained below

3 .IRC Class AA Loading

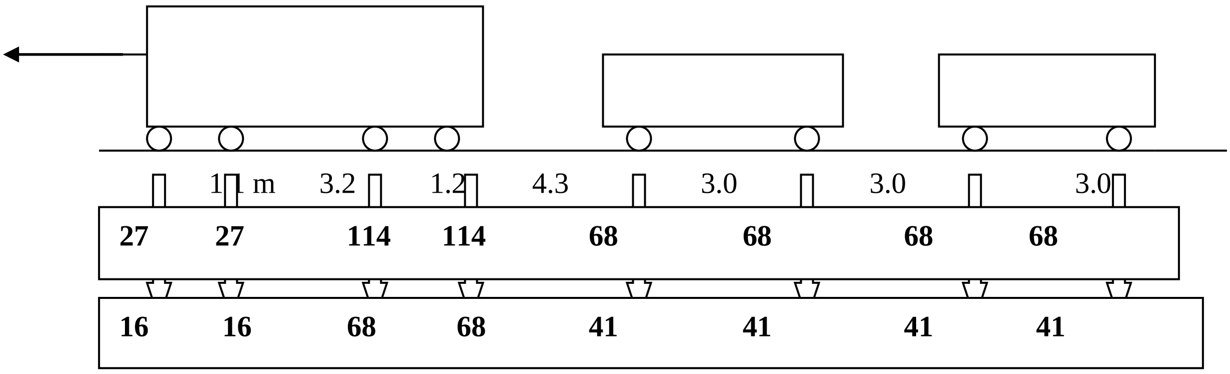
This is treated as heavy loading and is meant to be used for bridge for construction in certain industrial areas and other specified areas and highways. It is necessary that the bridge designed for IRC Class AA are checked for Class A Loading as well The IRC AA Loadings have two patterns: (a) tracked type, and (b) wheeled type. The details of their geometry are shown in Fig. 4.1

4. IRC Class A Loading :-

This is treated as standard loading. It is considered for all permanent in general. This loading has eight axles with a total length of about 25 m. The loading configuration is displayed in Fig.

5. IRC Class B Loading

This is considered light loading (Fig.) and is used in the design of temporary bridges (timber bridges). In addition to the above classes loading is a little different from class AA and is shown in Fig. 4.3. It has been reported [32] that IRC loading is severe for a single lane bridge, but less severe when compared with French, West German, Japanese and British standards for a two-lane bridge.



10.5.4 Impact Effect

In order to account for the dynamic effect of the sudden loading of a vehicle on to a bridge structure, an impact factor is used as a multiplier for loads on certain structural elements. From basic dynamics we know that a load that moves across a member introduces larger stresses than those caused by standstill load. However, the basis of impact factors predicted by IRC is not fully known. It has been felt by researchers [6] that the impact factors to a large extent depends on weight of the vehicle, its velocity, as well as surface characteristics of the road. It is pertinent to note that the live load increases on account of the consideration of the impact effect. For example, a span which is 9 m long would yield an impact factor of 0.10 (10%) and an impact multiplier of 1.10. The IRC specification for impact factors are computed as mentioned below.

For IRC Class A or Class B loading

If = A / (B + L)

Where

I_f = impact factor

A = constant, 4.5 for RCC bridges, 9.0 steel bridges

B = constant, 6.00 for RCC bridges, 13.50 steel bridges

L = span in m.

1 For IRC Class AA and 70R loading

1. Spans < 9 m
    (a) Tracked vehicle. 25% for spans up to 5 m linearly reducing to 10 % for spans up to 9 m.
    (b) Wheeled vehicle. For RCC bridges, 25% for spans up to 12 m, and in accordance with graph for spans > 12 m. For steel bridges, 25% for spans up to 23 m and as per graph (IRC 6) for spans exceeding 23 m.

Appropriate impact factors as mentioned below need be considered or substructures as well.

- At the bottom of the bed block: 0.5
- For the Top 3 m of the substructure: 0.5 to 0.0
- For portion of the substructure > 3 m. below the block: 0.0

2.Wind Loading

Wind loading offers a complicated set of loading conditions, which must be idealized in order to provide a workable design. The modeling of wind forces is dynamic one, with winds acting over a given time interval; these forces can be approximated to a static load uniformly distributed over the exposed region of the bridge. The exposed region of the bridge is taken as the aggregate surface areas of all elements (both superstructure and substructure) as seen in elevation (perpendicular to the longitudinal axis of the bridge). The wind forces may be selected from Ref. [21]

3 .Longitudinal Forces

These forces result from vehicles breaking or accelerating while traveling on a bridge. As a vehicle brakes, the load of the vehicle is transferred from its wheels to the bridge deck. The IRC specifies a longitudinal force of 20% of the appropriate lane load. This force is applied at 1.2 m above the level of the deck. The effect of the longitudinal forces on the superstructure is inconsequential; substructure elements, however, are affected more significantly. In general, the more stiff or rigid the structure is, the more severe the effects of forces will be [30].

4. Centrifugal forces

For bridge on horizontal curves, the effect of the centrifugal forces must also be calculated. Like longitudinal loading, centrifugal loading result from a vehicle traveling on a bridge and, in this instance, following a curvilinear path. This force is applied on a bridge and, in this instance, following a curvilinear path. This force is applied at 2 m above the level of the deck, and is defined as

$$C = Wv^2 / 127\ R$$

Where

C = Centrifugal force in KN, without impact
W= live load in KN
V = design speed in km/h
R = radius of the curve in m

Summery

The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam. The configuration assumed by the deformed neutral surface is known as the elastic curve of the beam. ... Deflection of Beam: Deflection is defined as the vertical displacement of a point on a loaded beam.

Deflection, in structural engineering terms, refers to the movement of a beam or node from its original position due to the forces and loads being applied to the member. Deflection, also known as displacement, can occur from external applied loads or from the weight of the structure itself, and the force of gravity in which this applies. It can occur in beams, trusses, frames and basically any other structure. To define deflection, let’s take a simple cantilevered beam deflection that has a person with weight (W) standing at the end:

Exercise

1 How to calculate the deflection beam

2 Define displacement

3 Explain the Macaulay’s Method with diagram

4 Explain the various type of loading

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Department of Civil Engineering

Name of Subject: - Strength of Materials	Subject Code:- BECVE302T
Unit-VI : Simple stress and strain	Semester: - III
Basic Introduction: State of stress in two dimensions, differential equation of equilibrium, Transformation of stresses, principal stresses, maximum shear stresses, Mohr's circle, Combined bending and torsion, Combined effect oftorsion and shear, Shear flow in thin walled section,	

- To make students learn and apply basic theories and concepts of equilibrium.
- To remember the basic fundamentals of mechanics and stress in two
- To apply the basic concept stress and strain
- To analysis the basic lows for problems solving
- To learn about compression and tension and flow in thin walled section

2.1 Introduction

Simple Stresses and Strains

In this chapter general meaning of stress is explained. Expressions for stresses and strains are derived with the following assumptions: For the range of forces applied the material is elastic *i.e.* it can regain its original shape and size, if the applied force is removed .Material is homogeneous *i.e.* every particle of the material possesses identical mechanical properties. Material is isotropic *i.e.* the material possesses identical mechanical property at any point in any direction. Presenting the typical stress-strain curve for typical steel, the commonly referred terms like limits of elasticity and proportionality, yield points, ultimate strength and strain hardening are explained. Linear elastic theory is developed to analyse different types of members subject to axial, shear, thermal and hoop stresses.

2.2 MEANING OF STRESS

When a member is subjected to loads it develops resisting forces. To find the resisting forces developed a section plane may be passed through the member and equilibrium of any one part maybe considered. Each part is in equilibrium under the action of applied forces and internal resisting forces. The resisting forces may be conveniently split into normal and parallel to the section plane. The resisting force parallel to the plane is called shearing resistance. The intensity of resisting force normal to the sectional plane is called intensity of Normal Stress

Stress:

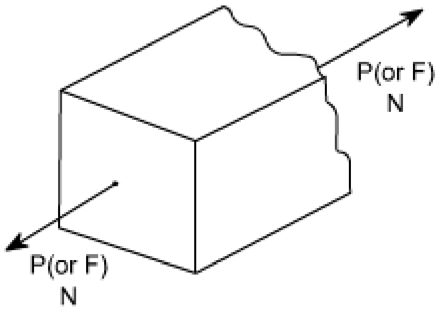


Fig 2. 1 Member subjected to load

Let us cor ed to some load or force (in Newton's). Let us imagine that the same rectangular bar is assumed to cut into two halves at section XX. The each portion of this rectangular bar is in equilibrium under the action of load P and the internal forces acting at the section XX has been shown in fig 2.2

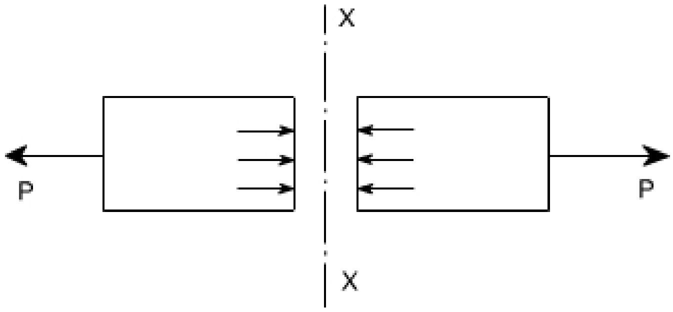


Fig 2.2 Variations in stresses

Now stress is defined as the force intensity or force per unit area. Here we use a symbol s to represent the stress.

$$\sigma = \frac{P}{A}$$

Where A is the area of the X – section

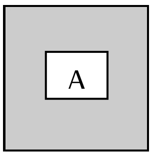


Fig 2.3 Cross section area

Here we are using an assumption that the total force or total load carried by the Rectangular bar is uniformly distributed over its cross – section. But the stress distributions may be for from uniform, with local regions of high stress known as stress concentrations. If the force carried by a component is not uniformly distributed over its cross – sectional area, A, we must consider a small area, 'dA' which carries a small load dP, of the total force 'P', Then definition of stress is

$$\sigma = \frac{\delta F}{\delta A}$$

As a particular stress generally holds true only at a point, therefore it is defined Mathematically as

$$\sigma = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$$

Units :

The basic units of stress in S.I units i.e. (International system) are N/m2 (or Pa)

MPa = 10^6 Pa

GPa = 10^9 Pa

KPa = 10^3 Pa

Some times N/mm^2 units are also used, because this is an equivalent to MPa. While US Customary unit is pound per square inch psi.

2.3 TYPES OF STRESSES:

Only two basic stresses exist: (1) Normal stress and (2) Shear stress. Other stresses either similar to these basic stresses or a combination of these e.g. bending stresses are a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.

Let us define the normal stresses and shear stresses in the following sections.

Normal stresses: We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter (б)

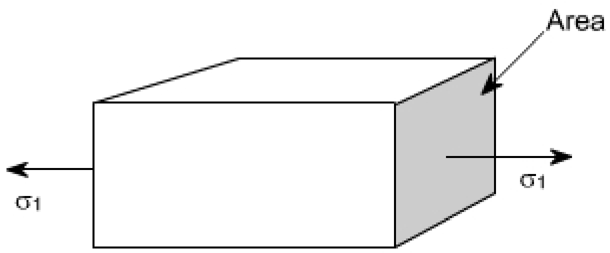


Fig 2.4 Tensile stresses

This is also l only in one direction however, such a state rarely exists, therefore we have biaxial and tri axial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :

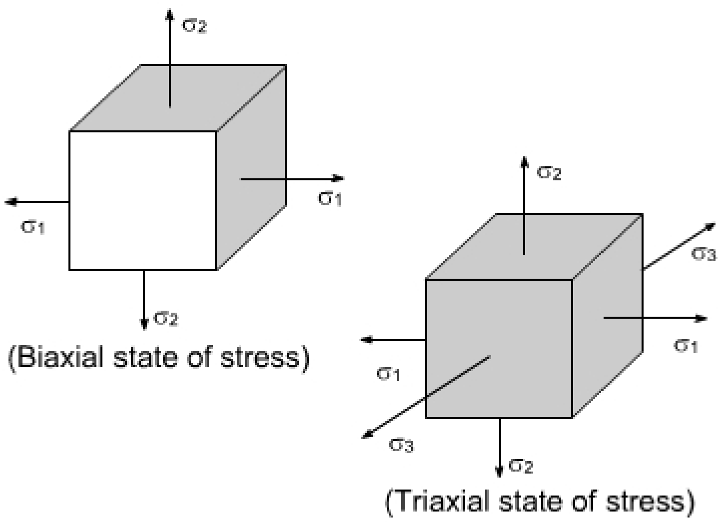


Fig 2.5 Stresses in various direction

Tensile or compressive str

The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

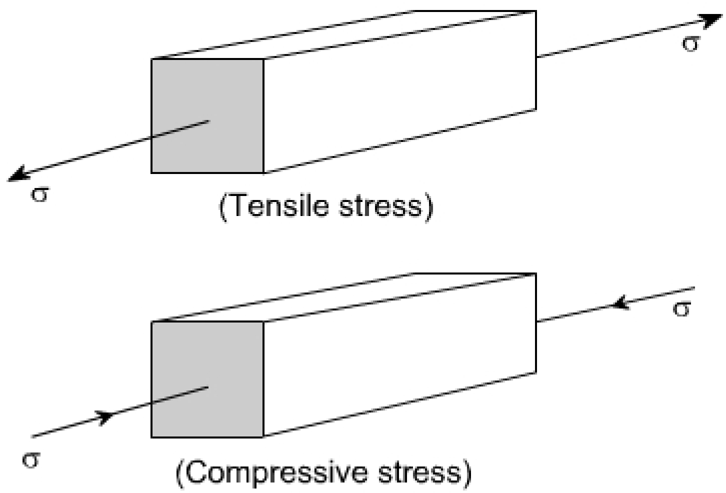
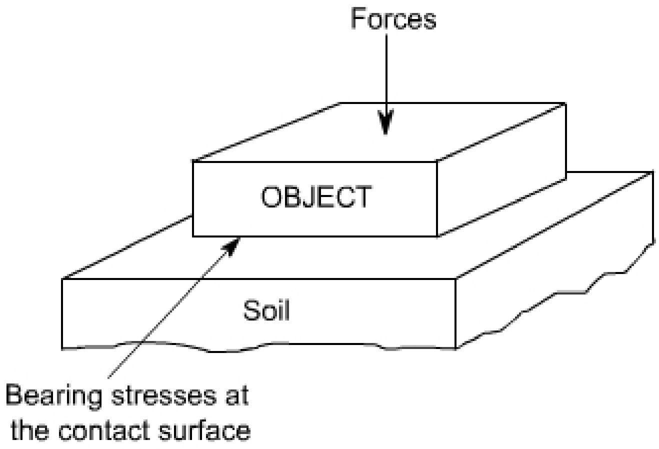


Fig 2.6
Tensile and compressive stresses

Bearing Stress: When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses)



Shear stresses: **Fig 2.7 Bearing stresses**

Let us consider now the situation, where the cross– sectional area of a block of materials subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force interest is known as shear stresses.

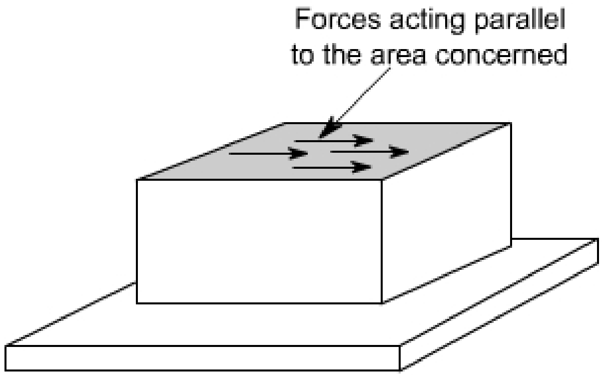


Fig 2.8 Shear stresses

The resulting force intensities are known as shear stresses, the mean shear stress being equal to

$\tau = F/A$

Where F is the total force and A the area over which it acts.

As we know that the particular stress generally holds good only at a point therefore we can define shear stress at a point as

$$\tau = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$$

The Greek symbol τ (tau) (suggesting tangential) is used to denote shear stress.

However, it must be borne in mind that the stress (resultant stress) at any point in a body is basically resolved into two components s and t one acts perpendicular and other parallel to the area concerned, as it is clearly defined in the following figure.

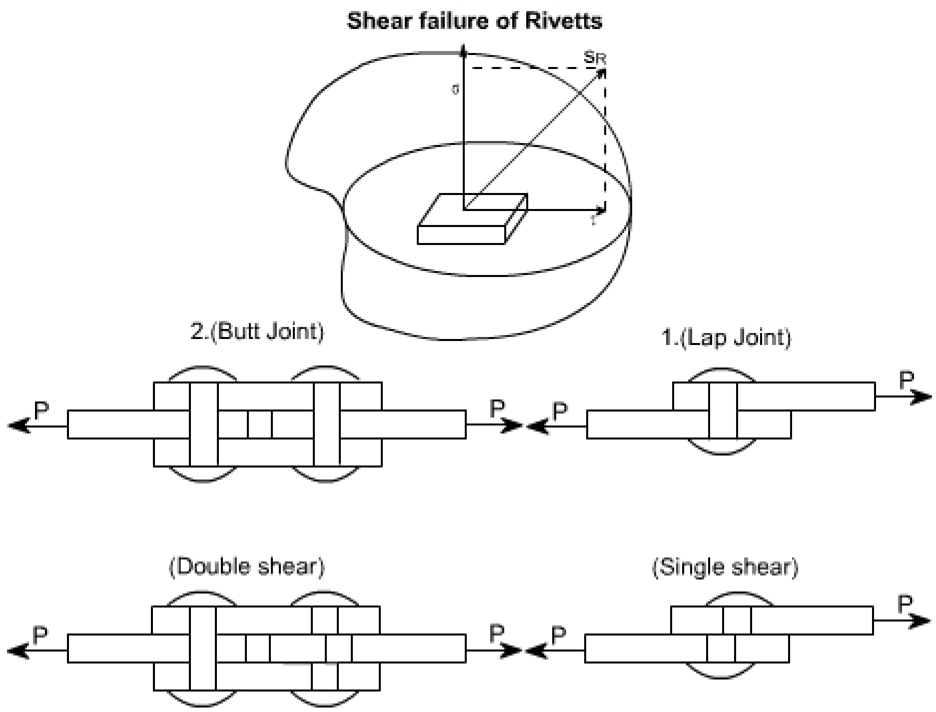


Fig2. 9 shear failure of rivet

The single shear takes place on the sir ctional of the rivet, whereas the double shear takes place in the case of Butt joints of rivet and the shear area is the twice of the X - sectional area of the rivet

2.4 CONCEPT OF STRAIN

Concept of strain: if a bar is subjected to a direct load, and hence a stress the bar will change in length. If the bar has an original length L and changes by an amount dL, the strain produce is defined as follows:

$$\text{strain}(\varepsilon) = \frac{\text{change in length}}{\text{orginal length}} = \frac{\delta L}{L}$$

Strain is thus, a measure of the deformation of the material and is a non dimensional Quantity i.e. it has no units. It is simply a ratio of two quantities with the same unit.

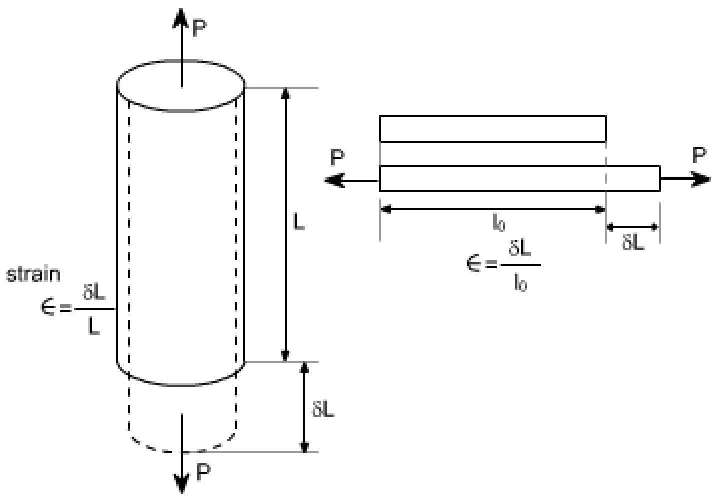


Fig 2.10 Concept of strain

Since in practice, the extensions of materials under load are very small, it is often convenient to measure the strain in the form of strain x 10^{-6} i.e. micro strain, when the symbol used becomes €

Sign convention for strain:

Tensile strains are positive whereas compressive strains are negative. The strain defined earlier was known as linear strain or normal strain or the longitudinal strain now let used fine the shear strain.

Definition: An element which is subjected to a shear stress experiences a deformation as shown in the figure below. The tangent of the angle through which two adjacent sides rotate relative to their initial position is termed shear strain. In many cases the angle is very small and the angle it self is used, (in radians), instead of tangent, so that g = Ð
AOB - Ð A'OB' = f
Shear strain: As we know that the shear stresses acts along the surface. The action of the stresses is to produce or being about the deformation in the body considers the distortion produced b shear sheer stress on an element or rectangular block.

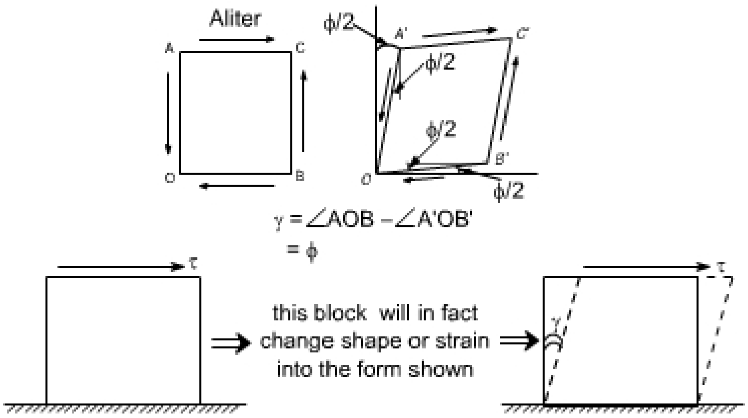


Fig 2.11 Shear strain

This shear strain or slide is f and can be defined as the change in right angle. or The angle deformation g is then termed as the shear strain. Shear strain is measured in radians & hence is non – dimensional i.e. it has no unit. So we have two types of strain i.e. normal stress & shear stresses.

2.5 STRESS-STRAIN RELATION The stress-strain relation of any material is obtained by conducting tension test in the laboratories on standard specimen. Different materials behave differently and their behaviour in tension and in compression differs slightly.

2.6 Behaviour in Tension
Mild Steel. Figure shows a typical tensile test specimen of mild steel. Its ends are gripped into universal testing machine. Extensometer is fitted to test specimen which measures extension over the length *L*, shown in Fig.2.12 The length over which extension is measured is called gauge *length.* The load is applied gradually and at regular interval of loads extension is measured. After certain load, extension increases at faster rate and the capacity of extensometer to measure extension comes to an end and, hence, it is removed before this stage is reached and extension is measured from scale on the universal testing machine. Load is increased gradually till the specimen breaks.

Fig. 2.12 Tension Test Specimen **Fig. 2.12a** Tension Test Specimen after Breaking

2.8 Hook's Law and numericals

A material is said to be elastic if it returns to its original, unloaded dimensions when load is removed. Hook's law therefore states that

Stress (σ) a strain(e)

$$\frac{\text{stress}}{\text{strain}} = \text{constant}$$

EXTENSION/SHORTENING OF A BAR

Consider the bars shown in Figure 2.15.

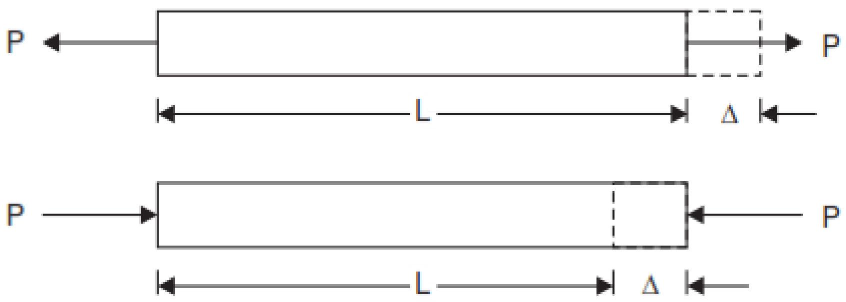


Fig 2.15 Bar subjected to tension and compression

From equation , Stress $\sigma = \frac{P}{A}$

From equation , Strain, $e = \Delta/L$

From Hooke's Law we have,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e} = \frac{P/A}{\Delta/L} = \frac{PL}{A\,\Delta}$$

$$\Delta = \frac{PL}{A\,E}$$

Numerical:

2.1. A circular rod of diameter 16 mm and 500 mm long is subjected to a tensile force 40kN. The modulus of elasticity for steel may be taken as 200 kN/mm^2. Find stress, strain and elongation of the bar due to applied load.

Solution: **Given Data:** Load P = 40 kN = 40 × 1000 N

E = 200 kN/mm= 200 × 10^3 N/mm^2

L = 500 mm

Diameter of the rod d = 16 mm

Therefore, sectional area $A = \frac{\pi d^2}{4} = \frac{\pi}{4} \times 16^2$

= = 201.06 mm^2

$$\text{Stress } \sigma = \frac{P}{A} = \frac{40 \times 1000}{201.06} = 198.94 \text{ N/mm}^2$$

$$\text{Strain } e = \frac{P}{E} = \frac{198.94}{200 \times 103} = 0.0009947$$

$$\text{Elongation } \Delta = \frac{PL}{AE} = \frac{4 \times 1000 \times 500}{201.06 \times 200 \times 10^3} = 0.497\text{mm}$$

2.2. A Surveyor's steel tape 30 m long has a cross-section of 15 mm × 0.75 mm. With this,line AB is measure as 150 m. If the force applied during measurement is 120 N more than the force applied at the time of calibration, what is the actual length of the line?

Take modulus of elasticity for steel as 200 kN/mm^2

Solution: **Given Data:** $A = 15 \times 0.75 = 11.25 \text{ mm}^2$

$P = 120$ N, $L = 30$ m $= 30 \times 1000$ mm

$E = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$

$$\text{Elongation } \Delta = \frac{PL}{AE} = \frac{120 \times 30 \times 1000}{11.25 \times 200 \times 10^3} = 1.600 \text{ mm}$$

Hence, if measured length is 30 m.

Actual length is 30 m + 1.600 mm = 30.001600 m

∴ **Actual length of line *AB*** $= \frac{150}{30} \times 30.001600 =$ **150.008 m**

2.3. A specimen of steel 20 mm diameter with a gauge length of 200 mm is tested to destruction. It has an extension of 0.25 mm under a load of 80 kN and the load at elastic limit is 102 kN. The maximum load is 130 kN.

The total extension at fracture is 56 mm and diameter at neck is 15 mm. Find

(i) The stress at elastic limit.
(ii) Young's modulus.
(iii) Percentage elongation.
(iv) Percentage reduction in area.
(v) Ultimate tensile stress.

Solution: **Given Data:** Diameter $d = 20$ mm

$$\text{Area } A = \frac{\pi d^2}{4} = 314.16 \text{ mm}^2$$

i. Stress at elastic limit $= \frac{\text{Load at elastic limit}}{\text{Area}}$

$$= \frac{102 \times 10^3}{314.16} = 324.75 \text{ N/mm}^2$$

ii. Young's modulus $E = \frac{\text{Stress}}{\text{Strain}}$ within elastic limit

$$= \frac{P/A}{\Delta/l} = \frac{80 \times 10^3/314.16}{0.25/200} = 203718 \text{ N/mm}^2$$

iii. Percentage elongation = $\frac{\text{Final Elongation}}{\text{Original Length}}$

$$= \frac{56}{200} \times 100 = 28$$

iv. Percentage reduction in area = $\frac{\text{Initial area} - \text{Final area}}{\text{Initial area}}$ x 100

$$= \frac{\frac{\pi}{4}x20^2 - \frac{\pi}{4}x15^2}{\frac{\pi}{4}x20^2} \text{ x } 100 = 43.75$$

v. Ultimate Tensile Stress = $\frac{\text{Ultimate Load}}{\text{Area}}$

$$= \frac{130 \times 10^3}{314.16} = 413.80 \text{ N/mm}^2$$

2.9 BARS WITH CROSS-SECTIONS VARYING IN STEPS

A typical bar with cross-sections varying in steps and subjected to axial load is as shown in Fig2.16.a Let the length of three portions be L_1, L_2 and L_3 and the respective cross-sectional areas of the portion be A_1,A_2,A_3 and E be the Young's modulus of the material and P be the applied axial load.

Fig2.16.(*b*) shows the forces acting on the cross-sections of the three portions. It is obvious that to maintain equilibrium the load acting on each portion is P only. Hence stress, strain and extension of each of these portions are as listed below:

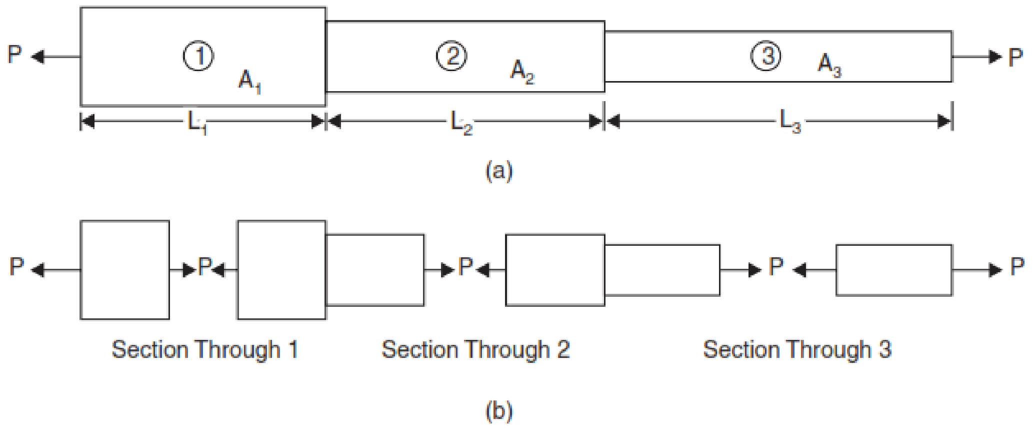


Fig. 2.16 Typical Bar with Cross-section Varying in Step

Portion	*Stress*	*Strain*	*Extension*
1	$p_1 = \frac{P}{A_1}$	$e_1 = \frac{p_1}{E} = \frac{P}{A_1E}$	$\Delta_1 = \frac{PL_1}{A_1E}$
2	$p_2 = \frac{P}{A_2}$	$e_2 = \frac{p_2}{E} = \frac{P}{A_2E}$	$\Delta_2 = \frac{PL_2}{A_2E}$
3	$p_3 = \frac{P}{A_3}$	$e_3 = \frac{p_3}{E} = \frac{P}{A_3E}$	$\Delta_3 = \frac{PL_3}{A_3E}$

Hence total change in length of the bar

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = \frac{PL1}{A1E} + \frac{PL2}{A2E} + \frac{PL3}{A3E}$$

2.10 NUMERICALS ON VARYING CROSS-SECTIONS

2.4 The bar shown in Fig. 2.17 is tested in universal testing machine. It is observed that a load of 40 kN the total extension of the bar is 0.280 mm. Determine the Young's modulus of the material.

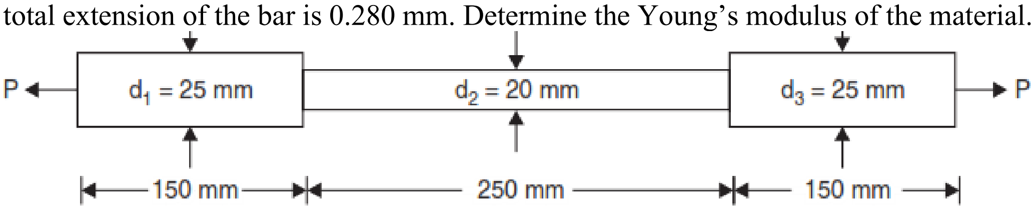


Fig 2.17 Tested Bar

Solution: Extension of portion 1, $= \frac{PL1}{A1E} = \frac{40x10^3x150}{490.87E}$

Extension of portion 2, $= \frac{PL2}{A2E} = \frac{40x10^3x250}{314.15E}$

Extension of portion 3, $= \frac{PL3}{A3E} = \frac{40x10^3x150}{490.87E}$

Total extension $= \frac{40x10^3}{E}$ x $\{\frac{150}{490.87} + \frac{250}{314.15} + \frac{150}{490.87}\}$

$0.280 = \frac{40x10^3}{E}$ X 1.120

E = 200990 N/mm^2

2.5 . The stepped bar shown in Figure2.18 is made up of two different materials. The material 1 has Young's modulus = 2 × 10^{5}N/mm^2.Find the extension of the bar under a pull of 30 kN if both the portions are 20 mm in thickness. while that of material2 is 1×10^{5}N/mm^2

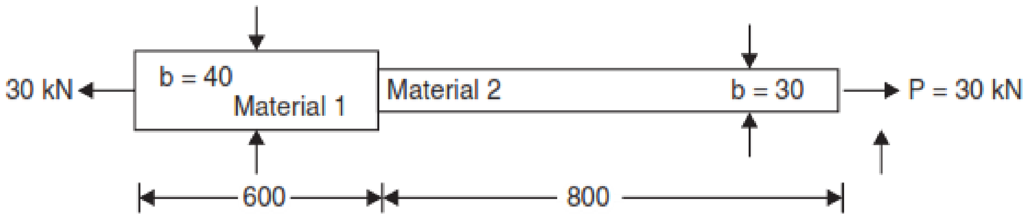


Fig 2.18 Stepped Bar

Solution: A_1 = 40 × 20 = 800 mm^2

A_2 = 30 × 20 = 600 mm^2

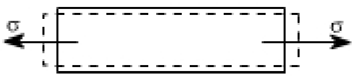
Extension of portion 1, $= \frac{PL1}{A1E} = \frac{30x10^3x600}{800x2x10^5}$ = 0.1125 mm.

Extension of portion 2, $= \frac{PL2}{A2E} = \frac{30x10^3x800}{600x1x10^5}$ = 0.4000 mm.

∴ Total extension of the bar = 0.1125 + 0.4000 = 0.5125 mm.

2.12 Poisson's Ratio:

If a bar is subjected to a longitudinal stress there will be a strain in this direction equal to σ / E . There will also be a strain in all directions at right angles to σ . The final shape being shown by the dotted lines.



It has been observed that for an elastic materials, the lateral strain is proportional to the longitudinal strain.The ratio of the lateral strain to longitudinal strain is known as the poison's ratio .

Poison's ratio (m) = - lateral strain / longitudinal strain

For most engineering materials the value of m his between 0.25 and 0.33.

2.13 Volumetric Strain:

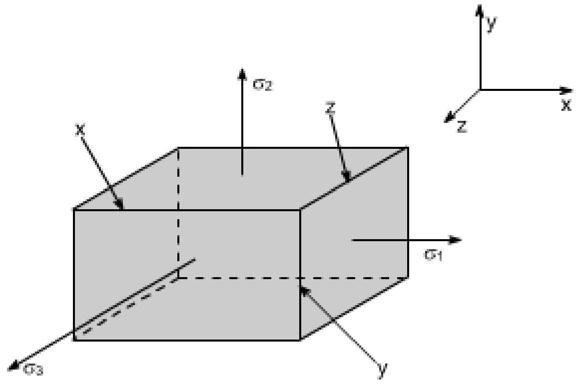


Fig 2.22 Strain in x y and z direction

Consider a rectangle solid of sides x, y and z under the action of principal stresses $\sigma_{1,}$ $\sigma_{2,}$ σ_3 respectively.

Then $\sigma_{1,}$ $\sigma_{2,}$ σ_3 respectively.

are corresponding linear strains, than the dimensions of the rectangle becomes

(x + €$_1$. x); (y + €$_1$. y); (z + €$_1$. z)

hence

$$\text{Volumetric strain} = \frac{\text{Increase in volume}}{\text{Original volume}}$$

$$= \frac{x(1+\epsilon_1)y(1+\epsilon_2)(1+\epsilon_3)z - xyz}{xyz}$$

$$= (1+\epsilon_1)y(1+\epsilon_2)(1+\epsilon_3) - 1 \cong \epsilon_1 + \epsilon_2 + \epsilon_3 \left[\text{Neglecting the products of } \epsilon\text{'s}\right]$$

: Let a cuboids of material having initial sides of Length x, y and z. If under some load system, the sides changes in length by dx, dy, and dz then the new volume (x+ dx) (y +dy) (z +dz)New volume = xyz + yzdx + xzdy + xydz Original volume = xyz

Change in volume = yzdx +xzdy + xydz

Volumetric strain = (yzdx +xzdy + xydz) / xyz = $б_x + б_y + б_z$

Neglecting the products of epsilon's since the strains are sufficiently small.

Volumetric strains in terms of principal stresses:

As we know that

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$\text{Futher Volumetric strain} = \epsilon_1 + \epsilon_2 + \epsilon_3$$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{E} - \frac{2\mu(\sigma_1 + \sigma_2 + \sigma_3)}{E}$$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}$$

hence the

$$\boxed{\text{Volumetric strain} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}}$$

2.14 ELASTIC CONSTANTS

In considering the elastic behaviour of an isotropic materials under, normal, shear and hydrostatic loading, we introduce a total of four elastic constants namely E, G, K, and g . It turns out that not all of these are independent to the others. In fact, given any two of them, the other two can be found out . Let us define these elastic constants

(i) E = Young's Modulus of Rigidity

= Stress / strain

(ii) G = Shear Modulus or Modulus of rigidity

= Shear stress / Shear strain

(iii) μ = Passion's ratio

= - lateral strain / longitudinal strain

(iv) K = Bulk Modulus of elasticity

= Volumetric stress / Volumetric strain

Where

Volumetric strain = sum of linear stress in x, y and z direction.

Volumetric stress = stress which cause the change in volume.

Let us find the relations between them

2.15 RELATION AMONG ELASTIC CONSTANTS

Relation between E, G and μ :

Let us establish a relation among the elastic constants E,G and u. Consider a cube of material of side 'a' subjected to the action of the shear and complementary shear stresses as shown in the figure and producing the strained shape as shown in the figure below.

Assuming that the strains are small and the angle A C B may be taken as 45^0

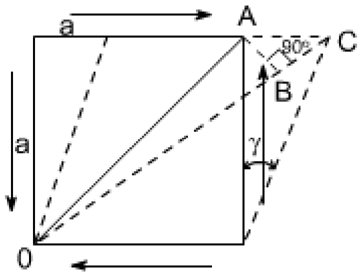


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Fig 2.23 Cube subjected to action of force

Therefore strain on the diagonal OA

= Change in length / original length

Since angle between OA and OB is very small hence OA @ OB therefore BC, is thechange in the length of the diagonal OA

Thus, strain on diagonal OA $= \frac{BC}{OA}$

$$= \frac{AC\cos 45^0}{OA}$$

$$OA = \frac{a}{\sin 45^0} = a.\sqrt{2}$$

hence strain $= \frac{AC}{a\sqrt{2}}.\frac{1}{\sqrt{2}}$

$$= \frac{AC}{2a}$$

but AC = aγ

where γ = shear strain

Thus, the strain on diagonal $= \frac{a\gamma}{2a} = \frac{\gamma}{2}$

From the definition

$$G = \frac{\tau}{\gamma} \text{ or } \gamma = \frac{\tau}{G}$$

thus, the strain on diagonal $= \frac{\gamma}{2} = \frac{\tau}{2G}$

Now this shear stress system is equivalent or can be replaced by a system of direct stresses at 45^0 as shown below. One set will be compressive, the other tensile, and both will be equal in value to the applied shear strain.

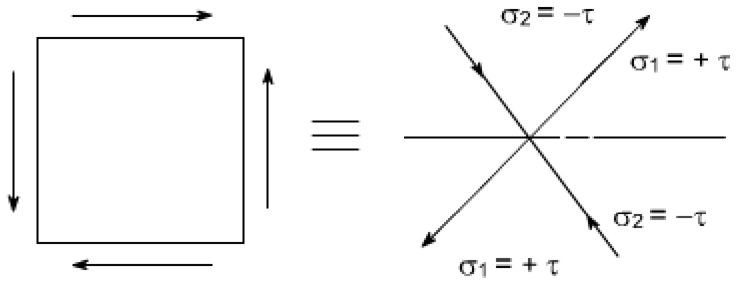

Fig 2.24 Direct stressess

Thus, for the direct state of stress system which applies along the diagonals?

$$\text{strain on diagonal} = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{\tau}{E} - \mu\frac{(-\tau)}{E}$$

$$= \frac{\tau}{E}(1+\mu)$$

equating the two strains one may get

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

or $$\boxed{E = 2G(1+\mu)}$$

We have introduced a total of four elastic constants, i.e E, G, K and μ. It turns out that not all of these are independent of the others. In fact given any two of then, the other two can be found.

Again $E = 3K(1-2\gamma)$

$$\Rightarrow \frac{E}{3(1-2\gamma)} = K$$

if $\gamma = 0.5$ $K = \infty$

$$\epsilon_v = \frac{(1-2\gamma)}{E}(\epsilon_x + \epsilon_y + \epsilon_z) = 3\frac{\sigma}{E}(1-2\gamma)$$

(for $\epsilon_x = \epsilon_y = \epsilon_z$ hydrostatic state of stress)

$\epsilon_v = 0$ if $\gamma = 0.5$

Irrespective of the stresses i.e, the material is incompressible. When g = 0.5 Value of k is infinite, rather than a zero value of E and volumetric strain is zero, or in other words, the material is incompressible.

Relation between E, K and μ:

Consider a cube subjected to three equal stresses s as shown in the figure below 2.25

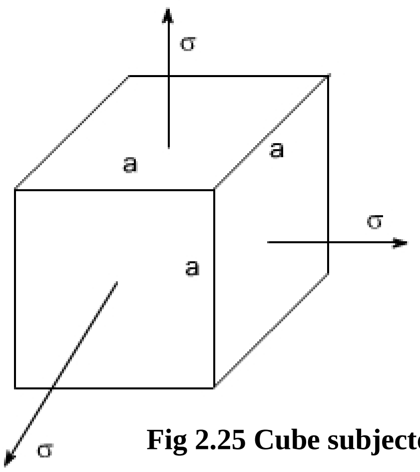

Fig 2.25 Cube subjected to three equal stresses

The total strain in one direction or along one edge due to the application of hydrostatic stress or volumetric stress s is given as

$$= \frac{\sigma}{E} - \gamma\frac{\sigma}{E} - \gamma\frac{\sigma}{E}$$

$$= \frac{\sigma}{E}(1 - 2\gamma)$$

volumetre strain = 3.linear strain

volumetre strain = $\epsilon_x + \epsilon_y + \epsilon_z$

or thus, $\epsilon_x = \epsilon_y = \epsilon_z$

$$\text{volumetric strain} = 3\frac{\sigma}{E}(1 - 2\gamma)$$

By definition

$$\text{Bulk Modulus of Elasticity (K)} = \frac{\text{Volumetric stress}(\sigma)}{\text{Volumetric strain}}$$

or

$$\text{Volumetric strain} = \frac{\sigma}{k}$$

Equating the two strains we get

$$\frac{\sigma}{k} = 3.\frac{\sigma}{E}(1 - 2\gamma)$$

$$\boxed{E = 3K(1 - 2\gamma)}$$

Relation between E, G and K :

The relationship between E, G and K can be easily determined by eliminating u from the already derived relations

E = 2 G (1 + μ) and E = 3 K (1 - μ)

Thus, the following relationship may be obtained

$$\boxed{E = \frac{9\,GK}{(3K + G)}}$$

Relation between E, K and G :

From the already derived relations, E can be eliminated

$$E = 2G(1+\gamma)$$

$$E = 3K(1-2\gamma)$$

Thus,we get

$$3k(1-2\gamma) = 2G(1+\gamma)$$

therefore

$$\gamma = \frac{(3K-2G)}{2(G+3K)}$$

or

$$\boxed{\gamma = 0.5(3K-2G)(G+3K)}$$

Summery

Introduction to Stress and Strain

You may have noticed that certain objects can stretch easily, but stretching objects like an iron rod sounds impossible, right? In this article, we will help you understand why few objects are more malleable than others. Mainly, we will be discussing Stress-strain curves because they are useful in understanding the tensile strength of a given material. We shall learn how force applied to a body generates stress. In this article, let us learn about stress and strain definition and relationship between the stress-strain.

What is Stress?

In mechanics, stress is defined as a force applied per unit area. It is given by the formula

$\sigma=FA$

where,

σ is the stress applied
F is the force applied
A is the area of force application

The unit of stress is N/m2

Stress applied to a material can be of two types. They are:

- Tensile Stress: It is the force applied per unit area which results in the increase in length (or area) of a body. Objects under tensile stress become thinner and longer.
- Compressive Stress: It is the force applied per unit area which results in the decrease in length (or area) of a body. The object under compressive stress becomes thicker and shorter.

What is Strain?

According to the strain definition, it is defined as the amount of deformation experienced by the body in the direction of force applied, divided by initial dimensions of the body. The relation for deformation in terms of length of a solid is given below.

$\epsilon=\delta lL$

where,

ϵ is the strain due to stress applied

δl is the change in length
L is the original length of the material.

The strain is a dimensionless quantity as it just defines the relative change in shape.

Depending on stress application, strain experienced in a body can be of two types. They are:

● Tensile Strain: It is the change in length (or area) of a body due to the application of tensile stress.
● Compressive Strain: It is the change in length (or area) of a body due to the application of compressive strain

When we study solids and their mechanical properties, information regarding their elastic properties is most important. These can be obtained by studying the stress-strain relationships, under different loads, in these materials.

Stress-Strain Curve

Exercise

1. Explain the tensile strain of rod
2. Define the principal stresses
3. What is Young's module . How it is calculated
4. Explain the relation between E G and μ