

Assignment model & Transportation model.

Assignment model:-

The assignment problem (model) concerned as to what happen to effectiveness function when we associate each of number of origin with each of number of destination. Each resource or facilities is to be associated with one and only one job (destination) and association are to be made in such a way that so as to maximize or minimize the total effectiveness.

There are two methods to solve assignment problem.

- 1) Hungarian method
- 2) Branch & Bound Technique

The condition which must be satisfy to solve assignment problem are

- 1) Problem must be a square matrix.
- 2) The objective must be of minimization
- 3) Hungarian method

Steps to solve assignment problem by Hungarian method are as follows

Step 1 Formulate the Problem

In this step we have to check both the

DATE: / /

PAGE NO:

2. Check for square matrix (If not given, converted into square matrix.)
 a) Check the objective of given problem.
 If maximization is given then converted into minimization.

Step 2:- Row Operation

It is perform to introduce at least a single zero in each row.

If the particular row already consist a single zero then no need to perform the row operation. For the row operation, it is perform by subtracting the lowest element from each element of that of that row.

Step 3:- Column Operation

It is perform to introduce at least a single zero in each column. The procedure is same as row operation.

Step 4: Row Assignment

Check each row for independent zero. (single zero). If two zero are there are called as dependent zero can not be assign.

Selected the independent zero by making square box around it and cancel it corresponding column.

A4 Long Book

DATE: / /

PAGE NO:

Step 5: Column assignment

Check each column for independent zero and assign it by making square box around it and cancel it corresponding row.

If the problem is not satisfy (not solve) then repeat step 4 and step 5 respectively again.

Step 6:- Reduce Matrix FurtherNumericals

Ques) There are 4 worker each of them performing there jobs according to there ability. The time required is given in the table. Assign a tasks to each worker to minimize the total time required.

Workers \ Job	1	2	3	4
A	18	26	17	(11)
B	(13)	28	19	26
C	38	19	18	(1)
D	19	26	24	(1)

A4 Long Book

DATE: / /

So, D) Step 1 formulate the problem
 No need to formulate the above problem.
 as i) It is a square matrix
 ii) Objective is of minimization

Step 2 Row operation

w \ T	1	2	3	4
A	7	15	6	0
B	0	15	(1)	13
C	23	(4)	3	0
D	9	16	14	0

Step 3 Column Operation

w \ T	1	2	3	4
A	7	11	5	(0)
B	(0)	11	0	13
C	23	(0)	(2)	0
D	9	12	13	0

Step 4 Row Assignment (Perform on above matrix)

Step 5 Column Assignment (Perform on above matrix)

Repeat step 4 & step 5

Step 6 Reduce mat. further
 + Select min. not cancelled element = 2

Not cancelled → subtract 2
 Single -11 → As it is
 Double -11 → Add 2

DATE: / /

w \ T	1	2	3	4
A	5	11	3	(0)
B	(0)	13	0	15
C	21	(0)	0	0
D	7	12	11	0

Repeat step 4 & step 5

Step 7 Reduce Mat. further & select min. not cancelled element = 3
 Not cancelled → sub 3
 Single -11 → As it is
 Double -11 → Add 3

w \ T	1	2	3	4
A	2	8	(0)	0
B	(0)	13	0	15
C	21	(0)	0	0
D	4	9	8	(0)

Repeat step 4 & step 5

w \ T	Task	Time
A	2	17
B		13
C	2	13
D	4	10

sq unit

DATE: / /

PAGE NO.:

The owner of a small machine shop has four machinist available for five jobs. The jobs are offered to assigned with the expected profit in 100 of RS.

For each machinist-job being as follows. Find the assignment of job to machinist that will result in maximum profit.

Machinist	A	B	C	D	E
1	6.2	7.8	*	10.1	8.2
2	7.0	8.4	6.5	7.5	6.0
3	3.7	9.2	11.1	7.0	8.2
4	*	6.4	8.7	7.7	8.0

Note: - '*' or '-' indicates that the job machinist combination is not possible.

1) For maximization problem, or maximize replace '*' or '-' with '-1'

2) For minimization problem, replace '*' or '-' with '∞'

Maximization:-

M	A	B	C	D	E
1	6.2	7.8	-1	10.1	8.2
2	7.0	8.4	6.5	7.5	6.0
3	3.7	9.2	(11.1)	7.0	8.2
4	-1	6.4	8.7	7.7	8.0

3) For converting maximization into minimization there are two methods

i) Multiply each element with -1
ii) Selecting highest element and subtract all the elements from it

A4 Long Book

DATE: / /

PAGE NO.:

Minimization:

(Select highest element = 11.1)

Machinist	A	B	C	D	E
1	4.3	3.3	12.1	(1)	2.9
2	4.1	(2.7)	4.6	3.6	5.1
3	2.4	1.3	0	4.1	2.9
4	12.1	4.7	(2.4)	3.4	3.1
Dummy Row	5	0	0	0	0

1) Formulate the problem:

As already formulate the problem by → (i) By adding dummy row

(ii) by converting maximization into minimization

2) Row Operation

M	A	B	C	D	E
✓ 1	3.9	2.3	11.1	(0)	1.3
✓ 2	1.4	(0)	1.9	3.3	2.4
✓ 3	2.4	1.3	(0)	4.1	2.9
4	3.7	2.3	0	1	0.7
-5	(0)	0	0	0	0

3) Column Operation:-

(No need to perform the column operation)

4) Row Assignment

(Perform on above matrix)

5) Column Assignment

(Perform on above matrix)

A4 Long Book

DATE: / /

PAGE NO.:

Repeat step 4 of step 5

⑤ Reduce matrix further.

select min. not cancelled element = 0.7

Not cancelled $\rightarrow$ Subtract 0.7

Single $\rightarrow$ As it is

Double $\rightarrow$ Add 0.7

M \ J	A	B	C	D	E
✓ 1	3.2	2.3	11.1	0	11.2
✓ 2	0.7	0	1.9	0.9	1.7
✓ 3	1.7	1.9	0	0	2.2
✓ 4	2	2.3	0	1	0
✓ 5	0	0.7	0.7	0.7	0

Repeat step 4 of step 5

Mechanist	Job	Profit
1	D	10.1
2	B	8.4
3	C	11.1
4	E	8.0
5	A	0
		37.6

The profit is hundreds of

$$Rs = 37.6 \times 100$$

$$= Rs. 3760/-$$

A4 Long Book

DATE: / /

PAGE NO.:

Q. 3

Assign a mechanic to job so that the total time required for completion of project should be minimum.

M \ J	1	2	3	4	5	6
A	20	23	18	10	16	20
B	50	20	17	16	15	11
C	60	30	40	55	8	7
D	6	7	10	20	100	9
E	18	19	28	17	60	70
F	9	10	20	30	40	55

soln.

step 1) formulate the problem

No. need to formulate the above problem

Step 2 Row Operation

M \ J	1	2	3	4	5	6
A	10	13	9	0	8	10
B	33	9	6	5	4	0
C	53	23	33	4	1	0
D	0	1	4	1	39	3
E	1	2	11	0	43	53
F	0	1	11	1	31	45

A4 Long Book

DATE: / /

PAGE NO.:

STEP 3 Column Operation

M	1	2	3	4	5	6
✓ A	10	12	4	0	5	10
✓ B	31	9	2	5	3	0
✓ C	53	22	29	48	0	0
D	0	0	0	11	53	2
E	1	1	7	0	42	53
F	0	0	7	41	50	46

STEP 4 Row Assignment

(Performed on above matrix)

STEP 5 Column Assignment

(Performed on above matrix)

Repeat step 4 & step 5

STEP 6 Reduce matrix further

select min. not cancelled element = 1

Not cancelled $\rightarrow$ sub.Single -11 $\rightarrow$ As it isDouble -11 $\rightarrow$ Add 1

A4 Long Book

DATE: / /

PAGE NO.:

M	1	2	3	4	5	6
✓ A	9	11	3	0	5	10
✓ B	38	7	1	5	3	0
✓ C	52	21	28	48	0	0
D	0	0	0	5	51	2
E	0	0	6	0	42	53
F	0	0	7	22	31	47

Repeat step 4 & step 5

Note: min. Not cancelled element = 0

 $\therefore$ We have to apply arbitrary zero method.

Note: When row + column assignment not satisfactory and min. not cancelled element is zero, then select any arbitrary zero and cancelled its corresponding row.

In arbitrary zero problem we always get two solution

M	1	2	3	4	5	6
✓ A	9	11	3	0	5	10
✓ B	38	7	1	5	3	0
✓ C	52	21	28	48	0	0
D	0	0	0	5	51	2
E	0	0	6	0	42	53
F	0	0	7	22	31	47

A4 Long Book

DATE: / /

PAGE NO.:

Soln 2

	1	2	3	4	5	6
A	7	4	3	10	5	10
B	39	0	1	5	3	10
C	32	31	28	48	0	0
D	0	0	0	15	34	4
E	0	0	6	0	42	53
F	0	0	7	22	31	47

Soln 1) Formulate the problem.

No need to formulate the above problem

Soln 1)

Men	Job	Time
A	4	10
B	6	11
C	5	8
D	3	10
E	1	18
F	2	10
		67 unit

Soln 2

Men	Job	Time
A	4	10
B	6	11
C	5	8
D	3	10
E	2	18
F	1	9
		67 unit

A4 Long Book

DATE: / /

PAGE NO.:

Q.4) If the ^{wages} of the worker are Rs 5 per hour for A, B, C each and Rs 3 per hour for D, E each the amount of time taken by each is given in the table. Determine min total time taken and minimize the total cost of getting 5 units of work done.

Job	A	B	C	D	E
1	7	3	3	3	(2)
2	6	(1)	6	6	5
3	(3)	4	9	10	7
4	(1)	5	2	2	4
5	6	6	9	4	(2)

Soln:

Wages = Rs 5/hr for A, B & C

Wages = Rs. 3/hr for D & E

Step 1 Formulate the problem

No need to formulate the above problem

Step 2 Row operation

	A	B	C	D	E
1	5	7	(1)	(1)	0
2	5	0	5	5	4
3	0	1	6	7	4
4	0	4	1	1	3
5	4	4	7	2	C

A4 Long Book

DATE: / /

PAGE NO.:

Step 4: Colm. operation

	A	B	C	D	E
1	0	7	0	0	0
2	5	0	0	4	0
3	0	0	0	0	4
4	0	5	0	0	3
5	4	0	0	0	0

Step 4) Row Assignment
(Perform on above matrix)Step 5) Colm Assignment
(Perform on above matrix)

Repeat Step 4 & Step 5

Step 6) Reduce matrix further

Here, the min. not cancelled
element is zero∴ we can not solve the problem
by this method

∴ Apply Arbitrary zero method.

0	0	0	0	0
0	0	0	0	0

A4 Long Book

DATE: / /

PAGE NO.:

Ex: 1) :-

Job	Employee	Time	wages	cost
1	C	3	5	15
2	B	1	5	5
3	A	3	5	15
4	D	2	3	6
5	E	2	3	6
11 hrs.				47/-

Ex: 2

Job	Employee	Time	wages	cost
1	D	3	3	9
2	B	1	5	5
3	A	3	5	15
4	C	2	5	10
5	E	2	3	6
11 hrs				45/-

A4 Long Book

DATE: / /

Step 2 Row operation

	A	B	C	D	E
1	10	3	0	12	0
2	0	16	13	19	4
3	0	8	7	10	5
4	15	2	0	(4)	4
5	33	0	21	28	23

Step 3 Colm operation. (perform on above matrix)

	A	B	C	D	E
1	10	3	0	12	0
2	0	16	13	19	4
3	0	8	7	10	5
4	15	2	0	4	4
5	33	0	21	24	23

Step 4 Row Assignment (perform above matrix)

Step 5 Colm assignment (p. above matrix)
Repeat step 4 + step 5

Step 6 Reduce matrix further.
Select min. not cancelled element = 4
Not cancelled → sub 4
Single → As 4 is
Double → Add 4

DATE: / /

R.5) A different machine can do any of the 5 required jobs with different profit resulting from each assignment as shown below. Find out maximum profit possible through optimum assignment.

	A	B	C	D	E
1	30	37	40	28	40
2	40	29	27	21	36
3	40	32	33	20	35
4	25	38	40	36	36
5	29	62	41	34	39

Self: Minimization:-

Job	A	B	C	D	E
1	32	25	(22)	34	22
2	(22)	38	35	41	26
3	(22)	30	29	32	27
4	27	24	(22)	26	26
5	33	0	21	28	23

Step 1 formulation the problem
No need to formulate

Step 2 Row operation

DATE: / /

PAGE NO.:

DATE: / /

PAGE NO.:

	A	B	C	D	E
1	11	7	8	9	
2	6	16	9	11	0
3	3	8	3	2	1
4	13	6	0	0	4
5	3/3	0	17	20	13

Again Repeat step 4 & 5

SOLN

Job	machine	Profit
1	C	40
2	E	36
3	A	40
4	D	36
5	B	62
Rs.		214/-

Q.6) Three buildings to be added in college campus. Find the assignment to minimize the total cost from construction.

	A	B	C
1	2.9	1.6	3.1
2	3.1	2.8	2.2
3	3.0	1.8	3.0
4	2.9	1.8	2.8
5	3.2	1.7	3.0

A4 Long Book

A4 Long Book

DATE: / /

PAGE NO.:

Minimization

H \ F	I	II	III	IV
A	30000	29750	27000	20000
B	31000	30500	28250	24000
C	31250	31250	32000	24000
D	31500	32000	32000	24000

Step 1 formulate the problem

No need to formulate as we already formulate it.

Step 2 Row operation

	I	II	III	IV
A	30000	29750	27000	0
B	11000	10500	8250	0
C	(7250)	(7250)	(8000)	0
D	7500	8000	8000	0

Step 3 Column operation

	I	II	III	IV
A	22750	22500	19000	[0]
B	8750	8250	250	0
C	[0]	0	0	0
D	250	750	[0]	0

Step 4 Row Assignment

(perform on above matrix)

Step 5 Column Assignment

(P.O.A.M)

A4 Long Book

DATE: / /

PAGE NO.:

Step 6 Reduce matrix further

* select min. Not cancelled element = 250

Not cancelled $\rightarrow$ sub. 250single -11 $\rightarrow$ as it isDouble -11 $\rightarrow$ Add 250

	I	II	III	IV
VA	22500	22250	19000	[0]
B	3500	3000	250	0
C	0	[0]	250	250
D	0	500	[0]	0

Repeat step 4 & step 5

Step 7) Reduce matrix further

* select min. not cancelled ele. = 250

Not cancelled $\rightarrow$ sub. 250Single -11 $\rightarrow$ as it isDouble -11 $\rightarrow$ Add 250

	I	II	III	IV
VA	22250	22000	18750	[0]
VB	3250	2750	[0]	0
C	0	[0]	250	500
VD	[0]	500	0	250

Repeat step 4 & step 5

A4 Long Book

Q.8

A race horse owner has four horses name ABCD & plans to enter them in a four rarer if he wish to enter them in each race. How should he enter them so as to make his expected profit (total purse winning) as great as possible. The data is given in the following table.

Horse	Probability of winning			
	Race I	Race II	Race III	Race IV
A	0.4	0.3	0.2	0.8
B	0.2	0.2	0.15	0.3
C	0.15	0.1	0.0	0.2
D	0.1	0.0	0.0	0.2
Purse amount	5000	7500	25000	40000

Soln:

Maximization

Horse	Race I	Race II	Race III	Race IV
A	5000×0.4 = 2000	7500×0.3 = 2250	25000×0.2 = 5000	40000×0.8 = 32000
B	5000×0.2 = 1000	7500×0.2 = 1500	25000×0.15 = 3750	40000×0.3 = 12000
C	5000×0.15 = 750	7500×0.1 = 750	25000×0.0 = 0	40000×0.2 = 8000
D	5000×0.1 = 500	7500×0.0 = 0	25000×0.0 = 0	40000×0.2 = 8000

DATE: / /

PAGE NO.

DATE: / /

PAGE NO.

13

House	Rate	Profit
A	IV	32000
B	III	3750
C	II	750
D	I	500

37000/-

Q.9)

A company has territories open and four sales map available for assignment. The territories are not rich to their potential. It is estimated that typical salesman operating in each territories to bring the following data

Territories	I	II	III	IV
Sales (in 1000's Rs)	60	50	40	35

For salesman also considering to before in their ability. It is estimated that the working under same condition there are as follows

Salesman	A	B	C	D
Proportion	7	5	5	4

Salesman → maximization

maximization →

Salesman	I	II	III	IV
A	7x60	7x50	7x40	7x35
B	5x60	5x50	5x40	5x35
C	5x60	5x50	5x40	5x35
D	4x60	4x50	4x40	4x35

S	I	II	III	IV
A	(420)	350	280	245
B	300	250	200	175
C	300	250	200	175
D	240	200	160	140

Minimization:-

S	I	II	III	IV
A	0	70	140	175
B	(120)	170	220	245
C	(120)	170	220	245
D	(120)	220	260	280

step 1) Formulate the problem
No need to formulate

step 2) Read operation

S	I	II	III	IV
A	0	70	140	175
B	0	50	100	125
C	0	50	100	125
D	0	(40)	(30)	(100)

A4 Long Book

A4 Long Book

DATE: / /

PAGE NO.:

Step 3 Column operation

	I	II	III	IV
✓ A	10	30	60	75
B	0	10	20	25
C	0	10	20	25
D	0	0	0	0

Step 4 Row Assignment

(Perform on above matrix)

Step 5 Column Assignment

(P. on a matrix)

Step 6 Reduce matrix further

Select min. not cancelled = 10

Not cancelled → Sub 10

Single cancelled → As it is

Double cancelled → Add 10

	I	II	III	IV
✓ A	0	20	50	65
✓ B	0	0	10	15
C	0	0	10	15
D	10	0	0	0

Repeat Step 4 & Step 5

A4 Long Book

DATE: / /

PAGE NO.:

Step 7 Reduce matrix further

Select min. not cancelled element = 10

Not cancelled → Sub 10

Single - 10 → As it is

Double - 10 → Add 10

	I	II	III	IV
✓ A	0	10	40	55
B	0	0	0	5
✓ C	0	0	0	5
D	20	0	0	10

Repeat Step 4 & Step 5

Sol (1):-

Salesman	Territories	Profit
A	I	420
B	II	250
C	III	200
D	IV	140
		1010

Sol (2):

Salesman	Territories	Profit
A	I	420
B	III	200
C	II	250
D	IV	140
		1010

∴ Sales in 1000's Rs.

$$= 1010 \times 1000 = 4209523 \text{ Rs.}$$

$$(7+5+5+9)$$

A4 Long Book

DATE: / /

PAGE NO.:

Q-10

A five swimmers are eligible to compete relay which is to be consist of four swimmers swimming in four different type i.e. backstroke, breaststroke, freestyle and butterfly. The time taken for five swimmers A, B, C, D, E to cover a distance of 100m in various swimming time given below in minutes and seconds. A swims backstroke in ~~1:07~~ 1:09, breaststroke in 1:05 and never compete in the freestyle butterfly.

→ 'B' is a freestyle specialist arranging 1:01 for 100m but can also swim breaststroke and butterfly 1:16, 1:20 respectively.

→ 'C' swims all style i.e. back, butterfly, free, breast in 1:10, 1:12, 1:05 and 1:20 resp.

→ 'D' swims only the butterfly 1:11 while 'E' swims backstroke 1:20, breaststroke 1:16, free style 1:06 and butterfly 1:10, which swimmers should be assigned to which style and who will not do a relay.

A4 Long Book

DATE: / /

PAGE NO.:

15

Swimmer	Back stroke	Breast stroke	Free stroke	Butterfly
A	1:09	1:05	—	—
B	—	1:16	1:01	1:20
C	1:10	1:20	1:05	1:12
D	—	—	—	1:11
E	1:20	1:16	1:06	1:10

* Replace (—) with ∞ as the problem is a minimum in dummy column minimization

	P	Q	R	S	T
A	(63)	(65)	∞	∞	0
B	∞	76	(61)	80	0
C	70	80	65	72	0
D	∞	∞	∞	71	0
E	30	76	66	(70)	0

Step 1 Formulate the problem

No need to formulate as we already introduce 1 dummy column

Step 2 Row operation

No need to perform

Step 3 Column operation

	P	Q	R	S	T
A	0	0	∞	∞	0
B	∞	11	0	10	0
C	1	15	4	2	0
D	∞	∞	∞	0	0
E	11	11	5	0	0

A4 Long Book

DATE: / /

PAGE NO.:

STEP 4:- Row Assignment
perform on above matrix

STEP 5:- Column Assignment
perform on above matrix

STEP 6 Reduce matrix Further

→ select min. Not cancelled element = 1
Not cancelled → Sub. 1
Single → → As it is
Double → → Add 1

	P	Q	R	S	T
A	0	10	0	0	0
B	0	11	9	11	1
C	10	14	5	2	0
D	0	0	0	1	10
E	10	10	9	10	0

Repeat step 4 & 5

swimmer	swimming style	Time
A	Q	65
B	R	61
C	P	70
D	T	0
E	S	70

• The swimming styles assigned with a swimmer to the swimmer are as follow:-

Swimmer A → Breast stroke

Swimmer B → Free style

Swimmer C → Back stroke

A4 Long Book

DATE: / /

PAGE NO.:

Swimmer E → Butterfly
If the swimmer D is not in relay.

6.11) X, Y, Z company has taken the third floor of a multi store building for rent with a crew to locate one of their zonal office there are five main rooms on the manager floor to be assign to five managers. Each room has its own advantages and disadvantages. The room different size and shape each of the five managers there ask to bank to room preferences among the room 301, 302, 303, 304, 305. This preferences will recorded in following table most of the manager did not like on the room, since they were not satisfy with some of this room and they have left this from the list.

As we mean this preferences can be confirm by number find out which manager should be assign which room so that there total preferences ranking is minimum.

Room (Preference)	m ₁	m ₂	m ₃	m ₄	m ₅
1	302	302	303	302	301
2	303	304	301	305	302
3	304	305	304	304	304
4	-	301	305	304	-
5	-	-	302	-	-

A4 Long Book

DATE: / /

DATE: / /

17

Room	m ₁	m ₂	m ₃	m ₄	m ₅
301	—	4	2	—	1
302	1	1	5	1	2
303	2	—	1	4	—
304	3	2	3	3	3
305	—	3	4	2	—

Manager	Room No.
m ₁	302
m ₂	304
m ₃	303
m ₄	305
m ₅	301

	m ₁	m ₂	m ₃	m ₄	m ₅
301	∞	4	2	∞	(1)
302	(1)	1	5	1	2
303	2	∞	(1)	4	∞
304	3	(2)	3	3	3
305	∞	3	4	(2)	∞

Step 1:- Formulate the problem
No need to formulate

Step 2:- Row operation

	m ₁	m ₂	m ₃	m ₄	m ₅
✓ 301	∞	3	1	∞	0
302	0	0	4	0	1
✓ 303	1	∞	0	3	∞
✓ 304	1	0	1	1	1
✓ 305	∞	1	2	0	∞

Step 3:- Col^m operation
No need to perform the col^m operation

Step 4:- Row Assignment

Step 5:- Col^m Assignment

A4 Long Book

A4 Long Book

Ques:- A solicitor firm has employees: 308: one of daily work. There are five typist and their charges different. According to learner understanding only one job is given to one typist and typist is paid for full hourly. even if one works for a fraction of an hour of an hourly find the least cost allocation for the following data.

Typist	Rate per hrs (Rs)	No. of pages per hr	Job	Rate per page
A	5	12	P	199
B	6	14	Q	175
C	3	8	R	245
D	7	10	S	298
E	4	11	T	178

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Q11) No. of pages x Rate Sheet $AP = 191 \times 5 = 955$ $AQ = 177 \times 5 = 885$ $AR = 145 \times 5 = 725$ $AS = 99 \times 5 = 495$ $= 16.52 \times 5 = 82.6$ $= 14.59 \times 5 = 72.95$ $= 12.09 \times 5 = 60.45$ $= 24.83 \times 5 = 124.15$ $= 17 \times 5 = 85$ $= 15 \times 5 = 75$ $= 13 \times 5 = 65$ $= 27 \times 5 = 135$ <table border="1" style="width: 100%; text-align: center;"> <tr> <th>Split/304</th> <th>P</th> <th>Q</th> <th>R</th> <th>S</th> <th>T</th> </tr> <tr> <td>A</td> <td>85</td> <td>75</td> <td>65</td> <td>135</td> <td>75</td> </tr> <tr> <td>B</td> <td>90</td> <td>74</td> <td>66</td> <td>122</td> <td>78</td> </tr> <tr> <td>C</td> <td>75</td> <td>68</td> <td>57</td> <td>114</td> <td>64</td> </tr> <tr> <td>D</td> <td>80</td> <td>72</td> <td>60</td> <td>150</td> <td>72</td> </tr> <tr> <td>E</td> <td>76</td> <td>69</td> <td>56</td> <td>112</td> <td>68</td> </tr> </table> Step 1 Formulate the problem No need to formulate the above problem Step 2 Row operation <table border="1" style="width: 100%; text-align: center;"> <tr> <th></th> <th>P</th> <th>Q</th> <th>R</th> <th>S</th> <th>T</th> </tr> <tr> <td>A</td> <td>20</td> <td>10</td> <td>0</td> <td>60</td> <td>10</td> </tr> <tr> <td>B</td> <td>24</td> <td>12</td> <td>0</td> <td>66</td> <td>12</td> </tr> <tr> <td>C</td> <td>18</td> <td>9</td> <td>0</td> <td>57</td> <td>12</td> </tr> <tr> <td>D</td> <td>20</td> <td>12</td> <td>0</td> <td>60</td> <td>12</td> </tr> <tr> <td>E</td> <td>20</td> <td>8</td> <td>0</td> <td>56</td> <td>12</td> </tr> </table>	Split/304	P	Q	R	S	T	A	85	75	65	135	75	B	90	74	66	122	78	C	75	68	57	114	64	D	80	72	60	150	72	E	76	69	56	112	68		P	Q	R	S	T	A	20	10	0	60	10	B	24	12	0	66	12	C	18	9	0	57	12	D	20	12	0	60	12	E	20	8	0	56	12	Step 3 Column operation <table border="1" style="width: 100%; text-align: center;"> <tr> <th></th> <th>P</th> <th>Q</th> <th>R</th> <th>S</th> <th>T</th> </tr> <tr> <td>A</td> <td>2</td> <td>1</td> <td>0</td> <td>4</td> <td>0</td> </tr> <tr> <td>B</td> <td>3</td> <td>1</td> <td>0</td> <td>11</td> <td>2</td> </tr> <tr> <td>C</td> <td>0</td> <td>1</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>D</td> <td>2</td> <td>1</td> <td>0</td> <td>4</td> <td>2</td> </tr> <tr> <td>E</td> <td>2</td> <td>0</td> <td>0</td> <td>0</td> <td>2</td> </tr> </table> Step 4 Row adjustment perform on above matrix Step 5 Column operation perform on above matrix Step 6 Reduce matrix further select min not cancelled element = 1 Not cancelled = -1 single -1 = As it is Unstable -1 = 1 <table border="1" style="width: 100%; text-align: center;"> <tr> <th></th> <th>P</th> <th>Q</th> <th>R</th> <th>S</th> <th>T</th> </tr> <tr> <td>A</td> <td>3</td> <td>2</td> <td>10</td> <td>4</td> <td>0</td> </tr> <tr> <td>B</td> <td>3</td> <td>2</td> <td>11</td> <td>3</td> <td>2</td> </tr> <tr> <td>C</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>2</td> </tr> <tr> <td>D</td> <td>2</td> <td>2</td> <td>0</td> <td>3</td> <td>2</td> </tr> <tr> <td>E</td> <td>2</td> <td>0</td> <td>0</td> <td>0</td> <td>2</td> </tr> </table> Repeat step 4 & 5		P	Q	R	S	T	A	2	1	0	4	0	B	3	1	0	11	2	C	0	1	0	1	2	D	2	1	0	4	2	E	2	0	0	0	2		P	Q	R	S	T	A	3	2	10	4	0	B	3	2	11	3	2	C	0	1	0	0	2	D	2	2	0	3	2	E	2	0	0	0	2
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C	0	1	0	1	2																																																																																																																																												
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D	2	2	0	3	2																																																																																																																																												
E	2	0	0	0	2																																																																																																																																												

DATE: / /

PAGE NO.

step 7) Reduce matrix further.

select min not cancelled element = 2

	A	B	C	D	E
A	0	1	2	3	4
B	1	0	1	2	3
C	2	1	0	1	2
D	3	2	1	0	1
E	4	3	2	1	0

repeat step 4 & 5

Hence, the min not cancelled element is zero
we apply arbitrary zero method.

soln:- 1)

Typist	Job	Cost	Typist	Job	Cost
A	T	75	A	T	75
B	A	66	B	R	66
C	A	66	C	S	114
D	P	80	D	P	80
E	S	112	E	Q	64
		Rs 394			Rs 894

A4 Long Book

DATE: / /

PAGE NO.

19

* * Branch And Bound Technique * *

OR
Travelling salesman CR Little's Method

Ques:-

	A	B	C	D	E
A	0	1	4	2	-
B	1	0	3	1	3
C	5	4	0	4	2
D	2	1	5	0	1
E	3	2	3	1	0

STEP 1 Replace 0 with ∞ and perform row operation

A4 Long Book

DATE: / /

PAGE NO.:

Step 2) column operation

	A	B	C	D	E
A	∞	0	1	1	∞
B	0	∞	0	0	1
C	0	2	∞	2	0
D	1	0	2	∞	0
E	0	1	0	0	∞

Starting Journey $C \rightarrow E$

Note: 1) Calculate the penalty for each row

- 2) penalty is calculated by adding the smallest value present in row and column of that given
- 3) select the max. penalty and cancel it corresponding row and column and again the path not the return journey i.e. $B \rightarrow C$ is not permitted, therefore, assign the maximum penalty (∞).

	A	B	C	D
A	∞	0	1	1
B	0	∞	0	0
C	1	0	2	∞
D	2	1	∞	0

Route $\rightarrow C-E-D$

	A	B	C
A	∞	0	1
B	0	∞	0
C	1	0	∞

A4 Long Book

DATE: / /

PAGE NO.:

Route $\rightarrow C-E-D-B$

	A	C
A	∞	1
B	0	∞

Row operation

	A	C
A	∞	0
B	0	∞

Route $\rightarrow C-E-D-B-A$

Final Route $\rightarrow C-E-D-B-A$
 Least distance $\rightarrow (1+1+2+1+1)+2+1$
 $= 9$ unit

A4 Long Book

same destination only.

DATE: / /

PAGE NO.:

Ques:- A salesman wants to visit the cities A, B, C, D, E. Find the least cost travelling route.

	A	B	C	D	E
A	0	2	5	7	1
B	5	0	3	8	2
C	8	7	0	4	7
D	12	4	6	0	5
E	1	3	2	8	0

Soln:- step I Replace 0 with ∞ and perform row operation.

	A	B	C	D	E
A	∞	2	5	7	(1)
B	6	0	3	8	(2)
C	8	7	∞	(4)	7
D	12	(4)	6	∞	5
E	(1)	3	2	8	∞

	A	B	C	D	E
A	∞	1	4	6	0
B	4	∞	5	0	3
C	4	5	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

A4 Long Book

PAGE NO.:

DATE: / /

step II Column operation

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	5	0
C	4	3	∞	0	3
D	8	0	1	∞	1
E	0	2	0	7	∞

step III

Calculate the penalty for each zero & select the max^m penalty and assign the path.

Route: $\overset{(R)}{C} \rightarrow \overset{(C)}{D}$ $\overset{(C)}{D} \rightarrow \overset{(C)}{E}$

after step III check row and col consist of zero if there is no zero then perform operation otherwise calculate penalty.

	A	B	C	E
A	∞	1	3	0
B	4	∞	0	0
C	4	0	∞	1
E	0	1	0	∞

Route: $C \rightarrow D \rightarrow E \rightarrow A$

A4 Long Book

DATE / /

PAGE NO.:

	B	C	E
A	1	3	∞
B	∞	0	0
D	0	∞	1

Row operation

	B	C	E
A	0	2	∞
B	0	0	0
D	0	∞	1

(Row A got being after that any diff. is add so if cut)

Route - $C \rightarrow D \rightarrow E \rightarrow A \rightarrow B$ (cont. cont. path being only one diff. add)

	C	E
B	0	∞
D	∞	1

Row operation

	C	E
B	0	0
D	∞	0

$\rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow A \rightarrow B \rightarrow C$

	E
D	∞

$\therefore$ Final Route = $C \rightarrow D \rightarrow E \rightarrow A \rightarrow B$

$\therefore$ least travelling cost = $(1+2+4+4+1)+1+1+1$
= 15 units

A4 Long Book

DATE / /

PAGE NO.:

max. time
+ + +
extra
added
daily

An airline that operates 7 days a week as a time table shown below. The crew must have a minimum lay over time of 5 hrs before flight after the pairing of flight that run on the lay over time away from home for any given pairing the crew will be gross at a city that result in smaller lay over. For each pairing also maintain the table where the crew will be less.

Flight No.	Delhi-Jaipur		Flight No.	Jaipur-Delhi	
	Departure	Arrival		Departure	Arrival
101	7:00 am	8:00 am	201	8:00 am	9:15 am
102	8:00 am	9:00 am	202	8:00 am	9:45 am
103	1:30 pm	2:30 pm	203	12:00 pm	2:45 pm
104	6:00 pm	7:30 pm	204	5:30 pm	6:45 pm

SOLN:

* Minimum Layover Time = 5 hrs.

$$(\text{Layover Time})_{\text{Jaipur}} = \left[\frac{\text{Arrival}}{\text{Time}} - \frac{\text{Departure}}{\text{Time}} \right]_{\text{Jaipur}}$$

$$(\text{Layover Time})_{\text{Delhi}} = \left[\frac{\text{Arrival}}{\text{Time}} - \frac{\text{Departure}}{\text{Time}} \right]_{\text{Delhi}}$$

1) Layover Time (Jaipur) $\Rightarrow$

$$\left\{ \begin{array}{l} 15 \text{ min} = 1 \text{ unit} \\ 1 \text{ hrs} = 4 \text{ unit} \end{array} \right\}$$

A4 Long Book

DATE: / /

PAGE NO.:

	201	202	203	204
101	24x4	24x4	28x4	(9x4)+2
102	23x4	23x4	27x4	(8x4)+2
103	17x4+2	17x4+2	21x4+2	27x4
104	12x4+2	12x4+2	16x4+2	22x4

	201	202	203	204
101	96	96	112	38
102	92	92	108	34
103	70	70	86	108
104	50	50	66	88

2) Layover time (Delhi) =>

	201	202	203	204
101	21x4+3=87	85	71	49
102	91	89	75	83
103	113	111	97	75
104	35	33	115	93

3) Minimum Layover Time

	201	202	203	204
101	87	85	71	(38)
102	91	89	75	(34)
103	70	(70)	86	75
104	35	(33)	66	88

A4 Long Book

DATE: / /

PAGE NO.:

23

4)	Row operation	201	202	203	204
101		49	47	33	0
102		57	55	(41)	0
103		0	0	(16)	5
104		2	0	33	55

Colm operation.

	201	202	203	204
✓ 101	49	47	17	19
102	57	55	25	9
- 103	(10)	0	0	5
✓ 104	2	0	17	55

Row Assignment (P.O.A.M.)

Colm Assignment (P.O.A.M.)

Now,

Reduce mat. further.

select min. Not cancelled element = 2

Not cancelled → sub. 2

single 11 → As it is

Delete 11 → Add 2

	201	202	203	204
✓ 101	47	47	15	(0)
102	55	55	23	0
- 103	0	2	(10)	7
- 104	0	(10)	15	55

A4 Long Book

Repeat Row assign. & Colm Assignment

Reduce matrix method further.

Select min not cancelled element = 15

Not cancelled $\rightarrow$ sub. 15

Single $\rightarrow$ As it is

Double $\rightarrow$ Add 15

	201	202	203	204
101	32	32	9	0
102	0	40	8	9
103	0	2	0	22
104	0	0	15	70

Repeat Row Assign. & Colm Assignment opposite.

Flight No.	Flight No.	lay out (time)	Head Quarter
101	203	71	Jairpur
102	204	34	Delhi
103	201	70	Delhi
104	202	33	Jairpur

Transportation Model

Transportation ^{deal with} of product ~~deal~~ deals with the transportation of product available at several sources to a number of different destination. It involve the movement of product from plant to ware house, warehouse to who-lesser, wholeseller to reseller, reseller to customer.

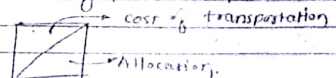
The condition of transportation model are :-
i) The problem must be a balance type i.e. supply = demand

If supply is not equal to demand then the problem is unbalance. Therefore balance the problem by adding dummy row and dummy column. Having first of transportation is zero.

ii) The objective must be minimization

Estimated origins	D ₁	D ₂	D ₃	Supply
01				10
02				20
Demand	5	20	10	35

Supply $\neq$ demand



DATE: / /

PAGE NO.:

There are three methods to find initial cost of transportation.

- 1) N-W corner Rule (North-west)
- 2) Least cost method
- 3) Vogel's approximation method (VAM)

Ques) Assign origin to destination to minimize the transportation cost.

	D ₁	D ₂	D ₃	Supply
O ₁	92	89	90	320
O ₂	91	91	95	1270
O ₃	87	90	92	190
Demand	100	180	350	

Solve the problem by N-W corner Rule

1) Least cost method.

2) Vogel's approximation method

to find initial cost.

Also find the optimum transportation cost by PIGDI'S method.

$$\text{Supply} = 320 + 1270 + 190 = 1780$$

$$\text{Demand} = 100 + 180 + 350 = 630$$

$$\text{i.e. Supply} \neq \text{Demand}$$

$$1780 \neq 630$$

∴ Supply ≠ demand, therefore the problem is unbalanced. So balance the problem by adding

A4 Long Book

DATE: / /

PAGE NO.:

dummy column i.e. (D₄) having cost of transportation zero.

$$\therefore \text{demand} = 1780 - 630 = 1150$$

Notes:-

1) Find the N-W corner and allocate the min of supply or demand.

origin \ destination	D ₁	D ₂	D ₃	D ₄	Supply
O ₁	92	89	90	0	320
O ₂	91	91	95	0	1270
O ₃	87	90	92	0	190
Demand	100	180	350	1150	1780

$$\therefore \text{Initial cost} = (92 \times 100) + (89 \times 180) + (90 \times 40) + (95 \times 310) + (0 \times 960) + (0 \times 190)$$

$$= \text{Rs. } 58,270/-$$

2) Least cost method.

origin \ destination	D ₁	D ₂	D ₃	D ₄	Supply
O ₁	92	89	90	0	320
O ₂	91	91	95	0	1270
O ₃	87	90	92	0	190
Demand	100	180	350	1150	1780

A4 Long Book

$$+ (87 \times 100) + (92 \times \dots)$$

$$= \text{Rs. } 57,000/-$$

Notes-

- 1) Select the min^m or least value for allocation
- 2) In case of tie always select maximum of (supply + demand)
- 3) Allocate the minimum of supply or demand.

2) VAM Method

	D ₁	D ₂	D ₃	D ₄	Supply	Column penalty	R _i	starting
O ₁	92	89	90	0	320	89, 90, 92	0	2000
	*	*	320	*	*	1, F		
O ₂	91	91	95	0	1270	91, 91, 95	3	
	*	120	*	1150	0			
O ₃	87	90	92	0	135	87, 90, 92	2	
	100	60	30	*	0	2, Z		
Demand	400	1150	1150	0	1780			
Row	4, 4	1, 1	2, 2	0, 0				
Rowby	F	1	2	0, 0				
KJ	85	88	90	-3				

$$\text{Initial cost} = (90 \times 320) + (91 \times 120) + (90 \times 1150)$$

$$+ (87 \times 100) + (90 \times 60) + (92 \times 30)$$

$$= \text{Rs. } 56,500/-$$

step 1) calculate the row penalty and column penalty by subtracting two minimum element of that column and row.

step 2) select the max^m penalty

step 3) enter into the matrix from the selected maximum penalty

step 4) select the minimum for allocation

step 5) allocate the min^m of supply and demand.

* optimization by using: moor's method

step 1) check the test of optimality

$$\text{No. of Allocation} = m + n - 1$$

$\downarrow$ $\downarrow$
 no. of rows no. of columns

$$= 3 + 4 - 1$$

$$= 6 - 1$$

No. of Allocation = 6.

Actual Allocation = 6

since both are equal test of optimality satisfied.

step 2) calculate R_i & K_j for stone's square only such that $C_{ij} = R_i + K_j$

C_{ij}	alloc.
----------	--------

Stone square

C_{ij}	*
----------	---

Water square

DATE: / /

PAGE NO.:

DATE: / /

PAGE NO.:

27

- 1) $O_1D_1 = 92 - (0 + 85) = 7$
- 2) $O_1D_2 = 89 - (0 + 88) = 1$
- 3) $O_1D_3 = 0 - (0 - 3) = 3$
- 4) $O_2D_1 = 91 - (3 + 85) = 3$
- 5) $O_2D_2 = 35 - (3 + 90) = 2$
- 6) $O_3D_3 = 0 - (2 - 3) = 1$

Since, all Δ 's are positive therefore No need to further improvement.
 $\therefore$ Optimum cost = Initial cost = ~~56,580~~ -

A manufacturer has the distribution centre at located at ABC. The centre has 40, 20, 40 unit of product available with them. The retail outlet required following numbers of unit. $P=25$, $Q=10$, $R=20$, $S=30$. The transportation cost per unit in rupees for each centre and outlet given in the following table. Find the optimum transportation assignment to minimize the transportation cost.

Supply	Demand			
	P	Q	R	S
A	50	30	40	50
B	35	30	100	45
C	40	60	95	35

$$\text{Supply (A, B, C)} = 40 + 20 + 40 = 100$$

$$\text{Demand} = 25 + 10 + 20 + 30 = 85$$

(P, Q, R, S)

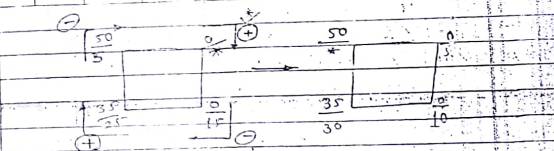
i.e. Supply $\neq$ demand

$\therefore$ The problem is unbalanced

$\therefore$ Balance the problem by adding dummy column having cost of transportation zero (and)

$$\text{Demand} = 100 - 85 = 15$$

	P	Q	R	S	T	Supply	column penalty	Rk
A	50	30	40	50	0	40	30, 30, 20	0
	5	10	20	5	*	20	0, 0	
B	35	30	100	45	0	20	39, 39, 5	-15
	20	*	*	*	*	0	10, 10	
C	40	60	95	35	0	40	35, 35	-15
	*	*	*	25	15	25	5, 5, 5	
Dummy	25	10	20	30	15	100		
	0	0	0	0	0	100		
Row penalty	5, 5	30, 30	25	10, 10	0, 0			
Column penalty	15	10	10	10	15			
Rk	50	0	40	50	25			



A4 Long Book

A4 Long Book

* Optimization by using MODI's method

STEP 1: Check the feasibility of optimality
 No. of Allocation = demand + supply - 1 = 7
 Actual Allocation = 7

DATE: / /

2. Test of optimality is satisfied. Horizontal & vertical line.

- 1) Give '+' sign to water sq.
- 2) Give Alternate sign (+, -, +, -)
- 3) select lowest Allocation from '-' sign.

	P	Q	R	S	T	Ri
A	50	30	40	50	0	0
B	35	20	100	45	0	-15
C	40	60	95	35	0	0
KT	50	30	40	35	0	

	P	Q	R	S	T	Ri
A	50	30	40	50	0	0
B	35	20	100	45	0	-15
C	40	60	95	35	0	0
KT	50	30	40	35	0	

Find New IF (from water sq. only)

$$A_S = 50 - (0 + 35) = 15$$

$$B_R = 30 - (-15 + 30) = 15$$

$$B_R = 100 - (-15 + 40) = 75$$

$$B_S = 45 - (-15 + 35) = 25$$

$$B_T = 0 - (-15 + 0) = 15$$

$$C_P = 40 - (0 + 50) = -10$$

$$C_R = 60 - (0 + 30) = 30$$

$$C_R = 95 - (0 + 40) = 55$$

$$C_T = 35 - (0 + 40) = -5$$

$$K_T = 50 - (0 + 40) = 10$$

$$K_T = 30 - (0 + 35) = -5$$

$$K_T = 40 - (0 + 50) = -10$$

$$K_T = 35 - (0 + 40) = -5$$

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$$K_T = 30 - (0 + 35) = -5$$

A4 Long Book

DATE: / /

PAGE NO.:

	P	Q	R	S	T	Ri
A	50	30	40	50	0	0
B	35	20	100	45	0	-15
C	40	60	95	35	0	0
KT	40	30	40	35	0	

Again find New I.P. (from water sq.)

$$A_P = 50 - (0 + 40) = 10$$

$$A_S = 50 - (0 + 35) = 15$$

$$B_Q = 30 - (-5 + 30) = 5$$

$$B_R = 100 - (-5 + 40) = 65$$

$$B_S = 45 - (-5 + 35) = 15$$

$$B_T = 0 - (-5 + 0) = 5$$

$$C_Q = 60 - (0 + 30) = 30$$

$$C_R = 95 - (0 + 40) = 55$$

$$C_T = 35 - (0 + 40) = -5$$

$$K_T = 40 - (0 + 40) = 0$$

$$K_T = 30 - (0 + 35) = -5$$

$$K_T = 40 - (0 + 50) = -10$$

$$K_T = 35 - (0 + 40) = -5$$

$$K_T = 30 - (0 + 35) = -5$$

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$$K_T = 30 - (0 + 35) = -5$$

$$K_T = 40 - (0 + 50) = -10$$

A4 Long Book

DATE: / /

PAGE NO.:

Soln:

$$\text{Supply} = 10 + 80 + 15 = 105$$

$$\text{Demand} = 75 + 20 + 50 = 145$$

since, supply $\neq$ demand, therefore problem is unbalanced

∴ Balance the problem by adding dummy row (or) having

Supply = $145 - 105 = 40$, with cost of transportation 5, 3 & 2 units.

	D	E	F	Supply	Column Penalty	Ri
A	5	10	3	10	4, 1, F	0
B	6	20	6	32	2, 2, 2, 2	3
C	3	15	5	23	1, 1, 1, F	0
G	5	3	2	10	1, 1, F	-1
Row Penalty	5	2	5	145		
	6	10	0	145		
Row	2, 2, 3	1, 1	3, 3			
Penalty		2	1			
	3	1	3			

• Optimizing by using modi's method
 ∴ check the cost of optimization

$$\text{No. of} = m + n - 1$$

$$\text{Allocation} = 4 + 3 - 1$$

$$= 6$$

$$\& \text{Actual Allocation} = 6$$

since, both are equal

∴ Test of optimality is satisfied

A4 Long Book

DATE: / /

PAGE NO.:

Step 2 :- calculate P_i & K_j for stones

Sq. only such that

$$C_{ij} = R_i + K_j$$

Step 3 :- Calculate I.P. for

costs sq. only such that

$$I.P. = C_{ij} - (R_i + K_j)$$

$$AD = 5 - (0 + 3) = 2$$

$$AF = 7 - (0 + 3) = 4$$

$$CE = 2 - (0 + 1) = 1$$

$$CF = 5 - (0 + 3) = 2$$

$$GD = 5 - (-1 + 3) = 3$$

$$GE = 3 - (-1 + 1) = 3$$

∴ since all I.P.'s are '+ve'

∴ No need of further Improvement

$$\therefore \text{Optimize cost} = (1 \times 10) + (6 \times 20) + (4 \times 10)$$

$$+ (6 \times 10) + (3 \times 15) + (2 \times 40) = 525$$

A4 Long Book

DATE: / /

PAGE NO.:

Q.6

Soln.

$$\text{Supply} = 5 + 6 + 2 + 9 = 22$$

$$\text{Demand} = 4 + 4 + 6 + 2 + 4 + 2 = 22$$

Since, Supply = Demand, problem is balanced.

	S_1	S_2	S_3	S_4	S_5	S_6	Supply	Cost penalty	R_i
w_1	7	10	7	4	7	9	50	5, 3, 3 2, 4, 0	0
w_2	5	1	5	5	3	3	54	2, 2, 2 4, 0, 1	3
w_3	4	1	2	2	1	5	20	2, 3, 4 1, 0, 0	0
w_4	4	3	6	9	0	9	42	4, 9, 4 1, 0, 0	0
Demand	4	4	6	2	4	2	22	8	
Row penalty	1, 1, 1, 1 3, 1	2, 4, 2, 5, 4, 4 2, 1, 2, 3, 0	4, 4, 1, 1 2, 3, 0	1, 1, 1, 1 5, 1	1, 1, 1, 1 5, 1	1, 1, 1, 1 5, 1			
K_j									

* Notes - 4 available (Colm + Rows) $\Rightarrow$ Give + mark in both Row & Colm.

Optimisation by using mod 2's method.

Step-1) Check the test of optimality

$$n_o. of = m + n - 1$$

$$\text{Allocation} = 4 + 6 - 1$$

$$= 9$$

$$P. \text{ Actual Allocation} = 8$$

Notes:-

Since, both are not equal the test of optimality fails and degeneracy exist.

To relieve the degeneracy, increase the number of allocation (stone square) by 1.

A4 Long Book

DATE: / /

PAGE NO.:

Allocate a value "E" infinite for small positive value (start to minimum water cost sale in such away that the no close loop should be form.

Note:- Here $\begin{bmatrix} 7 & 10 \\ 5 & 1 \end{bmatrix}$ is a minimum water cost cell (w_1, S_1) but from the cell a close loop is form, Hence rejected. Now, selecting another minimum water cost cell. (w_3, S_3) and no close loop form from this cell. (w_2, S_5). So replace.

$$\begin{array}{c} \text{water} \quad \text{stone sq.} \\ \begin{bmatrix} 7 & 10 \\ 5 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 10 \\ 5 & E \end{bmatrix} \end{array}$$

Step-2 Calculate R_i & K_j for stone sq. only such that

$$C_{ij} = R_i + K_j$$

Step-3 Calculate I.P. for water sq. only such that,

$$F.P = C_{ij} - (R_i + K_j)$$

$$w_1, S_1 = 7 - (0 + 0) = 7$$

$$w_1, S_2 = 10 - (0 + 2) = 12$$

$$w_1, S_4 = 4 - (0 + 0) = 4$$

$$w_1, S_5 = 7 - (0 + 0) = 7$$

$$w_2, S_1 = 5 - (3 + 0) = -2$$

$$w_2, S_3 = 5 - (3 + 2) = -5$$

$$w_2, S_4 = 5 - (3 + 0) = 2$$

$$w_3, S_2 = 3 - (0 + 2) = 5$$

$$w_3, S_5 = 9 - (0 + 0) = 9$$

$$w_3, S_6 = 1 - (0 + 0) = 1$$

$$w_4, S_1 = 9 - (0 + 0) = 9$$

A4 Long Book

DATE: / /

PAGE NO.:

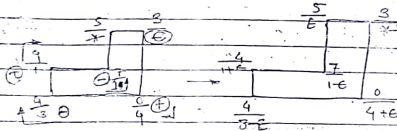
$$w_4 s_2 = 6 - (0 - 2) = 8$$

$$w_4 s_3 = 9 - (0 + 6) = 3$$

$$w_4 s_6 = 8 - (0 + 0) =$$

Since, J.P. values are (-ve) further improvement is Required.

∴ selecting highest negative value for improvement (Here, $w_4 s_3$).



	s_1	s_2	s_3
w_1	7	10	7
w_2	5	1	5
w_3	4	6	3
w_4	4	5	3
RHS	4	3	

DATE: / /

PAGE NO.:

31

Replacement Model

PAGE NO.:

Replacement Model

Individual Replacement

Group Replacement

The replacement of item is necessary because the efficiency of item decreases with the its time and maintenance cost i.e. running cost increases and salvage value decrease.

In the replacement model considering the capital cost, salvage value, running cost, discount rate, rate of interest. The replacement age of an item is decided. Items are classified as:

1) Deterative item

2) Non-deterative

For deterative items the replacement age is fixed and maintenance is possible. Mopet, car, bikes.

For non-deterative items the replacement age is not fixed and maintenance is not possible the item fails without any indication.

Electric bulb.

a) Deterative Item:-

$$P_{wn} = C - S_k \cdot \alpha^k + E R_k \cdot \alpha^{k-1} \quad \text{--- when ---}$$

running cost is incurred at the beginning of the year

$$P_{wn} = C - S_k \cdot \alpha^k + E R_k \cdot \alpha^{k-0.5}$$

--- when running cost is incurred at the mid of the year

$$P_{wn} = \frac{C - S_k \cdot \alpha^k + E R_k \cdot \alpha^k}{1 - \alpha^k} \quad \text{--- when ---}$$

at the end of the year

where,

P_{wn} = Present worth cost or Average Annual cost =

C = Capital cost

S_k = Salvage value or Resale value

R_k = Running cost i.e. maintenance cost

k = years

α = Discount Rate

$$\alpha = \frac{1}{1+i}$$

$i \Rightarrow$ rate of interest

PAGE NO.:

DATE: / /

DATE: / / Numericals



PAGE NO.:

A proposed cost Rupees. 12000/- when new the running cost and salvage value at the end of the given below. If the rate of interest on capital invested 15% per year and Running cost is assumed to have incurred at the mid of the year find when the Replacement should be done

Year (K)	1	2	3	4	5	6	7
Running cost (R _k)	1000	1200	1400	1500	2300	2400	3000
Salvage value (S _k)	9000	8000	6000	5000	4000	4000	4000

Data:- C = 12000/-
i = 15%

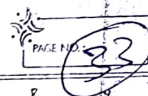
$$i.e. \alpha = \frac{1}{1+i} = \frac{1}{1+0.15} = 0.87$$

Find:- Replacement Age = ?

Ans:- Since Running cost is incurred at the mid of the year.

$$\therefore PWD = C - \sum_{k=1}^n S_k \alpha^k + \sum_{k=1}^n R_k \alpha^{k-0.5} \quad (\text{mid year})$$

DATE: / /



k	P _k	S _k	α^k	$P_k \alpha^{k-0.5}$	$S_k \alpha^k$	$P_k \alpha^{k-0.5} - S_k \alpha^k$	$1 - \alpha$	$\frac{C}{1-\alpha}$
1	1000	9000	$0.87 = 0.87$	$1000 \times 0.87 = 870$	$9000 \times 0.87 = 7830$	$870 - 7830 = -6960$	0.13	$\frac{12000}{0.13} = 92307.7$
2	1200	8000	$0.87^2 = 0.757$	6056	$8000 \times 0.757 = 6056$	0	0.13	92307.7
3	1400	6000	$0.87^3 = 0.658$	3948	$6000 \times 0.658 = 3948$	0	0.13	92307.7
4	1800	5000	$0.87^4 = 0.572$	2860	$5000 \times 0.572 = 2860$	0	0.13	92307.7
5	2300	4000	$0.87^5 = 0.498$	1992	$4000 \times 0.498 = 1992$	0	0.13	92307.7
6	2400	4000	$0.87^6 = 0.433$	1732	$4000 \times 0.433 = 1732$	0	0.13	92307.7
7	3000	4000	$0.87^7 = 0.377$	1508	$4000 \times 0.377 = 1508$	0	0.13	92307.7

As PWD goes on decreasing and does not reach minimum in 7 year therefore the replacement age for proposed is more than 7 years

Q.2

A luxury car purchase rupees 1200000/- The running cost for bus is given by $R_N = 10000 + 5000N$, where N = Age of bus in years. The resale value of bus in lakhs of rupees is given in the following table. The rate of interest is 15%. Find optimal replacement age of bus.

Year	1	2	3	4	5	6	7	8	9	10
Resale (Lakhs)	10	8	6	6	4	3	3	3	3	3

S_k

Soln: $C = 12,00,000/-$
 $R_k = 10000 + 5000N$ Age of bus
 $i = 15\% = 0.15$
 $i.e. \alpha = \frac{1}{1+i} = \frac{1}{1+0.15} = 0.87$
 Find: Replacement Age = ?
 com. Assuming the running cost is incurred at the beginning of the year.

$$P_{min} = \frac{C - S_k \alpha^k + \sum R_k \alpha^{k-1}}{1 - \alpha^k}$$

K	S_k	R_k	α^k	$S_k \alpha^k$	α^{k-1}	$R_k \alpha^{k-1}$	$\sum R_k \alpha^{k-1}$	$1 - \alpha^k$	P_{min}
1	10×10^3	10,000	0.87	8.7×10^3	1	10,000	10,000	0.13	26.15×10^3
2	8×10^3	15,000	0.757	6.056×10^3	0.87	13.05×10^3	23.05×10^3	0.243	25.4×10^3
3	6×10^3	20,000	0.658	3.948×10^3	0.757	15.14×10^3	38.19×10^3	0.344	24.02×10^3
4	4×10^3	25,000	0.572	2.288×10^3	0.658	16.94×10^3	54.57×10^3	0.428	21.24×10^3
5	2×10^3	30,000	0.498	1.000×10^3	0.572	17.16×10^3	71.75×10^3	0.502	20.37×10^3
6	0	35,000	0.433	0	0.498	17.43×10^3	89.18×10^3	0.567	19.60×10^3
7	0	40,000	0.377	0	0.433	17.32×10^3	106.5×10^3	0.623	19.15×10^3
8	0	45,000	0.328	0	0.377	16.74×10^3	123.4×10^3	0.672	18.8×10^3
9	0	50,000	0.285	0	0.328	16.4×10^3	139.55×10^3	0.715	18.5×10^3
10	0	55,000	0.248	0	0.285	15.67×10^3	155.22×10^3	0.752	17.03×10^3

$R_k = 10000 + 5000N$ ① $R_1 = 10000 + 5000 \times 0 = 10,000$
 When $k=1$, then $N=0$.
 When $k=2$, then $N=1$. ② $R_2 = 10,000 + 5000 \times 1 = 15,000$
 When $k=3$, then $N=2$. ③ $R_3 = 10,000 + 5000 \times 2 = 20,000$
 When $k=10$, then $N=9$. ④ $R_{10} = 10,000 + 5000 \times 9 = 55,000$

A4 Long Book

As P_{min} goes on decreasing and doesn't reach min in 10 years.

∴ Replacement age for bus is more than 10 years.

Q.3) A machine was purchased for Rs 150,000/- and has a maintenance cost $10,000 + 5000N$. The machine has no resale value but a scrap value of Rs 10,000. Determine optimum replacement age of m/c in year, where N is the no. of year of m/c used. What would be the replacement age if it has negligible resale value?

Soln: $C = 150,000$
 $R_k = 10,000 + 5000N$
 $S_k = 10,000$ ($S_k = \text{constant}$ so sign value is given)

Find: ① Replacement age = ? when $S_k = 10,000$
 ② Replacement age = ? when $S_k = 0$

Assume $k = 10$ year.
 Assume: $i = 15\% = 0.15$

$$i.e. \alpha = \frac{1}{1+i} = \frac{1}{1+0.15} = 0.87$$

Assuming Running cost is incurred at the beginning of the year.

A4 Long Book

DATE: / /

PAGE NO.:

$$P_{0n} = \frac{c - S_k \alpha^k + \sum R_k \alpha^{k-1}}{1 - \alpha^k}$$

S_k	R_k	α^k	$S_k \alpha^k$	α^{k-1}	$R_k \alpha^{k-1}$	$\sum R_k \alpha^{k-1}$	$1 - \alpha^k$	$P_{0n} = \frac{c - S_k \alpha^k + \sum R_k \alpha^{k-1}}{1 - \alpha^k}$
10000	10000	0.87	8700	1	10000	10000	0.13	11.53×10^5
10000	15000	0.757	7570	0.87	1305×10^3	2305×10^3	0.243	6.37×10^5
10000	2000	0.658	6580	0.757	1514×10^3	3819×10^3	0.344	5.32×10^5
10000	25000	0.572	5720	0.658	169×10^3	5454×10^3	0.428	4.44×10^5
10000	30000	0.498	4980	0.572	1716×10^3	7175×10^3	0.502	4.32×10^5
10000	35000	0.43	4300	0.498	1743×10^3	8818×10^3	0.567	4.76×10^5
10000	40000	0.377	3770	0.43	1922×10^3	1065×10^3	0.623	4.05×10^5
10000	45000	0.327	3270	0.377	1616×10^3	1234×10^3	0.672	4.02×10^5
10000	50000	0.285	2850	0.327	1640×10^3	1388×10^3	0.715	4.01×10^5
10000	55000	0.248	2480	0.285	1567×10^3	15584×10^3	0.752	4.03×10^5

24)

DATE: / /

PAGE NO.:

5.10.

$$C = 50000 + 10000 = 60000$$

$$P_1 + R_0 = 10000$$

thereby increases by Rs 3000 per year

$$\therefore R_0 = 13000, R_1 = 16000, R_2 = 19000, R_3 = 22000$$

$$R_{10} = 25000$$

$$i = 20\% = 0.20$$

$$\therefore \alpha = \frac{1}{1+i} = \frac{1}{1+0.20} = 0.83$$

find replacement age = ?

Assuming running cost is incurred at the beginning
assume $S_k = 0$ since not given.

$$P_{0n} = \frac{c - S_k \alpha^k + \sum R_k \alpha^{k-1}}{1 - \alpha^k}$$

S_k	R_k	α^k	$S_k \alpha^k$	α^{k-1}	$R_k \alpha^{k-1}$	$\sum R_k \alpha^{k-1}$	$1 - \alpha^k$	P_{0n}
0	10000	0.87	0	1	10000	10000	0.13	123×10^5
0	15000	0.757	0	0.87	1305×10^3	2305×10^3	0.243	7.12×10^5
0	20000	0.658	0	0.757	1514×10^3	3819×10^3	0.344	5.47×10^5
0	25000	0.572	0	0.658	169×10^3	5454×10^3	0.428	4.78×10^5
0	30000	0.498	0	0.572	1716×10^3	7175×10^3	0.502	4.41×10^5
0	35000	0.43	0	0.498	1743×10^3	8818×10^3	0.567	4.81×10^5
0	40000	0.377	0	0.43	1922×10^3	1065×10^3	0.623	4.11×10^5
0	45000	0.327	0	0.377	1616×10^3	1234×10^3	0.672	4.06×10^5
0	50000	0.285	0	0.327	1640×10^3	1388×10^3	0.715	4.05×10^5
0	55000	0.248	0	0.285	1567×10^3	15584×10^3	0.752	4.06×10^5

Since the problem without is in increase in
the 10th year therefore the replacement age
of this m/c is 9th year.

A4 Long Book

k	S_k	R_k	α^k	$S_k \alpha^k$	α^{k-1}	$R_k \alpha^{k-1}$	$\sum R_k \alpha^{k-1}$	$1 - \alpha^k$	P_{0n}
1	0	10000	0.83	0	1	10000	10000	0.17	4.11×10^5
2	0	15000	0.689	0	0.83	12300	15300	0.311	6.57×10^5
3	0	20000	0.571	0	0.689	13780	29080	0.429	1.33×10^6
4	0	25000	0.474	0	0.571	15700	44780	0.526	1.57×10^6
5	0	30000	0.393	0	0.474	17400	62180	0.607	1.57×10^6
6	0	35000	0.326	0	0.393	19000	81180	0.674	1.49×10^6
7	0	40000	0.271	0	0.326	22160	103340	0.723	1.42×10^6
8	0	45000	0.225	0	0.271	25125	128465	0.775	1.45×10^6
9	0	50000	0.186	0	0.225	28125	156590	0.814	1.42×10^6
10	0	55000	0.155	0	0.186	31500	188115	0.845	1.42×10^6

Q.10.

Group Replacement

DATE: / /



PAGE NO.:

Steps 1 Calculate overall cost of group Replacement (OCGR)

$$OCGR = C_m N + C_r \sum_k n_k$$

Step 2 Calculate overall cost of Individual Replacement.

$$OCIR = \frac{C_r N}{\sum_k p_k}$$

Step 3 Compare

① If $OCGR < OCIR$ group replacement policy is adopted

② If $OCGR > OCIR$, individual replacement policy is adopted

$$n_1 = N \cdot p_1$$

$$n_2 = N \cdot p_2 + n_1 \cdot p_1$$

$$n_3 = N \cdot p_3 + n_1 \cdot p_2 + n_2 \cdot p_1$$

$$n_4 = N \cdot p_4 + n_1 \cdot p_3 + n_2 \cdot p_2 + n_3 \cdot p_1$$

$$n_5 = N \cdot p_5 + n_1 \cdot p_4 + n_2 \cdot p_3 + n_3 \cdot p_2 + n_4 \cdot p_1$$

where,

C_1 = Cost of group Replacement per item.

C_2 = Cost of Individual Replacement per item

$N = N_0 \rightarrow$ No. of items under consideration

K = Period, p_k = Probability

p_k = Probability of failure in k^{th} period

n_k = No. of failures in k^{th} period

A4 Long Book

Q.8

DATE: / /

PAGE NO.:

Soln:

$$N = N_0 = 1000, C_r = ₹ 2 / \text{bulb}$$

$$C_m = 0.50 / \text{bulb}$$

Find 1 replacement policy = 2

② Find $L_{90} = 2$ when performed individual replacement policy.

K	% failure	p_k	n_k	$\sum n_k$	$OCGR = C_m N + C_r \sum n_k$
1	0.1	$p_1 = 0.1$	$n_1 = 100$	100	$0.50 \times 1000 + 2 \times 100 = 700$
2	0.25	$p_2 = 0.15$	$n_2 = 160$	260	$0.50 \times 1000 + 2 \times 260 = 920$
3	0.5	$p_3 = 0.25$	$n_3 = 281$	541	$0.50 \times 1000 + 2 \times 541 = 1082$
4	0.8	$p_4 = 0.3$	$n_4 = 377$	918	$0.50 \times 1000 + 2 \times 918 = 1336$
5	1.0	$p_5 = 0.2$	$n_5 = 350$	1268	$0.50 \times 1000 + 2 \times 1268 = 1536$

Comparative probability diagram

$$① n_1 = N \cdot p_1$$

$$= 1000 \times 0.1 = 100$$

$$② n_2 = N \cdot p_2 + n_1 \cdot p_1$$

$$= 1000 \times 0.15 + 100 \times 0.1$$

$$n_2 = 160$$

A4 Long Book

DATE: / /

PAGE NO.:

$$\textcircled{3} \quad n_3 = n_2 p_2 + n_1 p_1 + n_0 p_0$$

$$= 1000 \times 0.25 + 100 \times 0.15 + 160 \times 0.1$$

$$= 281$$

$$\textcircled{4} \quad n_4 = n_3 p_3 + n_2 p_2 + n_1 p_1 + n_0 p_0$$

$$= 1000 \times 0.3 + 100 \times 0.25 + 160 \times 0.15 + 281 \times 0.1$$

$$= 377$$

$$n_5 = n_4 p_4 + n_3 p_3 + n_2 p_2 + n_1 p_1 + n_0 p_0$$

$$= 1000 \times 0.2 + 100 \times 0.3 + 160 \times 0.25 + 281 \times 0.15 + 377 \times 0.1$$

$$= 350$$

$$\therefore \text{OCIR} = 510$$

Same as individual case
previous value of sudden large
value in descending order.

$$\text{OCIR} = C_u n$$

$$\sum K P_k$$

$$= 2 \times 100$$

$$(1 \times 0.1) + (2 \times 0.15) + (3 \times 0.25)$$

$$+ (4 \times 0.3) + (5 \times 0.2)$$

$$\text{OCIR} = 597$$

Since $\text{OCIR} < \text{OCIR}$

Therefore, group Replacement policy is adopted.

1) To find C_u

To perform individual Replacement policy

$$0 < C_u R \leq \text{OCIR}$$

$$C_u \cdot n + C_f \sum_{k=1}^{\infty} P_k \geq 597$$

$$C_u \times 1000 + 2(250) = 597$$

$$C_u = 0.67 \text{ Rs/wh.}$$

therefore, group replacement policy is
app. adopted.

A4 Long Book

DATE: / /

PAGE NO.:

Q.9

37

Soln.

$$N = n_0 = 1000$$

$$C_f = 5 \text{ Rs}$$

$$C_u = 2 \text{ Rs}$$

Find Replacement policy?

$\therefore$ Failure = $100 - \%$ survival

K	% Survival	% Failure	P_k	n_k	$\sum n_k$	OCIR
0	100	0	$P_0 = 0.0$	—	—	—
1	97	$3 = 0.03$	$P_1 = 0.03$	$n_1 = 30$	30	2150
2	90	$10 = 0.1$	$P_2 = 0.1$	$n_2 = 70$	100.9	1252
3	70	$30 = 0.3$	$P_3 = 0.3$	$n_3 = 204.2$	305.1	1175
4	30	$70 = 0.7$	$P_4 = 0.7$	$n_4 = 417$	722.6	1402
5	15	$85 = 0.85$	$P_5 = 0.85$	$n_5 = 203$	925	1325
6	0	$100 = 1$	$P_6 = 1$	$n_6 = 259$	1185	1320

Complementary
probability
always 1

$$n_1 = n_0 P_1$$

$$= 1000 \times 0.03$$

$$= 30$$

$$n_2 = n_0 P_2 + n_1 P_1$$

$$= 1000 \times 0.07 + 30 \times 0.03$$

$$= 70.9$$

$$n_3 = n_0 P_3 + n_1 P_2 + n_2 P_1$$

$$= 1000 \times 0.2 + 30 \times 0.07 + 70.9 \times 0.03$$

$$= 204.227$$

A4 Long Book

DATE: / /

PAGE NO.:

$$n_4 = n_0 p_1 + n_1 p_2 + n_2 p_3 + n_3 p_4$$

$$= 1000 \times 0.8 + 30 \times 0.2 + 70.9 \times 0.07 + 204.2 \times 0.03$$

$$= 417.081$$

$$n_5 = n_0 p_1 + n_1 p_2 + n_2 p_3 + n_3 p_4 + n_4 p_1$$

$$= 1000 \times 0.15 + 30 \times 0.4 + 70.9 \times 0.2 + 204.2 \times 0.07 + 417 \times 0.03$$

$$= 203$$

$$n_6 = n_0 p_1 + n_1 p_2 + n_2 p_3 + n_3 p_4 + n_4 p_1 + n_5 p_1$$

$$= 1000 \times 0.15 + 30 \times 0.4 + 70.9 \times 0.2 + 204.2 \times 0.07 + 417 \times 0.03 + 203 \times 0.03$$

$$= 259$$

$$O.C.U.R = 1175$$

$$\text{Now, } O.C.I.R = \frac{C_1 \cdot N}{\sum k p_k} = \frac{5 \times 1000}{1 \times 0.03 + 2 \times 0.07 + 3 \times 0.2 + 4 \times 0.4 + 5 \times 0.15 + 6 \times 0.15}$$

$$O.C.I.R = 1243.78$$

Since, $O.C.U.R < O.C.I.R$

group replacement policy is adopted

DATE: / /

PAGE NO.:

80101

Data: $C_1 = 15/\text{tube}$ $N = 19000$ (assume) $C_2 = 60/\text{tube}$ n_0

10 times of
last value
of age of
of failure

Period	Age of failure	End of hrs (k)	p_k	n_k	Sn_k	O.C.U.R
1	0-200	200	$p_1 = 0.10$	$n_1 = 1000$	1000	1000
2	201-400	400	$p_2 = 0.26$	$n_2 = 2700$	3700	2700
3	401-600	600	$p_3 = 0.35$	$n_3 = 4130$	7730	4130
4	601-800	800	$p_4 = 0.22$	$n_4 = 3655$	11385	3655
5	801-1000	1000	$p_5 = 0.07$	$n_5 = 3278$	14663	3278

$$① n_1 = n_0 p_1$$

$$= 1000 \times 0.10 = 1000$$

$$② n_2 = n_0 p_2 + n_1 p_1$$

$$= 1000 \times 0.26 + 1000 \times 0.10 = 2700$$

$$③ n_3 = n_0 p_3 + n_1 p_2 + n_2 p_1$$

$$= 1000 \times 0.35 + 1000 \times 0.26 + 2700 \times 0.10 = 4130$$

Since O.C.U.R is increased in the 3rd period (600 hr)
the best interval for group replacement
is 2nd period (end of 400 hr)

A4 Long Book

A4 Long Book

DATE: / /

PAGE NO:

(2.11)

Soln: Data: (1) $N = P_0 = 5000$ Find no. of poles to be replace
 (2) $C_1 = Rs. 1500$, $C_2 = Rs. 200$ and replacement age
 decide replacement plan

K P_k P_k

⇒ (1) $N = P_0 = 5000$ Find no. of poles to be Replace
 each year.

(2) $C_1 = Rs. 1500$, $C_2 = Rs. 200$ and find
 replacement age & decide replacement plan

K	P _k	n _k	=	Σ n _k	$C_2 \cdot n_k = \frac{C_2 \cdot n_k \cdot S_{n-1}}{K}$
1	$P_1 = 0.01$	$n_1 = 50$	50	50	$\frac{200 \times 50 \times (50+1)}{2} = 2525 \times 10^3$
2	$P_2 = 0.02$	$n_2 = 100$	100	150	$\frac{200 \times 100 \times (100+1)}{2} = 10100 \times 10^3$
3	$P_3 = 0.03$	$n_3 = 150$	150	300	$\frac{200 \times 150 \times (150+1)}{2} = 22575 \times 10^3$
4	$P_4 = 0.04$	$n_4 = 200$	200	500	$\frac{200 \times 200 \times (200+1)}{2} = 40200 \times 10^3$
5	$P_5 = 0.05$	$n_5 = 250$	250	750	$\frac{200 \times 250 \times (250+1)}{2} = 62525 \times 10^3$
6	$P_6 = 0.06$	$n_6 = 300$	300	1050	$\frac{200 \times 300 \times (300+1)}{2} = 90300 \times 10^3$
7	$P_7 = 0.07$	$n_7 = 350$	350	1400	$\frac{200 \times 350 \times (350+1)}{2} = 122535 \times 10^3$
8	$P_8 = 0.08$	$n_8 = 400$	400	1800	$\frac{200 \times 400 \times (400+1)}{2} = 160400 \times 10^3$
9	$P_9 = 0.09$	$n_9 = 450$	450	2250	$\frac{200 \times 450 \times (450+1)}{2} = 202545 \times 10^3$
10	$P_{10} = 0.10$	$n_{10} = 500$	500	2750	$\frac{200 \times 500 \times (500+1)}{2} = 252500 \times 10^3$

$$(1) P_1 = P_0 \cdot P_1 = 5000 \times 0.01 = 50$$

$$(2) P_2 = P_1 \cdot P_2 + P_0 \cdot P_2 = 5000 \times 0.02 + 50 \times 0.01 = 100.5$$

M Long Book

DATE: / /

PAGE NO:

$$O.C.G.R. = 10.77 \times 10^5$$

n₀ = 1000

$$O.C.I.R. = C_1 \cdot N = 1500 \times 5000$$

$$S.K. P_k = (1 \times 0.01) + (2 \times 0.02) + (3 \times 0.03) +$$

$$(4 \times 0.04) + (5 \times 0.05) + (6 \times 0.06) +$$

$$(7 \times 0.07) + (8 \times 0.08) + (9 \times 0.09) +$$

$$(10 \times 0.10)$$

$$O.C.I.R. = 11.2 \times 10^5$$

Since, $O.C.G.R. < O.C.I.R.$
 ∴ Group replacement policy is adopted

M Long Book

DATE: / /

Q.7

Soln:

$$C = 1,00,000 \text{ ₹}$$

To find :- i) Replacement Period = ?

ii) $P_{\text{w}A} = 36,000$ ₹ should we change
equip A with B?

Assuming Running cost is incurred in
the beginning of the year.

$$\therefore P_{\text{w}A} = \frac{C - S_k \cdot \alpha^k + S R_k \cdot \alpha^{k-1}}{1 - \alpha^k}$$

Assuming

$$i = 10\%$$

$$\alpha = \frac{1}{1+i}$$

$$= \frac{1}{1+0.10}$$

$$= 0.91$$

A4 Long Book

DATE: / /

1	2	3	4	5	6	7	8
K	R_k	S_k	α^k	$S_k \cdot \alpha^k$	α^{k-1}	$R_k \cdot \alpha^{k-1}$	$\sum R_k \cdot \alpha^{k-1}$
1	1500	5000	0.91	4550	1	1500	1500
2	1300	2500	0.828	2070	0.91	1179	3229
3	2300	1250	0.757	1745	0.828	1704	5133.4
4	2300	600	0.686	1576	0.757	2186.6	7320
5	3600	400	0.629	2268	0.686	2467.6	9789.6
6	4500	400	0.567	2548	0.629	2803	12537.6
7	5500	400	0.516	2839	0.567	3185	15716.6

(9)

(10)

$$1 - \alpha^k$$

$$P_{\text{w}A} = \frac{C - (S_k) + (S R_k)}{1 - \alpha^k}$$

$$0.09$$

$$77.22 \times 10^3$$

$$0.172$$

$$64.87 \times 10^3$$

$$0.241$$

$$57.75 \times 10^3$$

$$0.314$$

$$53.84 \times 10^3$$

$$0.376$$

$$51.36 \times 10^3$$

$$0.493$$

$$51.66 \times 10^3$$

$$0.484$$

$$52.70 \times 10^3$$

∴ since $P_{\text{w}A}$ is increase in the 7th year.

∴ The Replacement period for the
equipment A is 6th years.

$$\therefore P_{\text{w}A} = 51.66 \times 10^3$$

$$\text{ii) } P_{\text{w}A} = 51.66 \times 10^3$$

$$P_{\text{w}B} = 36,000 = 36 \times 10^3$$

Since, $P_{\text{w}A}$ is greater than $P_{\text{w}B}$

∴ Replace equipment A with B in the
6th year

A4 Long Book

DATE: / /

PAGE NO.

DATE: / /

PAGE NO. 47

Soln

Machine A

C = 50,000

R₁ to R₅ = 800/-

R₆ = 1000

R₇ = 1200

R₈ = 1400

R₉ = 1600

R₁₀ = 1800

Machine B

C = 2500/-

R₁ to R₆ = 200

R₇ = 400

R₈ = 600

R₉ = 800

R₁₀ = 1000

$$i.e. \alpha = \frac{1}{1+i} = \frac{1}{1+0.10} = 0.91$$

$$S_k = 0$$

To find which m/c should be purchased?

Soln: Assuming the R.C is in the beginning of the year

$$P_{un} = C \cdot S_k \cdot \alpha^k + \sum R_k \cdot \alpha^{k-1}$$

↓

Machine A:-

K	R _k	S _k	α^k	$S_k \cdot \alpha^k$	α^{k-1}	$R_k \cdot \alpha^{k-1}$	$\sum R_k \cdot \alpha^{k-1}$	$1 - \alpha^k$	P_{un}
1	200	0	0.91	0	1	200	200	0.09	50.00×10^3
2	800	0	0.828	0	0.91	728	1528	0.172	232.58×10^3
3	800	0	0.759	0	0.828	662.4	2190.4	0.246	212.156×10^3
4	800	0	0.686	0	0.759	603.2	2793.6	0.314	162.152×10^3
5	800	0	0.624	0	0.686	548.8	3342.4	0.376	141.868×10^3
6	1000	0	0.567	0	0.624	624	3966.4	0.433	124.632×10^3
7	1200	0	0.516	0	0.567	680.4	4646.8	0.484	112.306×10^3
8	1400	0	0.470	0	0.516	658	5304.8	0.529	101.546×10^3
9	1600	0	0.427	0	0.470	683.2	5988	0.572	92.78×10^3
10	1800	0	0.387	0	0.427	700.2	6689.2	0.610	82.93×10^3

Machine B:-

K	R _k	S _k	α^k	$S_k \cdot \alpha^k$	α^{k-1}	$R_k \cdot \alpha^{k-1}$	$\sum R_k \cdot \alpha^{k-1}$	$1 - \alpha^k$	P_{un}
1	200	0	0.91	0	1	200	200	0.09	32×10^3
2	200	0	0.828	0	0.91	182	382	0.172	15.76×10^3
3	200	0	0.759	0	0.828	165.6	547.6	0.246	12.615×10^3
4	200	0	0.686	0	0.759	150.8	698.4	0.314	10.12×10^3
5	200	0	0.624	0	0.686	137.2	835.6	0.376	8.56×10^3
6	200	0	0.567	0	0.624	124.8	960.4	0.433	8.0×10^3
7	400	0	0.516	0	0.567	226.8	1187.2	0.484	7.46×10^3
8	600	0	0.470	0	0.516	282	1469.2	0.529	7.55×10^3
9	800	0	0.427	0	0.470	341.6	1810.8	0.572	8.23×10^3
10	1000	0	0.387	0	0.427	330	2200.8	0.610	7.73×10^3

Compare P_{un}(A) is greater than P_{un}(B) as we purchased machine B.

$$\therefore P_{un}(A) > P_{un}(B)$$

DATE: / /

PAGE NO.:

Q.18

Soln:

$$C_L = 1.25 \text{ Rs/bulb}$$

$$N = N_0 = 1000$$

$$C_U = 0.30 \text{ Rs/bulb}$$

To find best interval for a replacement = ?

K	Cumulative probability	P _K	N _K	ΣN _K	$0.6:4:R = C_U \cdot N + C_L \cdot \Sigma N_K$
1	0.05	P ₁ = 0.05	N ₁ = 50	50	$0.30 \times 1000 + 1.25 \times 50 = 362.5$
2	0.13	P ₂ = 0.08	N ₂ = 82.5	132.5	$0.30 \times 1000 + 1.25 \times 132.5 = 423.75$
3	0.25	P ₃ = 0.12	N ₃ = 128.12	260.62	$0.30 \times 1000 + 1.25 \times 260.62 = 485.75$
4	0.43	P ₄ = 0.18	N ₄ = 181	441.62	$0.30 \times 1000 + 1.25 \times 441.62 = 541.51$
5	0.68	P ₅ = 0.25	N ₅ = 289.09	730.71	$0.30 \times 1000 + 1.25 \times 730.71 = 623.39$
6	0.88	P ₆ = 0.2	N ₆ = 213.9	944.61	$0.30 \times 1000 + 1.25 \times 944.61 = 696.76$
7	0.96	P ₇ = 0.08	N ₇ = 134.34	1078.95	$0.30 \times 1000 + 1.25 \times 1078.95 = 749.74$
8	1.00	P ₈ = 0.04	N ₈ = 134.66	1213.61	$0.30 \times 1000 + 1.25 \times 1213.61 = 798.51$

$$\textcircled{1} N_1 = N_0 \times P_1$$

$$= 1000 \times 0.05$$

$$N_1 = 50$$

$$\textcircled{2} N_2 = N_0 P_2 + N_1 P_1$$

$$= 1000 \times 0.08 + 50 \times 0.05$$

$$N_2 = 82.5$$

DATE: / /

PAGE NO.:

$$\textcircled{3} N_3 = N_0 P_3 + N_1 P_2 + N_2 P_1$$

$$= 1000 \times 0.12 + 50 \times 0.08 + 82.5 \times 0.05$$

$$N_3 = 128.12$$

$$\textcircled{4} N_4 = N_0 P_4 + N_1 P_3 + N_2 P_2 + N_3 P_1$$

$$= 1000 \times 0.18 + 50 \times 0.12 + 82.5 \times 0.08 + 128.12 \times 0.05$$

$$N_4 = 199.006$$

$$\textcircled{5} N_5 = N_0 P_5 + N_1 P_4 + N_2 P_3 + N_3 P_2 + N_4 P_1$$

$$= 1000 \times 0.25 + 50 \times 0.18 + 82.5 \times 0.12 + 128.12 \times 0.08 + 199.006 \times 0.05$$

$$N_5 = 289.09$$

$$\textcircled{6} N_6 = N_0 P_6 + N_1 P_5 + N_2 P_4 + N_3 P_3 + N_4 P_2 + N_5 P_1$$

$$= 1000 \times 0.2 + 50 \times 0.25 + 82.5 \times 0.18 + 128.12 \times 0.12 + 199.006 \times 0.08 + 289.09 \times 0.05$$

$$N_6 = 273.09$$

$$\textcircled{7} N_7 = N_0 P_7 + N_1 P_6 + N_2 P_5 + N_3 P_4 + N_4 P_3 + N_5 P_2 + N_6 P_1$$

$$= 1000 \times 0.08 + 50 \times 0.2 + 82.5 \times 0.25 + 128.12 \times 0.18 + 199.006 \times 0.12 + 289.09 \times 0.08 + 273.09 \times 0.05$$

$$N_7 = 194.34$$

$$\textcircled{8} N_8 = N_0 P_8 + N_1 P_7 + N_2 P_6 + N_3 P_5 + N_4 P_4 + N_5 P_3 + N_6 P_2 + N_7 P_1$$

$$= 1000 \times 0.04 + 50 \times 0.08 + 82.5 \times 0.2 + 128.12 \times 0.25 + 199.006 \times 0.18 + 289.09 \times 0.12 + 273.09 \times 0.08 + 194.34 \times 0.05$$

$$N_8 = 194.605$$

$$0.6:4:R = 208.59$$

Best interval is 208.59

A4 Long Book

DATE: / /

PAGE NO.:

Q. 12

$$N = n_0 = 10,000$$

$$C_1 = 1.5 \text{ Rs/register}$$

$$C_2 = 0.5 \text{ Rs/register}$$

life $\rightarrow$ Normal distribution

$$K = 6 \text{ week}$$

$$\sigma = 1 \text{ week}$$

B find:- Optimum Replacement policy = 2

Soln:- Note:- Always find k upto $f(z) = 4$,
since for $f(z) = 4$ the probability is 100%
and it is cumulative probability.

$$f(z) = \frac{k - k}{\sigma}$$

$$4 = \frac{k - 6}{1}$$

$$4 = k - 6$$

$$k = 10 \text{ weeks}$$

A4 Long Book

DATE: / /

PAGE NO.:

K	$f(z)$	Cumulative probability	P_k	n_k	$\sum n_k$	$OCR = \frac{C_1 \cdot N + C_2 \cdot \sum n_k}{K}$
1	-5	0	$P_1 = 0$	$n_1 = 0$	0	$\frac{1.5(10000) + 0}{1} = 5000$
2	-4	0	$P_2 = 0$	$n_2 = 0$	0	$\frac{1.5(10000) + 0}{2} = 2500$
3	-3	0.00135	$P_3 = 0.00135$	$n_3 = 13.5$	13.5	$\frac{1.5(10000) + 1.5(13.5)}{3} = 1673.41$
4	-2	0.02225	$P_4 = 0.02225$	$n_4 = 22.25$	22.25	$\frac{1.5(10000) + 1.5(22.25)}{4} = 1375.5$
5	-1	0.1587	$P_5 = 0.1587$	$n_5 = 1587$	1587	$\frac{1.5(10000) + 1.5(1587)}{5} = 1476.1$
6	0	0.5000	$P_6 = 0.5000$	$n_6 = 5000$	5000	$\frac{1.5(10000) + 1.5(5000)}{6} = 2083.33$
7	1	0.8413	$P_7 = 0.8413$	$n_7 = 8413$	8413	$\frac{1.5(10000) + 1.5(8413)}{7} = 2527.78$
8	2	0.9772	$P_8 = 0.9772$	$n_8 = 9772$	11631	$\frac{1.5(10000) + 1.5(9772)}{8} = 2805.81$
9	3	0.99865	$P_9 = 0.99865$	$n_9 = 3262$	14893	$\frac{1.5(10000) + 1.5(9772)}{9} = 205.90$
10	4	1.0	$P_{10} = 1.0$	$n_{10} = 3603$	18496	$\frac{1.5(10000) + 1.5(9772)}{10} = 2980.70$

450 sheet
extra copy
for answer

$$\textcircled{1} f(z) = f(-1) = 1 - f(z),$$

$$= 1 - 0.8413$$

$$= 0.1587$$

$$\textcircled{2} f(z) = f(-2) = 1 - f(z),$$

$$= 1 - 0.9772$$

$$= 0.02225$$

$$\textcircled{3} f(z) = f(-3) = 1 - f(z),$$

$$= 1 - 0.99865$$

$$= 0.00135$$

$$\textcircled{1} n_1 = n_0 \cdot P_1 = 10000 \times 0 = 0$$

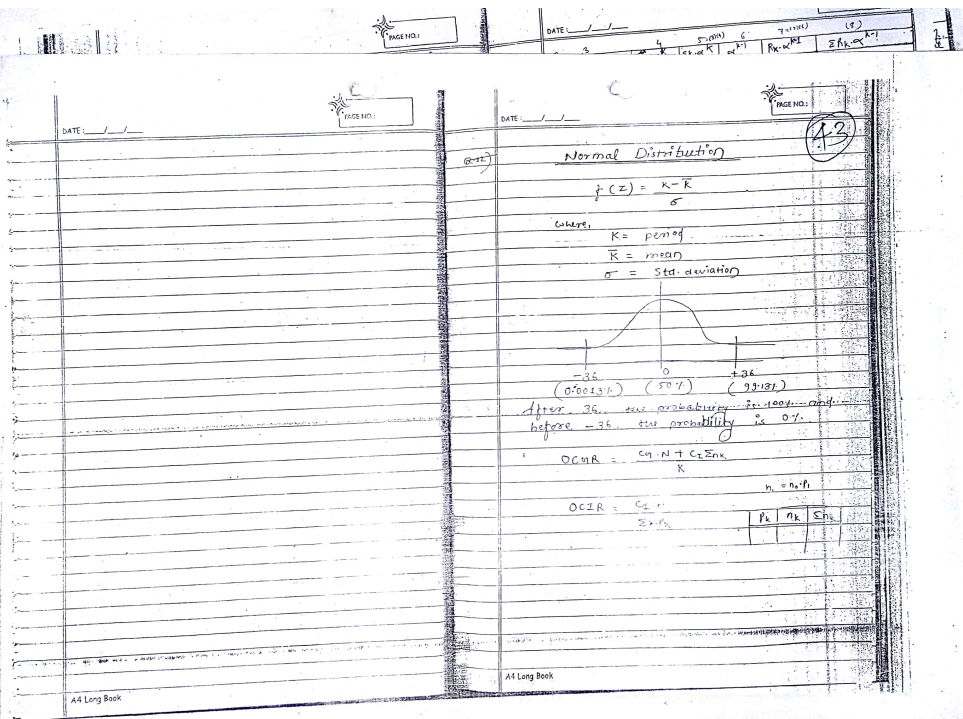
$$\textcircled{2} n_2 = n_0 \cdot P_2 + n_1 \cdot P_1 = 10000 \times 0 + 0 \times 0 = 0$$

$$\textcircled{3} n_3 = n_0 \cdot P_3 + n_1 \cdot P_2 + n_2 \cdot P_1$$

$$= 10000 \times 0.00135 + 0 \times 0 + 0 \times 0$$

$$= 13.5$$

A4 Long Book



DATE: / /

$n_1 =$

$n_5 =$

$n_6 =$

$n_7 =$

$n_8 =$

$n_9 =$

$n_{10} =$

$\therefore \text{OCIR} = 1335.5$

No. 42,

$\text{OCIR} = \frac{C_1 \cdot P_1}{\sum P_k}$

$= \frac{1.5 \times 10,000}{(1 \times 0) + (2 \times 0) + (3 \times 0.00135) + (4 \times 0.02145) + (5 \times 0.0135) + (6 \times 0.03413) + (7 \times 0.03413) + (8 \times 0.135) + (9 \times 0.02145) + (10 \times 0.0135)}$

DATE: / /

$\therefore \text{OCIR} = 2297.67$

Since,

$\text{OCUR} < \text{OCIR}$

$\therefore$ Group Replacement policy is

45

Q.13

Soln.

$N = n_0 = 10,000$

$C_f = Rs. 5$

life — Normal Distribution

$\bar{K} = 1000 \text{ hrs}$

$\sigma = 100 \text{ hrs}$

$C_u = Rs. 4$

$\text{Range} = 100 \text{ hrs.}$

To find: — Replacement Policy = ?

$\bar{z} = \frac{k - \bar{K}}{\sigma}$

$\therefore 4 = \frac{k - 1000}{100}$

$k = 1400 \text{ hrs.}$

Ans.

x	$P(x)$	Cumulative Probability	P_k	n_k	z_{n_k}	$OCMR = \frac{C_u n_k + C_p P_k}{P_k}$
100	0.00135	0	$P_1 = 0$	$n_1 = 0$	$z_1 = 0$	$\frac{4 \times 10000 + 5 \times 0}{0.00135} = 400$
200	0.00135	0	$P_2 = 0$	$n_2 = 0$	$z_2 = 0$	200
300	0.00135	0	$P_3 = 0$	$n_3 = 0$	$z_3 = 0$	133.33
400	0.00135	0	$P_4 = 0$	$n_4 = 0$	$z_4 = 0$	100
500	0.00135	0	$P_5 = 0$	$n_5 = 0$	$z_5 = 0$	80
600	0.00135	0	$P_6 = 0$	$n_6 = 0$	$z_6 = 0$	66.66
700	0.00135	0.00135	$P_7 = 0.00135$	$n_7 = 13.5$	13.5	57.23
800	0.0228	0.0228	$P_8 = 0.0228$	$n_8 = 21.5$	22.8	51.42
900	0.1587	0.1587	$P_9 = 0.1587$	$n_9 = 135.9$	158.7	53.26
1000	0.5000	0.5000	$P_{10} = 0.3413$	$n_{10} = 3413$	5000	55
1100	0.8413	0.8413	$P_{11} = 0.3413$	$n_{11} = 3463$	84.63	74.83
1200	0.9772	0.9772	$P_{12} = 0.4959$	$n_{12} = 3168$	115.9	81.79
1300	0.99865	0.99865	$P_{13} = 0.02145$	$n_{13} = 3262$	14.893	88.05
1400	1.0	1.0	$P_{14} = 0.00135$	$n_{14} = 3603$	18.496	94.62

$\therefore O.C.I.R. = 51.42$

Now,

$O.C.I.R. = \frac{C_u n}{\sum P_k}$

$= \frac{5 \times 10000}{0.00135}$

$(100 \times 0) + (200 \times 0) + (300 \times 0) + (400 \times 0) + (500 \times 0) + (600 \times 0) + (700 \times 0.00135) + (800 \times 0.00135) + (900 \times 0.00135) + (1000 \times 0.00135) + (1100 \times 0.02145) + (1200 \times 0.02145) + (1300 \times 0.02145) + (1400 \times 0.00135)$

$O.C.I.R. = 1050$

Since, $O.C.I.R. < O.C.I.R.$

check the problem. 1000

① $n_1 = n_0 P_1$
 $= 10,000 \times 0$
 $= 0$

② $n_2 = n_0 P_2 + n_1 P_1$
 $= 10000 \times 0 + 0 \times 0$
 $= 0$

$n_3 = n_0 P_3 + n_1 P_2 + n_2 P_1$
 $= 10000 \times 0.00135 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 0 \times 0 + 0 \times 0$
 $= 13.5$

PERT And CPM

DATE: _____

(Project Management)

47

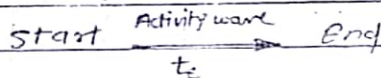
PERT: Programme Evaluation Review Technique

CPM: Critical Path Method.

Terms use in network Diagram.

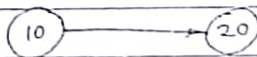
1) Activity:-

An activity is an element of a project which must be carried out to complete the project. It is represented by arrow ($\rightarrow$)



2) Event:-

The begining end point of "activity" are called as event. It is represented by end number circle.



3) Path.

An unbroken chain of activity connecting the initial event to some other event is called as path.

4) Network

It is the graphical representation of logically and sequentially connected arrows and events.

The Rules for developing network diagram:

"Each activity" represented by only one arrow

One or more activity can end at one event



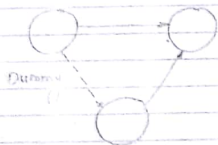
One or more activity can begin from one event

There should be only one starting event and ending event

Between any pair of events there should be only one activity

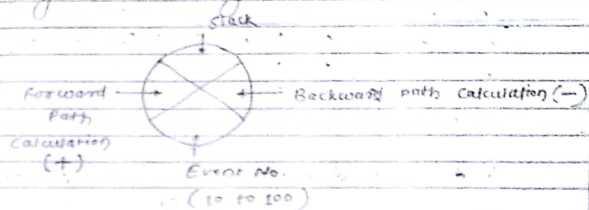


Faulty Network (wrong diagram)



• Critical Path (define):

critical path is the longest path in the network from the starting event to the ending event having slack zero or constant.



• Formulae:-

1) Expected time (t_e):

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

where,

t_o = optimistic time (shortest time)

t_p = pessimistic time (longest time)

t_m = most likely time

t_e = Expected time

2) Standard Deviation (σ):

$$\sigma = \frac{t_p - t_o}{6}$$

3) Variance:-

$$\text{Variance} = \sigma^2$$



PAGE NO.:

DATE: / /

The project has following 12 activity. The time estimates in a days of different activities are given in the table.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Predecessor	-	A	A	B	C	C	D	E	F	G	H	I
t_o	1	1	1	3	2	3	4	6	2	4	1	3
t_m	2	2	3	4	3	5	5	7	4	6	2	5
t_p	3	3	5	5	4	7	6	8	6	8	3	7

- 1) Construct a network to represent the above project.
- 2) Number the event using Fulkerson I-J
- 3) Determine standard deviation for each activity
- 4) Determine expected time (t_e)
- 5) Find the critical path
- 6) Determine the expected project length
- 7) The probability of completing the project in 30 days and 27 days
- 8) The probability of time in which the project can be completed with a probability of 30%.
- 9) Calculate total float (Tf), Free float (Ff) and independent float (If)
- 10) Calculate EST, LST, EFT, LFT
- 11) Calculate slack for each event
- 12) Calculate variance for each activity

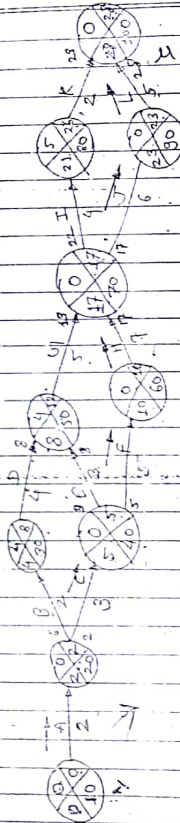
A4 Long Book

DATE: / /

PAGE NO.:

40

	I	J	K	L	I	J	K	L	I	J	K	L
t_o	1	1	1	3	2	3	4	6	2	4	1	3
t_m	2	2	3	4	3	5	5	7	4	6	2	5
t_p	3	3	5	5	4	7	6	8	6	8	3	7
t_e	2	2	3	4	3	5	5	7	4	6	2	5
σ	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33	0.33
σ^2	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11



A4 Long Book

DATE: / /

PAGE NO.:

Q critical path:- A-C-F-H-J-L

C project length = 28 days

P probability calculation:-

$$Z = \frac{K - \bar{X}}{\sigma_p}$$

where, K = duration for which probability to be calculated $\bar{X}$ = project duration σ_p = prob. of std. deviation

$$\therefore \sigma_p = \sqrt{(Z\sigma)^2} \text{ critical activity}$$

$$= \sqrt{0.11^2 + 0.44^2 + 0.11^2 + 0.44^2 + 0.44^2}$$

$$= 1.487$$

Case-I

Calculate probability for 30 days

$$K = 30$$

$$\bar{X} = 28$$

$$\sigma_p = 1.487$$

$$\therefore Z = \frac{K - \bar{X}}{\sigma_p}$$

$$= \frac{30 - 28}{1.487}$$

$$= 1.34$$

$$P(Z) \text{ prob.}$$

$$1.34 / 0.9099$$

Therefore, the prob. for completing the project in 30 days is 90.99%

A4 Long Book

DATE: / /

PAGE NO.:

Case-II

Calculate the probability for 26 days

$$K = 26$$

$$Z = \frac{K - \bar{X}}{\sigma_p} = -1.420$$

$$\therefore P(-1.42) = 1 - P(1.42)$$

$$= 1 - 0.9222$$

$$= 0.0778$$

Therefore, the prob. for completing the project in 26 days is 7.78%

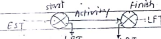
3) find duration of time for probability 90%

$$Z = \frac{K - \bar{X}}{\sigma_p}$$

$$1.29 = \frac{K - 28}{1.487}$$

$$\therefore K = 29.215 \text{ days}$$

5) Total Float (T.F.), Free Float (F.F.), Independent Float (I.F.)



EST - Earliest start time

LST - Latest start time

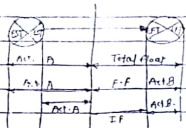
EFT - Earliest finish time

LFT - Latest finish time

A4 Long Book

DATE: / /

PAGE NO.:



$$① TF = LFT - t_e - EST$$

$$② FF = EFT - t_e - EST$$

$$③ IF = EFT - t_e - LST$$

①	②	③	④	⑤	⑥	⑦	⑧	⑨
Act	t_e	EST	LST	EFT	LFT	TF	F.F	IF
A	2	0	0	2	2	0	0	0
B	2	2	2	4	9	4	0	0
C	3	2	2	5	5	0	0	0
D	4	4	9	8	12	4	0	-4
E	3	5	5	8	12	4	0	0
F	5	5	5	10	10	0	0	0
G	5	8	12	17	17	0	4	0
H	7	10	10	17	17	0	0	0
I	4	17	17	21	26	5	0	0
J	6	17	17	23	23	0	0	0
K	2	21	26	28	28	0	5	0
L	5	23	23	28	28	0	0	0

A4 Long Book

DATE: / /

PAGE NO.:

Q.2) A project consist of 9 job with the following relationship.

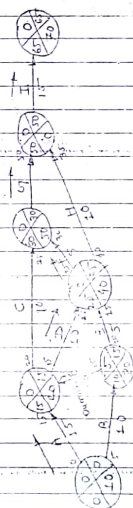
Draw the network and find the critical path.

Activities	A	B	C	D	E	F	G	H	I
Predecessors	-	-	A,B	A,B	B	D,E	C,F	D,E	G,H
time	15	10	10	10	5	5	20	10	15

Also calculate Total Float (TF), Free Float (FF), Independent Float.

for critical activity only.

Soln:-



A4 Long Book

DATE: / /

PAGE NO:

critical path:- 1) A-C-U-I
2) A-D-F-U-I
3) A-D-H-I

Critical path :- A-D-F-U-I

critical activity

①	②	③	④	⑤	⑥	⑦	⑧	⑨
Act.	t_e	EST	LST	EFT	LFT	T.F.	F.F.	IF
A	15	0	0	15	15	0	0	0
D	10	15	15	25	25	0	0	0
F	5	25	25	30	30	0	0	0
U	20	30	30	50	50	0	0	0
I	15	50	50	65	65	0	0	0

DATE: / /

PAGE NO:

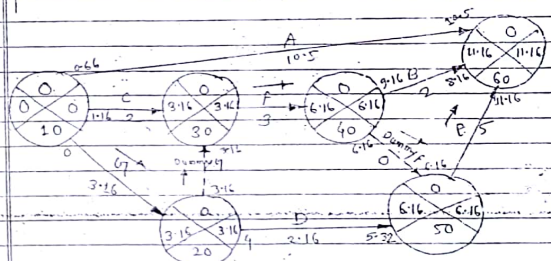
Q.9

Find the expected length for the given project.

Activity	A	B	C	D	E	F	U
Depends on	-	F	-	U	D, F	U, C	-
t_e in hr	8-10-5	1-2-3	1-2-3	1-2-4	4-5-6	2-3-4	2-3-5

Soln:

Activity	A	B	C	D	E	F	U
t_e	10.5	2	2	2.16	5	3	3.16



- 1) C-F-B
- 2) A-D-E
- 3) C-F-Dummy-F
- 4) U-Dummy-U-F-Dummy-F-E
- 5) A-Dummy-U-F-B

∴ Critical path:- A-Dummy-F-Dummy-F-E
(10-20-30-40-50-60)

∴ The expected project length = 11.16 days

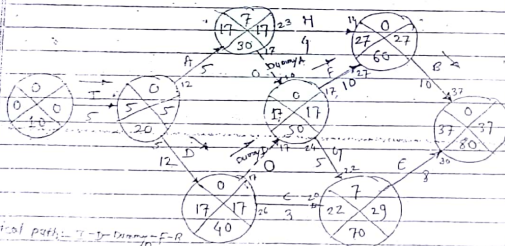
DATE: / /

PAGE NO.:

Q4) Construct the network to represent above project and find critical path. Calculate Total float (TF), FF, IF for non-critical activity.

Activity	A	B	C	D	E	F	G	H	I
Depends on	-	H, F	B, D	I	A, C	A, D	A	-	-
Duration (cc)	5	20	3	12	8	10	5	4	5

Soln: Network diagram...



critical path: A-D-Dummy-F-H

Activity	1	2	3	4	5	6	7	8	9
Activity	A	B	C	D	E	F	G	H	I
ES	0	5	5	5	10	17	7	0	0
LS	0	5	5	5	10	17	7	0	0
EF	5	25	8	17	18	27	12	4	5
LF	5	25	8	17	18	27	12	4	5
TF	0	0	0	0	0	0	0	0	0
FF	0	0	0	0	0	0	0	0	0
IF	0	0	0	0	0	0	0	0	0

A4 Long Book

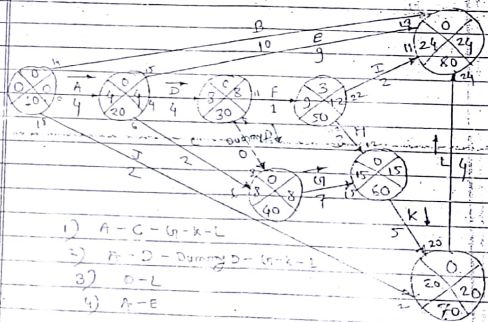
DATE: / /

PAGE NO.:

Q5) A event of activity along with there depending and time duration for the project given below. find out critical path and give the time scale version.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Depends on	-	-	A	A	A	D	C, D	F	F	-	G, H	J, K
Duration (cc)	4	10	2	4	9	1	7	3	2	2	5	9

Soln:



- 1) A-C-G-K-L
- 2) A-D-Dummy-D-G-K-L
- 3) D-L
- 4) A-E

critical path: A-D-Dummy-D-G-K-L

A4 Long Book

Q1. The preliminary data for a different activity of a project is as follows:

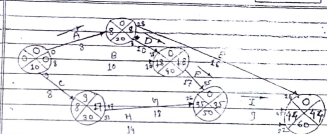
$A < D$, $E > A$, $D < F$, $C < G$, $H < G$, $F < I$,
 $I > G$, $B < F$
 Activity A, B, C are concurrent activity. Time (in days) for completion of each activity is given below.

Activity	A	B	C	D	E	F	G	H	I
Time (days)	8	10	8	10	14	17	14	9	9

Draw the network to represent the above project. Find the critical path, Free float (FF) and total float (TF) for critical activities.

- $A < D$ Activity D depends on A
- $E > A$ Activity E depends on A
- $D < F$ Activity F depends on D
- $C < G$ Activity G depends on C
- $H < G$ Activity H depends on G
- $E < I$ Activity I depends on E
- $F < I$ Activity I depends on F
- $H < I$ Activity I depends on H

Activity	A	B	C	D	E	F	G	H	I
Duration (days)	8	10	8	10	14	17	14	9	9



Critical path is $A \rightarrow E \rightarrow I$
 $A \rightarrow D \rightarrow F \rightarrow I$

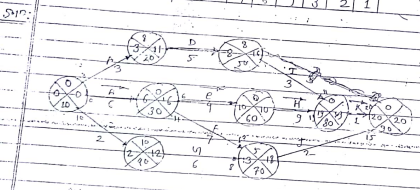
Critical path is $A \rightarrow D \rightarrow F \rightarrow I$

Activity	A	D	F	I
ES	0	8	18	35
LS	0	8	18	35
EF	8	18	35	44
LF	8	18	35	44
TF	0	0	0	0
FF	0	0	0	0

DATE: / / PAGE NO.:

Q.7 For the project draw the time scale version and calculate the expected project net MDD. Find total float (TF), free float (FF) and independent float (IF) critical activity.

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	-	-	A	B	B	C	E	D	H	I	J
Duration	3	6	2	5	4	7	6	9	3	2	1

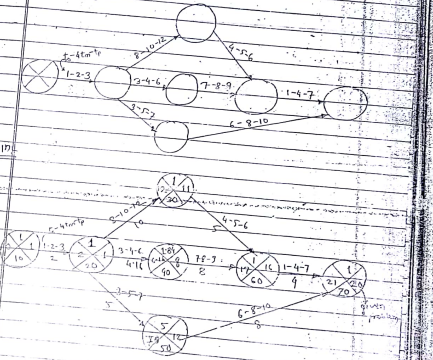


Critical path: A-B-H-K

Activity	A	B	H	K
ES	0	3	19	20
LS	0	3	19	20
EF	3	9	28	29
LF	3	9	28	29
TF	0	0	0	0
FF	0	0	0	0
IF	0	0	0	0

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Q.8 For the given network identify the critical path. The completion time for the project is 20 weeks. Given diagram:-



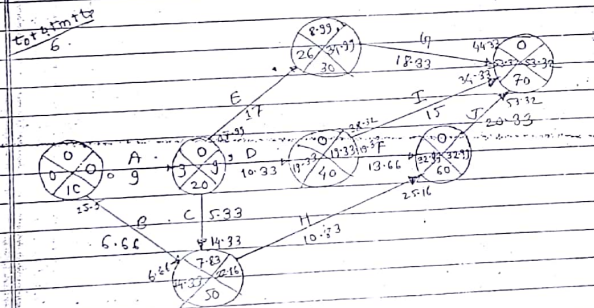
Critical path: 1-2-3-4-5-6-7-8-9-10

Sub critical path: 1-3-4-5-6-7-8-9-10

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Determine the standard deviation and expected time for each activity. find the duration of project for which the probability completion project during estimated duration is 70%.

Activity	10-20	10-50	20-50	20-40	20-30	40-60	30-70	50-60	40-70	60-70
t_o	6	4	3	7	14	10	13	9	12	17
t_m	9	7	5	10	37	14	18	11	15	20
t_p	12	8	9	15	20	16	25	12	18	25



DATE: / /

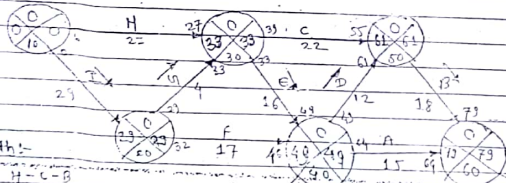
PAGE NO:

Q-5.10

Prepare a network to represent the project and establish the critical path also find free and independent float for non critical activity.

Activity	A	B	C	D	E	F	G	H	I
depends	-	A, B	A, B	E, F	A, H	I	I	-	-
duration	12	18	22	12	16	17	9	27	23

Soln:-



Critical path:-

1. A-B-C-D-E-F-G-H-I
2. I-E-A
3. I-H-A
4. I-F-D-B
5. I-F-D-S

A4 Long Book

DATE: / /

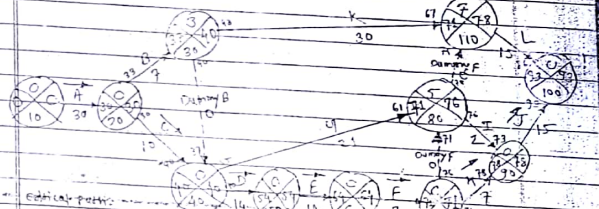
PAGE NO:

Q-11

Draw the network and find Free Float CF, T.F, IF for all activity.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
depends	-	A	A	B, C	D	E	B, C	F	F, G	H, I	J	K, L
duration	30	7	10	14	10	7	21	7	2	15	30	15

Soln



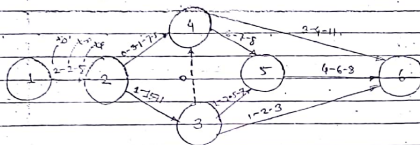
Critical path:-

A-C-D-E-F-H-I-J-K-L

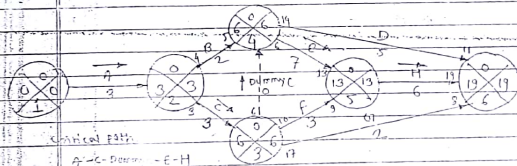
Act	te	EST	LST	EFT	LFT	T.F	P.F	I.F
A	30	0	0	30	30	0	0	0
B	7	30	30	37	37	0	0	0
C	10	30	30	40	40	0	0	0
D	14	40	40	54	54	0	0	0
E	10	40	40	50	50	0	0	0
F	7	50	50	57	57	0	0	0
G	21	50	50	71	71	0	0	0
H	7	57	57	64	64	0	0	0
I	2	57	57	59	59	0	0	0
J	15	64	64	79	79	0	0	0
K	30	79	79	109	109	0	0	0
L	15	109	109	124	124	0	0	0

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Q.13. Find the given network. Find critical path, t_E , t_F , t_P



So, $t_E = \frac{t_0 + 4t_M + t_P}{6}$
 $t_E = \frac{2 + 4 \times 2 + 3}{6} = \frac{2 + 8 + 3}{6} = \frac{13}{6} = 2.16$

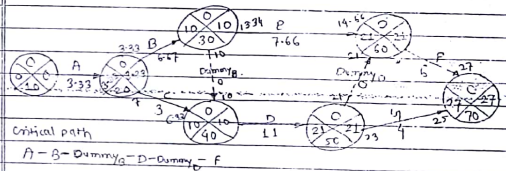


Critical path:
 A-C-Dummy-E-H
 A-B-D

Q.13. Represent PERT on a graph

Activity	A	B	C	D	E	F	G
from	1	2	3	4	5	6	7
to	2	3	4	5	6	7	8
duration	2	4	3	1	2	3	4

Find (1) Project duration for probability 95%
 (2) Determine probability if project duration is reduced by 3 days



Critical path:
 A-B-Dummy-D-E-F

DATE: ___/___/___

PAGE NO.: 59

● Critical Path Method (CPM) (CPT)

(Ques) A project consist of 6 activities as shown below. If the indirect costs is Rs 75 per day find ① Draw the activity network of the project ② Find optimum (duration) and optimum project cost ③ Find the critical path

Activity	Depends on	Normal		Crash	
		Time	Cost	Time	Cost
A	—	8	100	6	200
B	—	4	150	2	350
C	B	2	50	2	50
D	A	10	100	5	400
E	A	5	160	1	200
F	E	3	80	1	100

Soln: Step-1 Draw the network diagram and find the critical path.

Step-2) Find the total cost

Step-3) Calculate the cost slope $\left(\frac{\Delta C}{\Delta T}\right)$

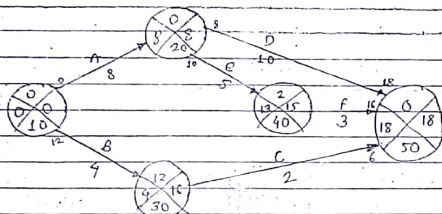
Step-4) Draw the time scale version on graph

Step-5) Prepare crashing table

DATE: / /

PAGE NO:

Step 1 Draw the network & find critical path



critical path :- A-D
(10-20-50)

Step 2:- find total cost = 2

Total cost = Direct cost + Indirect cost

$$\therefore \text{Direct cost} = \sum \text{Normal cost} = 100 + 150 + 50 + 100 + 100 + 80 = 580 /-$$

$$\therefore \text{Indirect cost} = \text{Ind. cost/day} \times \text{Project Duration} = 11 \times 18 = 198 /-$$

$$\therefore \text{Total cost} = 580 + 198 = 778 /-$$

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Formula: $\text{net saving} = (\text{Saving} - \text{Inc. in cost})$

DATE: / /

PAGE NO:

Step 3 Calculate the cost slope (AC/AT) = ?

Act	AC	AT	AC/AT	AC/AT
A	100	2-0	50	Normal cost
B	200	2	100	Crash cost
C	0	0	0	
D	300	5-8	60	Normal cost
E	100	4	25	Normal time
F	20	2	10	Crash time

Step 4 Draw the time scale time version (on graph)

Step 5 Crashing table:-

Act	Days crash	Cost slope	Increase in cost	New project duration	Saving	Net saving	Project duration
A	2	50	100	18-2=16	75x2=150	150-100=50	16-2=14
D	2	60	120	16-2=14	75x2=150	150-120=30	14-2=12
D-F	2	70	140	14-2=12	75x2=150	150-140=10	12-2=10

* Crashing Rule

- Any critical activity is to be crash (A-D)
 - Cost slope of critical activity must be $\leq$ Ind. cost/day
- i.e. $\left(\frac{AC}{AT} \right)_{A-D} \leq \text{Ind. cost/day}$

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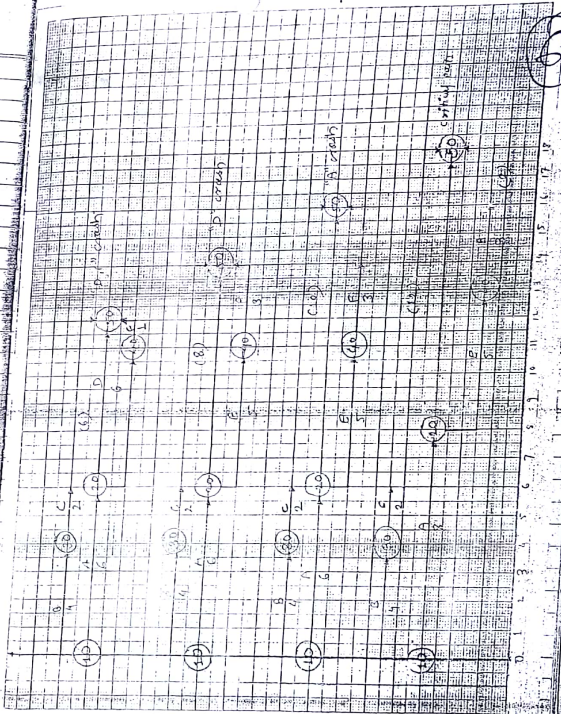
DATE: / /

PAGE NO:

- 2) Minimum cost slope is to be crash
 3) ~~then parallel~~ check for parallel crashing.

parallel crashing :-

- 1) $B - F \Rightarrow (60+10) = 70 < 75$
 2) $B - E \Rightarrow (60+25) = 85$ (X) Reject due to cost/slope
 3) $F - C \Rightarrow (X)$
 4) $A - C - B \Rightarrow (X)$ } Reject due to time.



61

WJD (1.0)

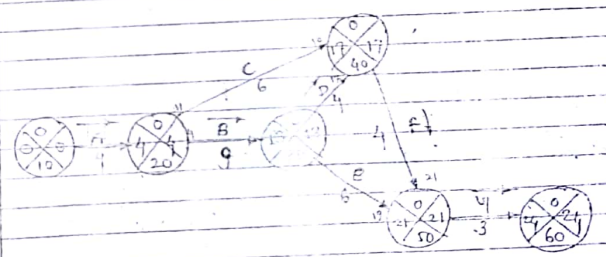
DATE: / /

Q.2) A project consist of 7 activity and dependency data is given in the table. If the indirect cost per day is Rs. 150 per day. And the optimum project duration.

Activity	A	B	C	D	E	F	G
Depends on	-	A	A	B	B	C,D	E,F
Duration	4	3	6	4	6	4	3

Activity	Depends on	Normal		Crashing	
		Time	Cost	Time	Cost
A	-	4	300	2	450
B	A	3	600	5	960
C	A	6	300	4	460
D	B	4	300	3	390
E	B	6	1440	3	1980
F	C,D	4	350	2	470
G	E,F	3	270	2	350

Soln:-



Critical path
 ⇒ A - B - D - F - G

DATE: / /

PAGE NO.:

STEP 2) Find Total cost = ?

Total cost = Direct cost + Indirect cost

* Direct cost = $\sum \text{Normal cost} = 3560/-$

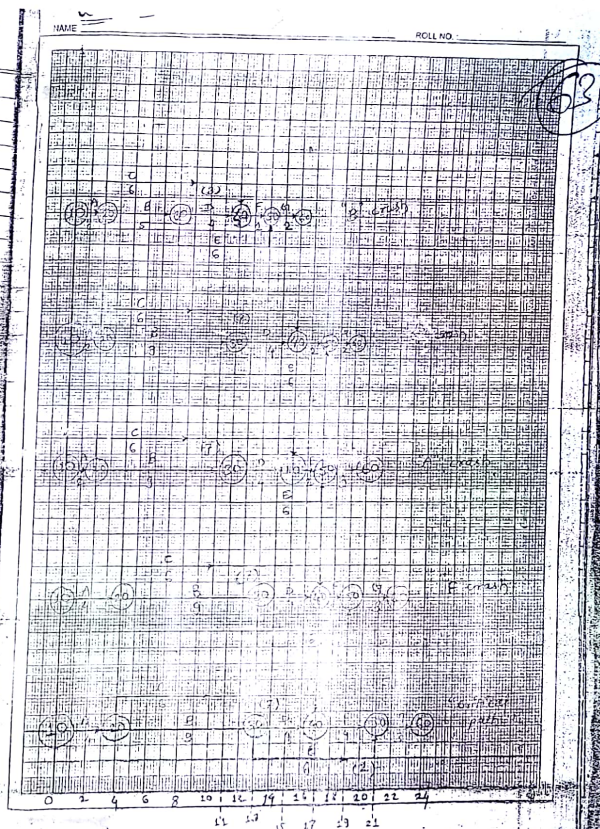
* Indirect cost = Ind. Cost/day $\times$ Project Duration
 $= 150 \times 24$
 $= 3600$

$\therefore$ Total cost = $3560 + 3600$
 $= 7160/-$

STEP 3 Calculate cost slope $\left(\frac{\Delta C}{\Delta T} \right)$

Activity	ΔC	ΔT	$\frac{\Delta C}{\Delta T}$
A-B	150	2-0	75 ✓ (2)
B-C	360	4-0	90 ✓ (4)
C-D	160	2	80
D-E	90	1	90 ✓ (5)
E-F	540	3	180
F-G	320	2-0	60 ✓ (3)
G	80	1-0	80 ✓ (3)

STEP 4 Draw the time scale region (on graph)



DATE / /

PAGE NO.

Step 5: Crashing Table

Activity	days crash	cost slope	Increasing cost	Normal duration	Saving	Net Saving	Normal project cost
F	2	60	120	$24 \times 2 = 48$	$100 \times 2 = 200$	$200 - 120 = 80$	$710 - 100 \times 2 = 510$
A	2	75	150	$22 \times 2 = 44$	$150 \times 2 = 300$	$300 - 150 = 150$	$710 - 150 \times 2 = 410$
U	1	80	80	$20 \times 1 = 20$	$150 \times 1 = 150$	$150 - 80 = 70$	$710 - 150 \times 1 = 560$
B	4	90	360	$15 \times 4 = 60$	$150 \times 4 = 600$	$600 - 360 = 240$	$710 - 150 \times 4 = 110$

* parallel crashing

$$\textcircled{1} D-E = 70 + 180 = 250 > 150 \quad X$$

$$\textcircled{2} E-D = 90 + 90 + 180 = 360 > 150 \quad X$$

∴ parallel crashing is not possible because the values are greater than Ind. cost/day

∴ Optimum project duration = 15 days
— cost = 652.0/-

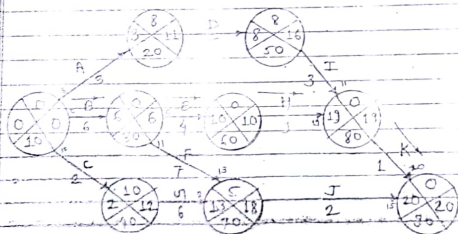
DATE: / /

Q.3

Calculate the optimum time and cost for the given project for which indirect cost per day is Rs. 35.

Activity	Depends on	Normal		Crashing	
		Time	Cost	Time	Cost
A	—	3	50	2	100
B	—	6	130	3	310
C	—	2	60	1	120
D	A	5	90	4	150
E	B	4	50	4	50
F	B	7	120	4	210
G	C	6	100	3	250
H	E	3	170	5	450
I	D	3	40	1	140
J	F, G	2	20	1	50
K	H, I	1	50	1	50

Self: STEP 1 - Draw the network diagram and find the critical path.



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DATE: / /

critical path:- B-E-H-K

Step 2 find total cost = ?

Total cost = Direct cost + Indirect cost

• Direct cost = $\sum \text{Normal cost} = 880/-$

• Indirect cost = Indirect cost/day $\times$ project duration
 $= 35 \times 20$
 $= 700/-$

∴ Total cost = $880 + 700$
 $= 1580/-$

Step 3 Calculate cost slope $\left(\frac{\Delta C}{\Delta T}\right)$

Activity	ΔC	ΔT	$\frac{\Delta C}{\Delta T}$	
A	50	1	50	
B	180	3	60	✓ ①
C	60	1	60	
D	60	1	60	
E	0	0	0	✓
F	90	3	30	
G	150	3	50	
H	280	4	70	✓ ②
I	100	2	50	
J	30	1	30	
K	0	0	0	✓

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DATE: / /

Step 4 Draw the time scale version on graph

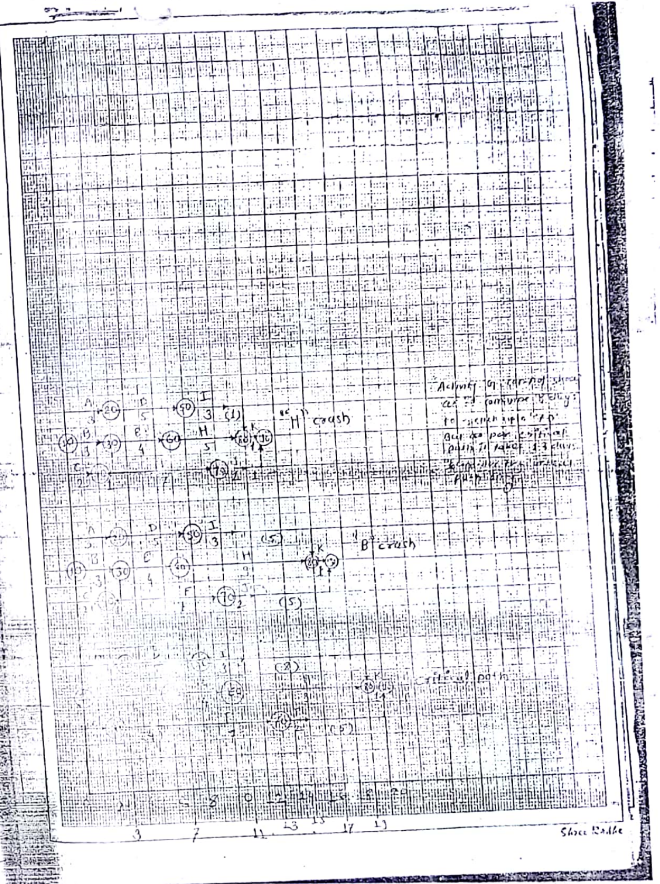
Step 5 prepare crashing table

Activity	days crash	Cost slope	Increase in cost	New project duration	Saving	Net saving	New Project cost
B	3	60	180	$20 - 3 = 17$	$75 \times 3 = 225$	45	$2380 - 45 = 2335$
H	4	70	280	$17 - 4 = 13$	$75 \times 4 = 300$	$300 - 280 = 20$	$2335 - 20 = 2315$

parallel cost crash

H-J X

H-I X

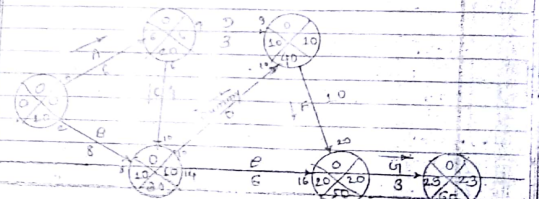


DATE: / /

- B-4) The indirect cost of a project is Rs 300 per day. Find out normal cost and duration of the project. e) Optimum duration and optimum cost of the project.
- 3) If all activity are crash, what will be the project duration and cost.

Activity	Normal		Crash	
	Time	cost	Time	cost
10-20	6	1400	4	1900
10-30	8	2000	5	2800
20-30	4	1100	2	1500
20-40	3	800	2	1400
30-40	dummy	2000	—	—
30-50	6	900	3	1600
40-50	10	2500	6	3500
50-60	3	300	2	800

Step 1) Draw the network & find critical path



Critical path = 10-20-30-50-60

A4 Long Book

DATE: / /

PAGE NO: 104

Step 2 Calculate Total cost

$$\begin{aligned} \text{Total cost} &= \text{Direct cost} + \text{Indirect cost} \\ &= \sum \text{Normal cost} + (\text{Ind. cost/day} \times \text{project duration}) \\ &= 9200 + (300 \times 23) \\ &= 16,100 /- \end{aligned}$$

Step 3 calculate cost slope ($\frac{\Delta C}{\Delta T}$)

Activity	ΔC	ΔT	$\frac{\Delta C}{\Delta T}$	
A	500	2	250	(2)
B	800	3	266.66	
C	400	2	200	(1)
D	600	1	600	
Dummy	—	—	—	
E	700	3	233.33	
F	1000	4	250	(3)
G	300	1	300	(4)

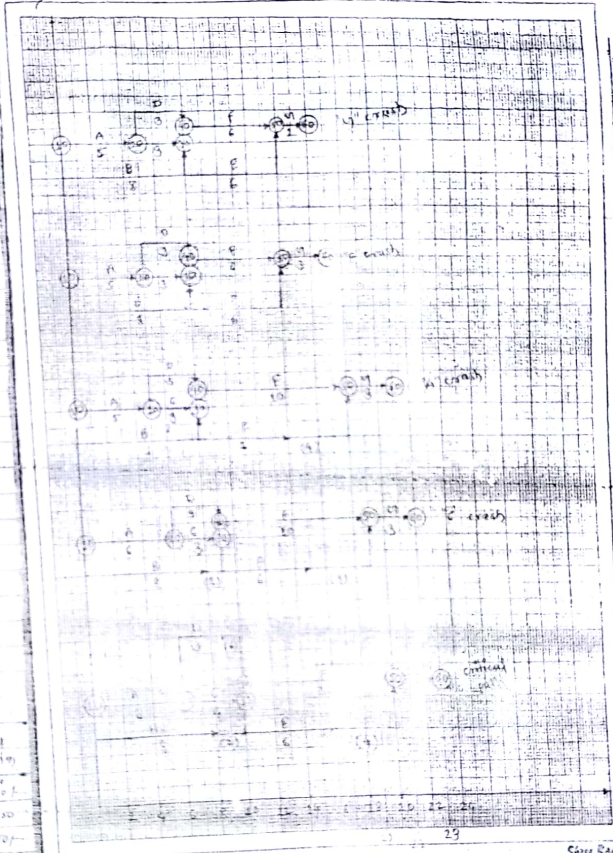
Step 4

Draw the time cost version of P.M.P.

Step 5 crashing table

Activity	days crash	cost slope	Increase in cost	new project duration	Save	Net saving	Final project duration
C	2	200	200	23-1=22	300	100	22
A	1	250	250	22-1=21	300	50	21
G	1	300	300	21-1=20	1200	900	20

104



Save Rs. 900

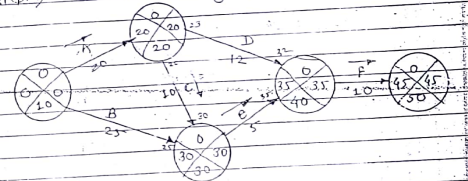
DATE: / /

PAGE NO: 29

5. Analysis the project for optimum time and optimum cost

Activity	Depends on	Normal		Crashing	
		Time	Cost	Time	Cost
A	—	20	600	17	720
B	—	25	200	25	200
C	A	10	300	8	400
D	A	12	400	6	700
E	B, C	5	300	2	420
F	B, D, E	10	300	5	600

Step 1) Draw network diag. & find critical path.



critical path is A-C-E-F

Step 2 Calculate Total Cost

Total cost = Direct cost + Indirect cost

$$\therefore \text{Direct Cost} = \sum_{\text{cost}} \text{Normal} = 600 + 200 + 300 + 400 + 300 + 300 = 2100/-$$

DATE: / /

PAGE NO.:

* Assume, Indirect cost/day = maximum of $\left(\frac{\Delta C}{\Delta T}\right)$ from critical path

step 3) Calculate cost slope $\left(\frac{\Delta C}{\Delta T}\right) = ?$

Activity	ΔC	ΔT	$\frac{\Delta C}{\Delta T}$	
A	120	3-0	40	①
B	0	0	0	
C	140	2	70	④
D	300	6	50	
E	120	3-0	40	②
F	300	5-0	60	③

* Assume, Indirect cost/day
= 70 Rs/day

* Indirect cost = Indirect cost/day $\times$ Project duration
= 70×45
= 3150

Now,

Total cost = Direct cost + Indirect cost
= $2100 + 3150$
= 5250

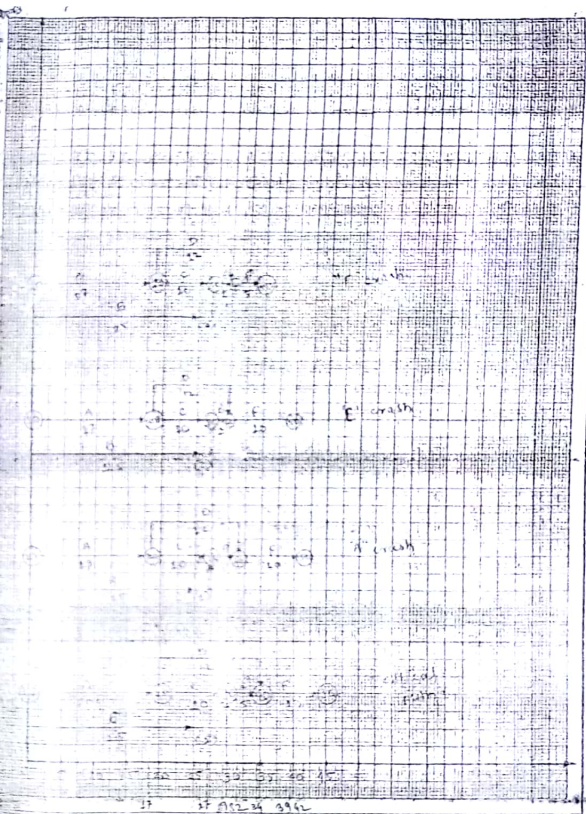
* Review the direct cost, variation (for 30/25/5)

STEP 5: checking To let

Activity	Days	Cost	Increase in cost	New project duration	Saving	Net saving	New project duration
	crash	slope	in cost				
A	3	40	120	$45-3=42$	210	$210-120=90$	$42-3=39$
E	3	40	120	$42-3=39$	210	$210-120=90$	$39-3=36$
F	5	60	300	$39-5=34$	350	$350-300=50$	$34-5=29$

Optimum Price = 34

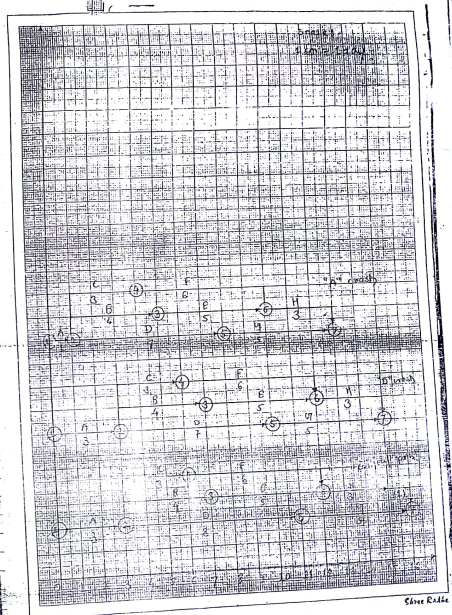
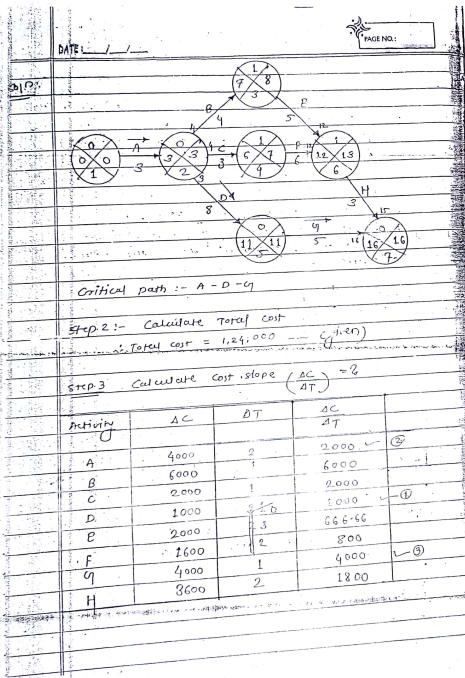
Minimum cost = 5020



DATE: / / PAGE NO: 71

Q.5) The XYZ company offered a contract to construct and deliver 9 automatic press to the ABC bottling plant. The contract price is Rs. 150,000 on meeting a specific delivery time. The bonus for earlier delivery. The marketing department has established this cost and time relationship as shown in the table. The normal delivery time is 16 weeks. The contract price of Rs. 150,000. Based on the probability for each of the following specify delivery time, recommend dead the delivery schedule should follow to maximum profit.

Activity	Normal		Crash	
	Time	Cost	Time	Cost
1-2	3	15000	1	19000
2-3	4	18000	3	24000
2-4	3	14000	2	16000
2-5	8	15000	7	16000
3-6	5	13000	2	15000
4-6	6	12000	4	13600
5-7	5	20000	4	24000
6-7	3	17000	1	20000
Contract delivery time (weeks)	15	14	13	12
Contract Amount	142500	143800	150000	152500



DATE: / /

PAGE NO. 83

STEP 4 Draw the time scale version (on graph)

STEP 5 Crashing Table:-

Act.	days	Cost	Increasing in Cost	New project duration	Saving	Net Saving	New Cost Profit
	Crash	Slope					
D	1	1000	1000	16-1=15	-	-	$125000 - 1000 = 124000$
A	2	2000	4000	13-2=11	-	-	$125000 - 4000 = 121000$

Contract Amt	Profit	plus contract - contract Amt
142500	17500	
150000	21500	

Unit - 1 Simplex

Linear Programming

PAGE NO.:

DATE: / /

Theory, Duality in L.P.P., Formulation, Graphical

OR

Simplex (13)

3 stages

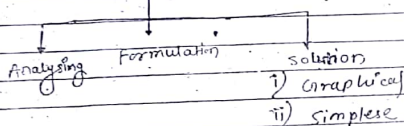


Table I:-

Type of constraints	Extra variable needed	objective
$\leq$	$+s$ (slack variable)	max. min
$\geq$	$-s$ $+A$	$-m$ m
$=$	A (Artificial variable)	$-m$ m

Table II:-

objective	self is optimum	key column(K_c)	key row(K_r)
Max	when $[C_j - E_j]$ row containing constant 0 & -ve	most +ve value of $(C_j - E_j)$	min (+ve) value $\theta = \frac{b}{a_{rk}}$
Min	$(C_j - E_j)$ row containing constant 0 & +ve value	most -ve value of $C_j - E_j$	min (+ve) value $\theta = \frac{b}{a_{rk}}$

PAGE NO.:

DATE: / /

- 1) key row is called as a pivot row.
- 2) intersection element of K_c & K_r is called as key element.
- 3) Make the key element unity (1) always.

$$\text{New Row element} = \left[\begin{array}{c} \text{element of the} \\ \text{Row to be} \\ \text{replaced} \end{array} \right] \div \left[\begin{array}{c} \text{corresponding} \\ \text{key column} \\ \text{element} \end{array} \right] = \text{New pivot Row element}$$

Q.1) Max $Z = 5x + 10y$

Sub to,

$$5x + 8y \leq 48$$

$$5x + 3y \leq 48$$

$$x + 9y \leq 48$$

$$x, y \geq 0$$

Solve by simplex method

Soln: Convert the canonical form into standard form

$$\text{max } Z = 5x + 10y + 0s_1 + 0s_2 + 0s_3$$

Sub to,

$$5x + 8y + s_1 = 48$$

$$5x + 3y + s_2 = 48$$

$$x + 9y + s_3 = 48$$

$$x, y \geq 0$$

	C_j	5	10	0	0	0	0	0	0
C_B	X_B	x	y	s_1	s_2	s_3	b	$\theta = \frac{b}{a_{rk}}$	
	s_1	5	8	1	0	0	48	$\frac{48}{8} = 6$	
	s_2	5	3	0	1	0	48	$\frac{48}{3} = 16$	
	s_3	1	9	0	0	1	48	$\frac{48}{9} = 5.33$	$\leftarrow K_r$
$E_j = \sum C_B \cdot a_{ij}$									
	$C_j - E_j$	0	0	0	0	0	0	0	
At Long Run $(0.5x + 0.5y + 0.5s_1 + 0.5s_2 + 0.5s_3)$									

DATE: / /

PAGE NO.:

Degeneracy: $\rightarrow$ Some elements in the 1st & 3rd Row

Identity matrix:-

1st row:- $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\frac{0}{8}$ $\frac{0}{8}$
 2nd row:- $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\frac{0}{8}$ $\frac{1}{8}$ $\leftarrow K_0$
 3rd row:- $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $\frac{0}{8}$ $\frac{0}{8}$

3rd Row $\Rightarrow$ pivot Row (0-S₃) $\Rightarrow$

$\frac{1}{8}$ $\frac{0}{8}$ $\frac{0}{8}$ $\frac{0}{8}$ $\frac{1}{8}$ $\frac{45}{8}$
 $= \frac{1}{8}$ $= 1$ $= 0$ $= 0$ $= \frac{1}{8}$ $= 6$

1st row $-(0-S_1) \Rightarrow$

$5 - (8 \times \frac{1}{8}) = 4$
 $8 - (8 \times 1) = 0$
 $1 - (8 \times 0) = 1$
 $0 - (8 \times 0) = 0$
 $0 - (8 \times \frac{1}{8}) = -1$
 $48 - (8 \times 6) = 0$

2nd row $(0-S_1) \Rightarrow$

$5 - (3 \times \frac{1}{8}) = \frac{37}{8}$
 $3 - (3 \times 1) = 0$
 $0 - (3 \times 0) = 0$
 $1 - (3 \times 0) = 1$
 $0 - (3 \times \frac{1}{8}) = -\frac{3}{8}$
 $48 - (3 \times 6) = 30$

C _j	5	10	0	0	0	0	0	0	0
C _B	S ₁	S ₂	S ₃	b	4	30	6	4	48
0	S ₁	(1)	0	1	0	-1	0	0	0
0	S ₂	$\frac{37}{8}$	0	0	1	$-\frac{3}{8}$	0	0	0
10	Y	$\frac{1}{8}$	1	0	0	$\frac{1}{8}$	0	0	0
E _T = $\sum C_j \cdot a_{ij}$	$\frac{1}{8}$	10	0	0	0	$-\frac{10}{8}$	0	0	0
C _j - E _T	$(\frac{1}{8} - 10)$	0	0	0	0	0	0	0	0

$(2 \times 4 + 0 \times \frac{37}{8} + 10 \times \frac{1}{8})$

DATE: / /

PAGE NO.:

Pivot Row (1st Row) (0-S₁) $\Rightarrow$

$\frac{4}{4} = 1$ $\frac{0}{4} = 0$ $\frac{1}{4}$ $\frac{0}{4} = 0$ $\frac{-1}{4}$ $\frac{0}{4} = 0$

2nd Row (0-S₁) $\Rightarrow$

$\frac{37}{8} - (\frac{37}{8} \times 1) = 0$

$0 - (\frac{37}{8} \times 0) = 0$

$0 - (\frac{37}{8} \times \frac{1}{4}) = -\frac{37}{32}$

$1 - (\frac{37}{8} \times 0) = 1$

$-\frac{3}{8} - (\frac{37}{8} \times \frac{1}{4}) = -\frac{25}{32}$

$30 - (\frac{37}{8} \times 0) = 30$

3rd Row (10-Y) $\Rightarrow$

$\frac{1}{8} - (\frac{1}{8} \times 1) = 0$

$1 - (\frac{1}{8} \times 0) = 1$

$0 - (\frac{1}{8} \times \frac{1}{4}) = -\frac{1}{32}$

$0 - (\frac{1}{8} \times 0) = 0$

$\frac{1}{8} - (\frac{1}{8} \times \frac{1}{4}) = \frac{5}{32}$

$6 - (\frac{1}{8} \times 0) = 6$

C _j	5	10	0	0	0	0	0	0
C _B	X	Y	S ₁	S ₂	S ₃	b	4	30
5	X	1	0	$\frac{1}{4}$	0	$-\frac{1}{4}$	0	0
0	S ₂	0	0	$-\frac{37}{32}$	1	$\frac{25}{32}$	0	0
10	Y	0	1	$-\frac{1}{32}$	0	$\frac{5}{32}$	0	6
E _T = $\sum C_j \cdot a_{ij}$	5	10	$-\frac{37}{32}$	0	0	$\frac{15}{32}$	0	0
C _j - E _T	0	0	$-\frac{37}{32}$	0	0	$-\frac{15}{32}$	0	0

Since, (C_j - E_j) contains constant n^o of 've' value

$\therefore$ soln is optimal

$\therefore x = 0$

$y = 6$

$\therefore \text{Max} = 5x + 10y = 5 \times 0 + 10 \times 6$

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Max Z = 60

$$Z_{\max} = 2x_1 + 3x_2$$

Sub. to,

$$\begin{aligned} x_1 - x_2 &\geq 4 \rightarrow -s_1 + A \\ x_1 + x_2 &\leq 6 \rightarrow s_2 \\ x_1 &\leq 2 \rightarrow s_3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Check the objective function

Convert the canonical form into std. form.

$$Z_{\max} = 2x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3 - mA$$

Sub. to,

$$\begin{aligned} x_1 - x_2 + s_1 + A &= 4 \\ x_1 + x_2 + s_2 &= 6 \\ x_1 + s_3 &= 2 \end{aligned}$$

	C_j	2	3	0	0	0	-M			
C_B	C_B	x_1	x_2	s_1	s_2	s_3	A	b	$\theta = \frac{b}{a_{ij}}$	
-M	A	1	-1	-1	0	0	1	4	$\frac{4}{1} = 4$	
0	s_2	1	1	0	1	0	0	6	$\frac{6}{1} = 6$	
0	s_3	1	0	0	0	1	0	2	$\frac{2}{1} = 2$	
$E_j = \sum C_B \cdot a_{ij}$		-M	M	M	0	0	-M			θ
$C_j - E_j$		2+M	3-M	-M	0	0	0			

$$-M(1+0+0) \rightarrow K_c$$

Big M method

$$M=4$$

Note: Take the value of M greater than actual greatest value.

Pivot Row ($0-s_3$) 3rd Row:-

$$= 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 2$$

1st Row ($-M, A$) :-

$$(1 \times 1) = 0$$

$$-1 - (1 \times 0) = -1$$

$$-1 - (1 \times 0) = -1$$

$$0 - (1 \times 0) = 0$$

$$0 - (1 \times 1) = -1$$

$$1 - (1 \times 0) = 1$$

$$4 - (1 \times 2) = 2$$

2nd Row ($0-s_2$) :-

$$1 - (1 \times 1) = 0$$

$$1 - (1 \times 0) = 1$$

$$0 - (1 \times 0) = 0$$

$$1 - (1 \times 0) = 1$$

$$0 - (1 \times 1) = -1$$

$$0 - (1 \times 0) = 0$$

$$6 - (1 \times 2) = 4$$

	C_j	2	3	0	0	0	-M			
C_B	C_B	x_1	x_2	s_1	s_2	s_3	A	b		
-M	A	0	-1	-1	0	-1	1	2		
0	s_2	0	1	0	1	-1	0	4		
2	x_1	1	0	0	0	1	0	2		
$E_j = \sum C_B \cdot a_{ij}$		2	M	M	0	M+2	-M			
$C_j - E_j$		0	3-M	-M	0	-M-2	0			

since $C_j - E_j$ row constant contain constant no -ve value,

Soln is optimal

$$\therefore x_1 = 2$$

$$x_2 = 0$$

$$\therefore \text{Max } Z = 2x_1 + 3x_2$$

$$= 2 \times 2 + 3 \times 0$$

$$\text{Max } Z = 4$$

A4 Long Book

DATE: / /

PAGE NO.:

Minimize $Z = 2x_1 + x_2$ solve by simplex method

Sub. to,

$$3x_1 + x_2 = 3 \rightarrow +A_1$$

$$4x_1 + 3x_2 \geq 6 \rightarrow -S_1 + A_2$$

$$x_1 + 2x_2 \leq 3 \rightarrow S_2$$

$$x_1, x_2 \geq 0$$

Solve by simplex method.

Convert canonical form into std. form.

Minimize $Z = 2x_1 + x_2 + 0S_1 + 0S_2 + MA_1 + MA_2$

Sub. to,

$$3x_1 + x_2 + A_1 = 3$$

$$4x_1 + 3x_2 - S_1 + A_2 = 6$$

$$x_1 + 2x_2 + S_2 = 3$$

	C_j	2	1	0	0	M	M		
C_i	C_B	x_1	x_2	S_1	S_2	A_1	A_2	b	$\theta = \frac{b}{a_{ik}}$
M	A_1	3	1	0	0	1	0	3	$\frac{3}{1} = 3 \rightarrow K_1$
M	A_2	4	3	-1	0	0	1	6	$\frac{6}{3} = 2$
0	S_2	1	2	0	1	0	0	3	$\frac{3}{2} = 1.5$
$E_2 = 2E_1 - 4E_2$									
$E_3 = E_2 - E_1$									
$E_4 = E_3 - E_1$									

Big M method.
 $M=8$

DATE: / /

PAGE NO.:

(1) Pivot Row (1st Row) :-

$$\frac{3}{3} = 1 \quad \frac{1}{3} \quad \frac{0}{3} = 0 \quad \frac{0}{3} = 0 \quad \frac{1}{3} \quad \frac{0}{3} = 0 \quad \frac{3}{3} = 1$$

(2) 2nd Row (MA_2) :-

$$4 - (4 \times 1) = 0$$

$$3 - (4 \times \frac{1}{3}) = \frac{5}{3}$$

$$-1 - (4 \times 0) = -1$$

$$0 - (4 \times 0) = 0$$

$$0 - (4 \times \frac{1}{3}) = -\frac{4}{3}$$

$$1 - (4 \times 0) = 1$$

$$6 - (4 \times 1) = 2$$

(3) 3rd Row ($0-S_2$) :-

$$1 - (1 \times 1) = 0$$

$$2 - (1 \times \frac{1}{3}) = \frac{5}{3}$$

$$0 - (1 \times 0) = 0$$

$$1 - (1 \times 0) = 1$$

$$0 - (1 \times \frac{1}{3}) = -\frac{1}{3}$$

$$0 - (1 \times 0) = 0$$

$$3 - (1 \times 1) = 2$$

	C_j	2	1	0	0	M	M		
C_i	C_B	x_1	x_2	S_1	S_2	A_1	A_2	b	$\theta = \frac{b}{a_{ik}}$
2	x_1	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	1	3
M	A_1	0	$\frac{2}{3}$	-1	0	$-\frac{1}{3}$	1	2	$\frac{2}{2/3} = 3$
0	S_2	0	$\frac{1}{3}$	0	1	$-\frac{1}{3}$	0	2	$\frac{2}{1/3} = 6$
$E_2 = E_2 - 2E_1$									
$E_3 = E_3 - E_1$									
$E_4 = E_4 - E_1$									

K_C

3rd Row (Pivot Row) $0-S_2$:-

$$\frac{0}{1/3} = 0 \quad \frac{1}{1/3} = 3 \quad \frac{0}{1/3} = 0 \quad \frac{1}{1/3} = 3 \quad \frac{0}{1/3} = 0 \quad \frac{2}{1/3} = 6$$

DATE: / /

PAGE NO.:

1st Row $:- (2 \times x_1) :-$

$$1 - \left(\frac{1}{3} \times 0\right) = 1$$

$$\frac{1}{3} - \left(\frac{1}{3} \times 1\right) = 0$$

$$0 - \left(\frac{1}{3} \times 0\right) = 0$$

$$0 - \left(\frac{1}{3} \times \frac{3}{5}\right) = -\frac{1}{5}$$

$$\frac{1}{3} - \left(\frac{1}{3} \times \frac{1}{5}\right) = \frac{2}{5}$$

$$0 - \left(\frac{1}{3} \times 0\right) = 0$$

$$1 - \left(\frac{1}{3} \times \frac{6}{5}\right) = \frac{3}{5}$$

2nd Row $:- (MA_2)$

$$0 - \left(\frac{5}{3} \times 0\right) = 0$$

$$\frac{5}{3} - \left(\frac{5}{3} \times 1\right) = 0$$

$$-1 - \left(\frac{5}{3} \times 0\right) = -1$$

$$0 - \left(\frac{5}{3} \times \frac{3}{5}\right) = -1$$

$$-\frac{4}{3} - \left(\frac{5}{3} \times -\frac{1}{5}\right) = -1$$

$$1 - \left(\frac{1}{3} \times 0\right) = 1$$

$$-\left(\frac{5}{3} \times \frac{6}{5}\right) = 0$$

DATE: / /

PAGE NO.:

C _B	2	1	0	0	M	M	
C _B	x_1	x_2	s_1	s_2	A_1	A_2	b
x_1	1	0	0	$-\frac{1}{5}$	$\frac{2}{5}$	0	$\frac{3}{5}$
A_2	0	0	-1	-1	-1	1	0
x_2	0	1	0	$\frac{3}{5}$	$-\frac{1}{5}$	0	$\frac{6}{5}$
$E_1 = \text{Eti. obj}$	2	1	-M	$\frac{1}{5} - M$	$\frac{2}{5} - M$	-M	
$C_B - E_1$	0	0	M	$-\frac{1}{5} + M$	$M - \frac{2}{5}$	0	
	0	0	0	$\frac{1}{5}$	$\frac{1}{5}$	0	

Since, $C_B - E_1$ row contains constant 0 & -ve value. $\therefore$ Soln is optimal.

$$x_1 = \frac{3}{5}$$

$$x_2 = \frac{6}{5}$$

$$\therefore \text{Min } Z = 2 \times \frac{3}{5} + \frac{6}{5}$$

$$\text{Min } Z = \frac{12}{5}$$

DATE: / /

PAGE NO:

+ Duality in L.P.P. * (4/5 marks)

Primal	Dual
max.	min
min	Max
Constraint coeff.	constraint
constraints	const. coefficient

check? $\Rightarrow$ max $\rightarrow \leq$
 min $\rightarrow \geq$

9.1) max $Z = 20x + 30y$

sub to,

$$5x + 2y \leq 210 \quad \text{--- (a)}$$

$$2x + 4y \leq 300 \quad \text{--- (b)}$$

$$y \leq 65 \quad \text{--- (c)}$$

$$x, y \geq 0$$

Convert this primal into duality

Soln. min $Z^* = 210a + 300b + 65c$

sub to,

$$3a + 2b \geq 20 \quad \text{--- (Coefficient of } x)$$

$$2a + 4b + 1c \geq 30 \quad \text{--- (Coefficient of } y)$$

$$a, b, c \geq 0$$

==

DATE: / /

PAGE NO:

9.2

max $Z = 4x + 5y$

sub to,

$$6x + 5y \leq 250 \quad \text{--- (a)}$$

$$4x + 6y \leq 200 \quad \text{--- (b)}$$

$$6x + 5y \geq 150 \quad \text{--- (c) (multiply with -1)}$$

$$9x + 5y \geq 130 \quad \text{--- (d) (multiply with -1)}$$

$$x, y \geq 0$$

Soln.

Convert the primal into duality

Soln.

multiply eqn (3) & eqn (4) by -1, to convert the greater than sign into less than sign, as the problem is of maximization.

$$\therefore 6x + 5y \geq 150 \quad \text{--- multiply with -1}$$

$$\therefore -6x - 5y \leq -150 \quad \text{--- (3) --- (c)}$$

$$9x + 5y \geq 130 \quad \text{--- multiply with -1}$$

$$\therefore -9x - 5y \leq -130 \quad \text{--- (4) --- (d)}$$

min $Z^* = 250a + 200b - 150c - 130d$

sub to,

$$6a + 4b - 5c - 9d \geq 4$$

$$5a + 6b - 5c - 5d \geq 5$$

$$a, b, c, d \geq 0$$

DATE: / /

PAGE NO.:

Q3) Min $Z = 30x + 40y$

Sub. to

$$x + y \leq 250 \quad \text{--- (a)}$$

$$x \leq 200$$

$$y \geq 100$$

$$x, y \geq 0$$

Soln:

multiply eqn (1), (2) with -1

$$\therefore -x - y \geq -250 \quad \text{--- (1) (a)}$$

$$-x \geq -200 \quad \text{--- (2) (b)}$$

$$y \geq 100 \quad \text{--- (3) (c)}$$

$$\max Z^* = -250a - 200b + 100c$$

Sub. to

$$-a - b \leq 30$$

$$-a + c \leq 40$$

$$a, b, c \leq 0$$

DATE: / /

PAGE NO.:

Q4)

$$\min Z_1 = 30x - 40y$$

Sub. to

$$-x - y \geq -250 \quad \text{--- (a)}$$

$$-x \geq -200 \quad \text{--- (b)}$$

$$y \geq 100 \quad \text{--- (c)}$$

$$x, y \geq 0$$

Convert the primal into duality.

Soln:

$$\max Z^* = -250a - 200b + 100c$$

Sub. to

$$-a - b \leq 30$$

$$-a + c \leq 40$$

$$\text{i.e. } a - c \geq 40$$

$$a, b, c \geq 0$$

Q5)

$$\min Z = 3x_1 - 2x_2 + 4x_3$$

Sub. to

$$3x_1 + 5x_2 + 4x_3 \geq 7 \quad \text{--- (a) 1}$$

$$6x_1 + x_2 + 3x_3 \geq 4 \quad \text{--- (b) 2}$$

$$7x_1 - 2x_2 - x_3 \leq 10 \quad \text{--- (c) 3}$$

$$x_1 - 2x_2 + 5x_3 \geq 3 \quad \text{--- (d) 4}$$

$$4x_1 + 7x_2 - 2x_3 \leq 3 \quad \text{--- (e) 5}$$

$$x_1, x_2, x_3 \geq 0$$

Convert the primal into duality.

Soln:

multiply eqn (3) & (5) with -1, to convert $\leq$ sign into $\geq$ sign as the objective function is of minimum

$$7x_1 - 2x_2 - x_3 \leq 10 \quad \text{--- multiply with (-1)}$$

$$-7x_1 + 2x_2 + x_3 \geq -10 \quad \text{--- (3) (c)}$$

A4 Long Book

DATE: / /

PAGE NO.:

(Q4) Min $Z = 3x_1 + 8x_2$

s.t.

$x_1 \leq 80$

$x_2 \geq 60$

$$\begin{cases} \rightarrow S_1 \\ \rightarrow -S_2 + A_1 \\ \rightarrow A_2 \end{cases}$$

$x_1 + x_2 = 200$

$x_1, x_2 \geq 0$

Solve by simplex method.

Soln. Convert canonical form into std. form.

Min $Z = 3x_1 + 8x_2 + 0S_1 + 0S_2 + M A_1 + M A_2$

s.t.

$x_1 + S_1 = 80$

$x_2 - S_2 + A_1 = 60$

$x_1 + x_2 + A_2 = 200$

Convert std. form into tabular form.

	C_j	3	8	0	0	M	M		
C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	b	$\theta = \frac{b}{a_{ik}}$
0	S_1	1	0	1	0	0	0	80	$\frac{80}{1} = 80$
M	A_1	0	1	0	-1	1	0	60	$\frac{60}{1} = 60$
M	A_2	1	1	0	0	0	1	200	$\frac{200}{1} = 200$
Z_j		3M	8M	0	0	M	M		
$Z_j - C_j$		3-M	8-2M	0	0	M	0		
		(-6)	(-10)	0	(1)				

↑ Key By m method
 $\therefore M=3$ Pivot Row:- 2nd Row ($M A_2$):-

No change

$$0 \quad 1 \quad -20 \quad -1 \quad 0 \quad 0 \quad 60$$

DATE: / /

PAGE NO.:

2nd Row ($M A_2$):-

$1 - (1 \times 0) = 1$

$1 - (1 \times 1) = 0$

$0 - (1 \times 0) = 0$

$0 - (1 \times 0) = 0$

$0 - (1 \times -1) = 1$

$0 - (1 \times 1) = -1$

$-1 - (1 \times 0) = -1$

$200 - (1 \times 60) = 140$

1st Row ($0 - S_1$)

$1 - (0 \times 0) = 1$

$0 - (0 \times 1) = 0$

$1 - (0 \times 0) = 1$

$0 - (0 \times -1) = 0$

$0 - (0 \times 1) = 0$

$0 - (0 \times 0) = 0$

$80 - (0 \times 60) = 80$

	C_j	3	8	0	0	M	M		
C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	b	$\theta = \frac{b}{a_{ik}}$
0	S_1	1	0	1	0	0	0	80	$\frac{80}{1} = 80$
8	x_2	0	1	0	-1	1	0	60	$\frac{60}{1} = 60$
M	A_2	1	1	0	0	1	-1	140	$\frac{140}{1} = 140$
Z_j		3M	8	0	M-8	8-M	M		
$Z_j - C_j$		3-M	0	0	8-M	2M-8	0		
		(-6)			(-1)	10			

1st Row (-Pivot) ($0 - S_1$):-

No change

$$1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 80$$

2nd Row ($8 - x_2$):-

$0 - (0 \times 1) = 0$

$1 - (0 \times 0) = 1$

$0 - (0 \times 1) = 0$

$-1 - (0 \times 0) = -1$

$1 - (0 \times 0) = 1$

$0 - (0 \times 0) = 0$

$60 - (0 \times 60) = 60$

A4 Long Book

DATE: / /

	C_j	3	8	0	0	M	M		
	C_{B_j}	x_1	x_2	s_1	s_2	A_1	A_2	b	$\theta = \frac{b}{a_{ik}}$
1	x_1	1	0	1	0	0	0	80	$\frac{80}{1} = 80$
2	x_2	0	1	0	-1	1	0	60	$\frac{60}{-1} = -60$
3	A_2	0	0	-1	1	-1	1	60	$\frac{60}{1} = 60$
4	M								
	$B_j - C_{B_j}$	3	8	3-M	-8-M	3-M	M		
	$C_j - E_j$	0	0	M-3	3-M	2M-8	0		

Pivot Row (M_{A_2}) :- 3rd Row :-
No change
0 0 -1 1 -1 1 60

1st Row ($3-x_1$) :-
No change
0 1 0 0 0 80

2nd Row ($8-x_2$) :-
 $0 - (-1 \times 0) = 0$
 $1 - (-1 \times 1) = 1$
 $0 - (-1 \times -1) = -1$
 $-1 - (-1 \times 1) = 0$
 $1 - (-1 \times -1) = 0$
 $0 - (-1 \times 1) = 1$
 $60 - (-1 \times 60) = 120$

DATE: / /

	C_j	3	8	0	0	M	M		
	C_{B_j}	x_1	x_2	s_1	s_2	A_1	A_2	b	
1	x_1	1	0	1	0	0	0	80	
2	x_2	0	1	-1	0	0	1	120	
3	s_2	0	0	-1	1	-1	1	60	
	$B_j - C_{B_j}$	3	8	-5	0	0	-8		
	$C_j - E_j$	0	0	5	0	M	M-6		

Simplex end.

DATE: / /

PAGE NO:

$$4x_1 + 7x_2 - 2x_3 \leq 3 \quad \text{--- multiply with } (-1)$$

$$\therefore -4x_1 - 7x_2 + 2x_3 \geq -3 \quad \text{--- (5) - (4)}$$

$$\therefore \text{Max } Z^* = 7a + 4b - 10c + 3d - 3c$$

sub. to,

$$\checkmark 8a + 6b - 7c + 1d - 4e \leq 3$$

$$\checkmark 5a + 1b + 2c - 2d - 7e \leq -2$$

$$\checkmark -5a - 1b - 2c + 2d + 7e \geq 2$$

$$\checkmark 4a + 3b + c + 5d + 2e \leq 4$$

$$a, b, c, d, e \geq 0$$

Q.6) $\text{Min } Z = 3x + 8y$

sub. to,

$$x \leq 80$$

$$y \geq 60$$

$$x + y = 200$$

Convert this primal into duality.

$$x \leq 80 \rightarrow -x \geq -80 \rightarrow (a)$$

$$y \geq 60 \rightarrow y \geq 60 \rightarrow (b)$$

$$x + y = 200 \rightarrow \begin{cases} x + y \geq 200 \rightarrow c' \\ x + y \leq 200 \rightarrow c'' \end{cases}$$

$$\therefore -x - y \geq -200 \rightarrow c''$$

$$\text{where } c = c' = c''$$

$$\text{Max } Z^* = -80a + 60b + 200c' - 200c''$$

$$80a + 60b + 200c' - 200c''$$

DATE: / /

PAGE NO:

$$\text{Max } Z^* = -80a + 60b + 200c$$

sub. to,

$$-a + c' - c'' \leq 3$$

$$\text{i.e. } -a + c \leq 3$$

$$b + c' - c'' \leq 8$$

$$\text{i.e. } b + c \leq 8$$

$$a, b, c \geq 0$$

Q.1) Theory = $\frac{m}{n} - (3 \times 2)$
Graphical Method / Duality / Formulation (7)

OR

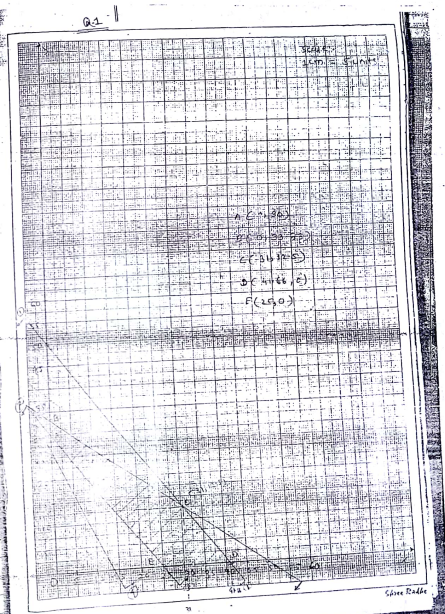
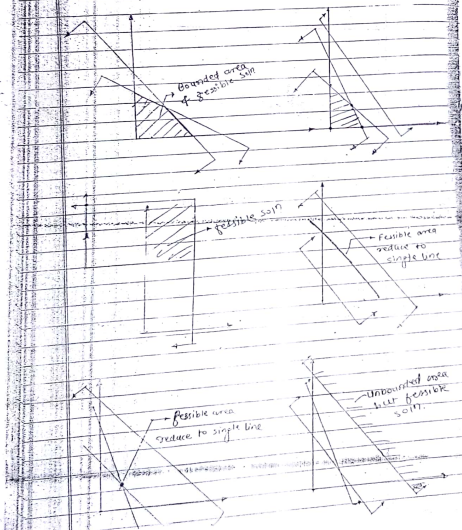
Q.2) Simplex /

Formulation with simplex

--- " --- with Graphical

A4 Long Book

Graphical Method *



DATE: / /

- $\leq$ → Arrow Bend towards origin
- $\geq$ → Arrow Bend Away from origin
- $=$ → No bend

Q.1) $\text{Max } Z = 4x + 5y$
 Sub to,
 $6x + 5y \leq 250$
 $4x + 6y \leq 200$
 $6x + 5y \geq 150$
 $9x + 5y \geq 130$
 $x, y \geq 0$

Solve by graphical method.

Soln: Split the given equation to form
 and put one time time
 $(x=0)$ and other time $(y=0)$

$\therefore$ ① $6x + 5y \leq 250$ $\begin{cases} x=0, y=50 (0, 50) \text{ ①} \\ y=0, x=41.66 (41.66, 0) \text{ ②} \end{cases}$

② $4x + 6y \leq 200$ $\begin{cases} x=0, y=33.33 (0, 33.33) \text{ ③} \\ y=0, x=50 (50, 0) \text{ ④} \end{cases}$

③ $6x + 5y \geq 150$ $\begin{cases} x=0, y=30 (0, 30) \text{ ⑤} \\ y=0, x=25 (25, 0) \text{ ⑥} \end{cases}$

④ $9x + 5y \geq 130$ $\begin{cases} x=0, y=26 (0, 26) \text{ ⑦} \\ y=0, x=14.4 (14.4, 0) \text{ ⑧} \end{cases}$

Q. 10

A (0, 30) $\rightarrow \therefore \max Z = 4 \times 0 + 5 \times 30 = 150$

B (0, 33.33) $\rightarrow \max Z = 4 \times 0 + 5 \times 33.33 = 166.65$

C (31, 12.5) $\rightarrow \max Z = 4 \times 31 + 5 \times 12.5 = 186.5$

D (41.66, 0) $\rightarrow \max Z = 4 \times 41.66 + 5 \times 0 = 166.65$

E (25, 0) $\rightarrow \max Z = 4 \times 25 + 5 \times 0 = 100$

$\therefore \max Z = 186.5$

Q. 2) $\max Z = 5x + 10y$

sub to,

$5x + 8y \leq 48$

$5x + 3y \leq 48$

$x + 8y \leq 48$

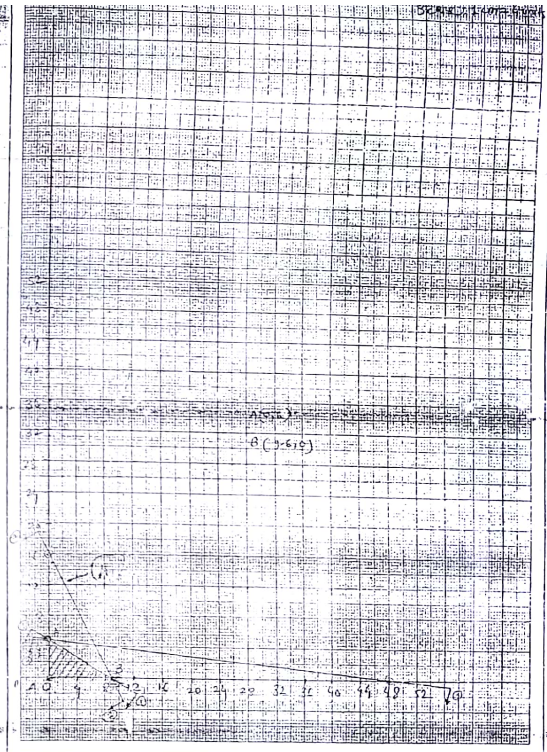
Solve by graphical method.

Soln:

① $5x + 8y \leq 48$ $\begin{cases} x=0, y=6 (0, 6) \\ y=0, x=9.6 (9.6, 0) \end{cases}$

② $5x + 3y \leq 48$ $\begin{cases} x=0, y=16 (0, 16) \\ y=0, x=9.6 (9.6, 0) \end{cases}$

③ $x + 8y \leq 48$ $\begin{cases} x=0, y=6 (0, 6) \\ y=0, x=48 (48, 0) \end{cases}$



DATE / /

PAGE NO.

$$\therefore \max Z = 5 \times 0 + 10 \times 6 = 60 \quad \checkmark$$

$$\max Z = 5 \times 9.6 + 10 \times 0 = 48$$

$$Z = 12x_1 + 16x_2$$

$$Z_A(0,0) = 0$$

$$Z_B(0,2) = 32$$

$$Z_C(8,0) = 96 \quad \checkmark$$

$$Z_D(4,2) = 80$$

A4 Long Book

Sub to,

$$\textcircled{1} \quad 3x + 2y \leq 210$$

$$2x + 4y \leq 300$$

$$y \leq 65$$

$$x, y \geq 0$$

$$\textcircled{1} \quad 3x + 2y \leq 210$$

$$x=0, y=105 \quad (0, 105) \textcircled{A}$$

$$y=0, x=70 \quad (70, 0) \textcircled{B}$$

$$\textcircled{2} \quad 2x + 4y \leq 300$$

$$x=0, y=75 \quad (0, 75) \textcircled{C}$$

$$y=0, x=150 \quad (150, 0) \textcircled{D}$$

$$\textcircled{3} \quad y \leq 65 \rightarrow x=0, y=65 \quad (0, 65) \textcircled{E}$$

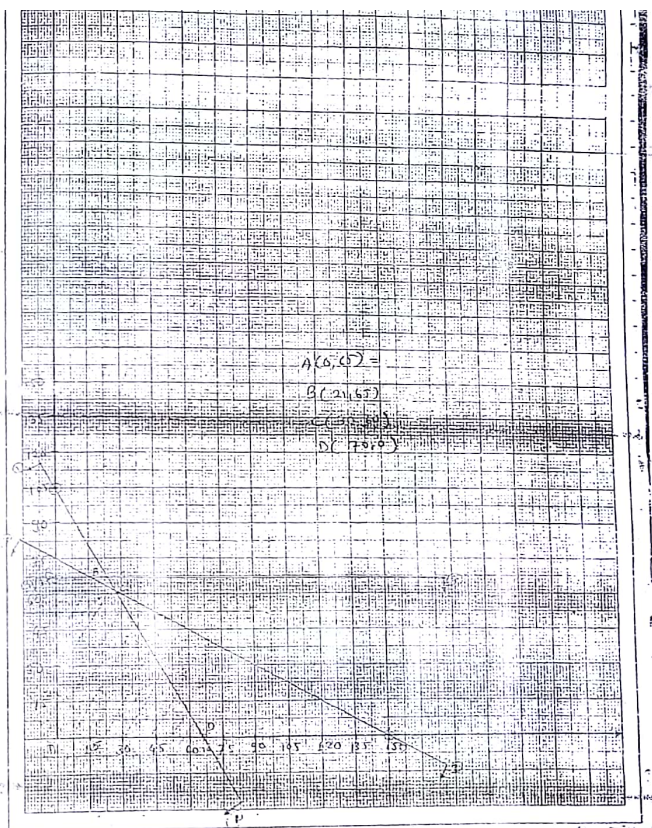
$$\text{max } Z = 20 \times 0 + 30 \times 65 = 1950$$

$$\text{max } Z = 20 \times 21 + 30 \times 65 = 2370$$

$$\text{max } Z = 20 \times 80 + 30 \times 60 = 2400$$

$$\text{max } Z = 20 \times 70 + 30 \times 0 = 1400$$

$$\therefore \text{max } Z = 2370$$



Shree Radhe

DATE: / / PAGE NO: 89

Q4) $\min Z = 10x_1 + 4x_2$
 Sub. to,
 $4x_1 + x_2 \geq 50$
 $2x_1 + x_2 \geq 60$

Solve

① $4x_1 + x_2 \geq 50$ $\rightarrow x_1 = 0, x_2 = 50 \rightarrow (0, 50)$
 $x_2 = 0, x_1 = 12.5 \rightarrow (12.5, 0)$

② $2x_1 + x_2 \geq 60$ $\rightarrow x_1 = 0, x_2 = 60 \rightarrow (0, 60)$
 $x_2 = 0, x_1 = 30 \rightarrow (30, 0)$

$\min Z = 10 \times 0 + 4 \times 50 = 200$
 $\min Z = 10 \times 12.5 + 4 \times 0 = 125$
 $\min Z = 10 \times 30 + 4 \times 0 = 300$
 $\therefore \min Z = 125$

Q5) $\min Z = 2x_1 + 8x_2$
 Sub. to,
 $x_1 \leq 80$
 $x_2 \geq 60$
 $x_1 + x_2 = 200$

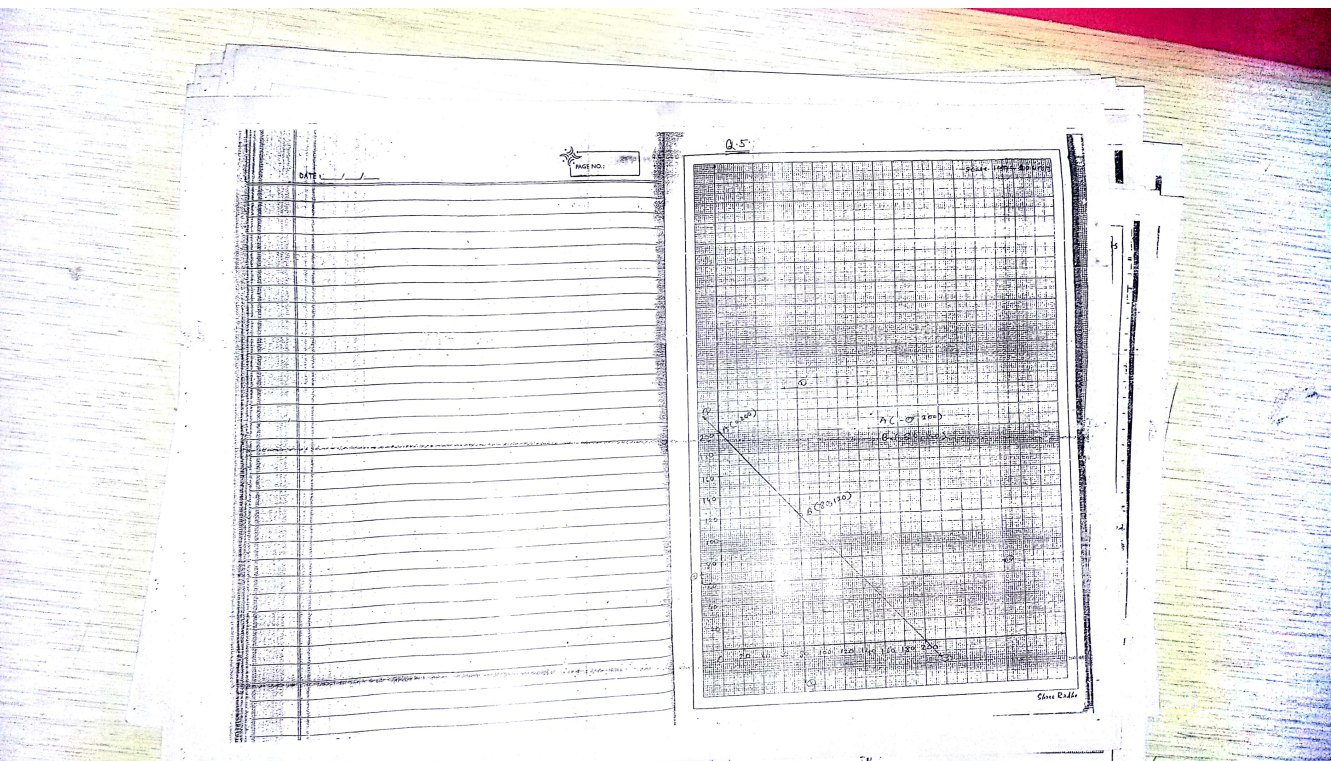
① $x_1 \leq 80$ $\rightarrow x_1 = 80, x_2 = 0 \rightarrow (80, 0)$

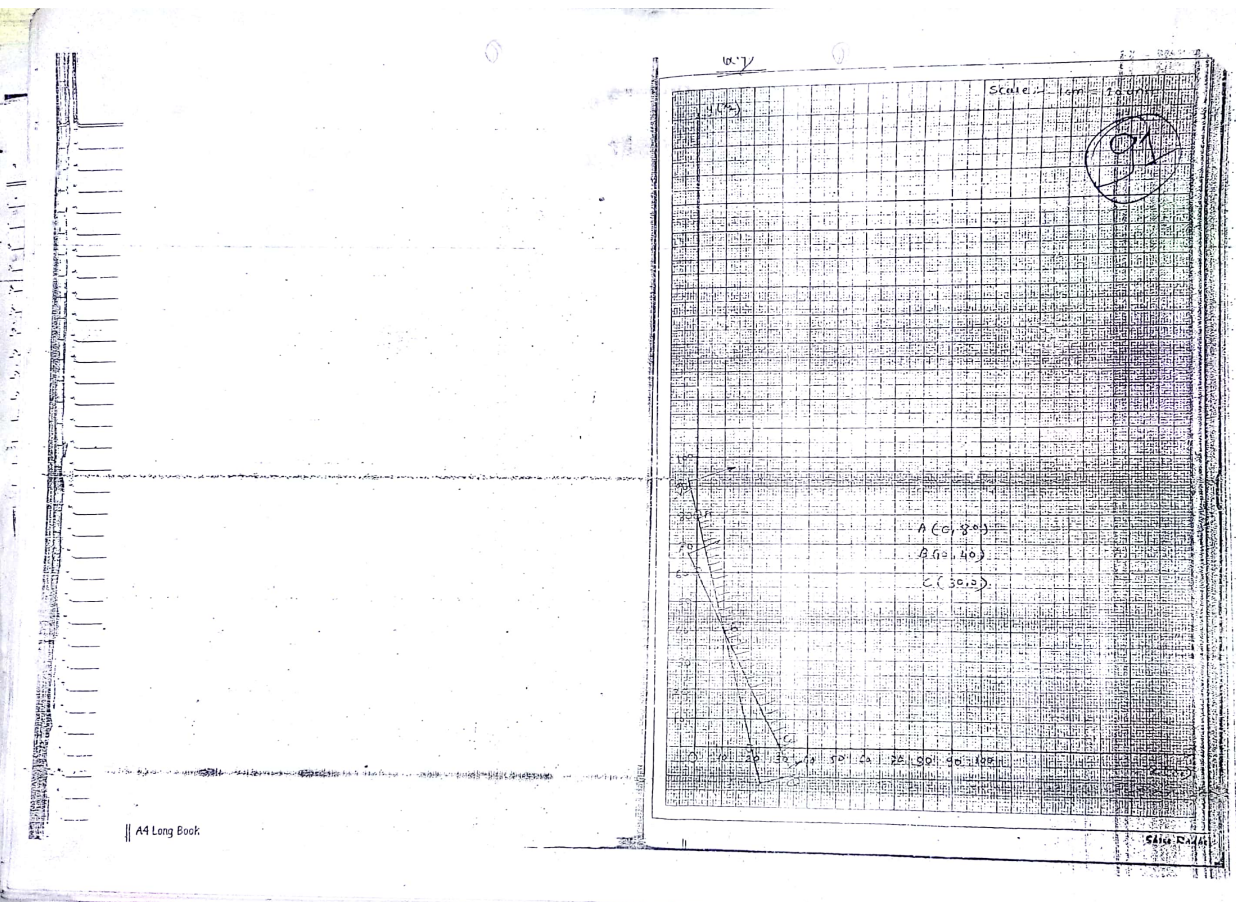
② $x_2 \geq 60$ $\rightarrow x_1 = 0, x_2 = 60 \rightarrow (0, 60)$

③ $x_1 + x_2 = 200$ $\rightarrow x_1 = 0, x_2 = 200 \rightarrow (0, 200)$
 $x_2 = 0, x_1 = 200 \rightarrow (200, 0)$

$\min Z = 2 \times 80 + 8 \times 200 = 1680$
 $\min Z = 2 \times 0 + 8 \times 60 = 480$
 $\min Z = 2 \times 200 + 8 \times 0 = 400$
 $\therefore \min Z = 400$

44 Long Book $\min Z = 1200$





Unit No. 6

PAGE NO. _____

DATE: / /

* Queing Theory *
(waiting line model) } Q. 11
OR
Simulation } Q. 12

Formulas :-

1) Mean or Avg. No. of units in the system

$$\bar{n} = \frac{\lambda}{\mu - \lambda}$$

2) Average waiting time for customers

$$\bar{t}_w = \frac{\lambda}{\mu(\mu - \lambda)}$$

3) The probability that No. customers in service

$$P_0 = 1 - \frac{\lambda}{\mu}$$

4) Probability that all the job will leave the unit

$$1 - P_0 = \frac{\lambda}{\mu}$$

5) Probability that there are 'n' customers in service

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right)$$

2.6.1 mm Book

DATE: / /

PAGE NO. _____

6) Mean Queue length or Avg. Q. length for waiting line

$$= \frac{\lambda^2}{\mu(\mu - \lambda)}$$

7) Equipment utilization or

$$= \frac{\lambda}{\mu}$$

8) Minimum cost of service rate :-

$$= \lambda + \sqrt{\lambda \frac{C_w}{C_f}}$$

9) Total expected system cost/hr.

$$= C_w \left(\frac{\lambda^2}{\mu - \lambda}\right) + C_f \left(\frac{\lambda}{\mu}\right)$$

where,

λ = Arrival rate, unit/time unit/unit time

μ = Service rate unit/unit time

C_w = cost of waiting / period

C_f = cost of servicing / unit

n = no. of units in the system

A4 Long Book

Numerical

PAGE NO.:

DATE: / /

- Q.1) Vehicle arrival at petrol pump having 1 petrol unit in a poisson's passion with an average of 10 unit/hr. the service time distribution is potential with a mean of 3m. Find
- 1) No. of units in the system
 - 2) Waiting time for each customer
 - 3) Avg. length of queue
 - 4) Probability that the customer arriving will have to wait.
 - 5) Utilization factor
 - 6) Probability that the number units in a system two.

Given: $\lambda = 10$ unit/hr, arrival rate.

Service time = 3 min

$\therefore 1$ vehicle = 3 min

$\therefore 10$ vehicles = 60 min (1hr)

$\therefore \mu = 60 \times 1$

$\therefore \mu = 60$

$\therefore \mu = 20$ unit/hr

(1) No. of units in the system is given by

$$\bar{n} = \frac{\lambda}{\mu - \lambda} = \frac{10}{20 - 10} = \frac{10}{10} = 1 \text{ unit}$$

(2) Waiting Time for customers

$$T_w = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{10}{20(20 - 10)} = \frac{10}{20(10)} = \frac{10}{200} = 0.05 \text{ hr}$$

(3) Avg. length of queue:-

$$\begin{aligned} &= \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{10^2}{20(20 - 10)} \\ &= \frac{100}{20(10)} \\ &= \frac{100}{200} \\ &= 0.5 \end{aligned}$$

(4) Probability that the customer arriving will have to wait.

$$\begin{aligned} 1 - P_0 &= \frac{\lambda}{\mu} \\ &= \frac{10}{20} \\ &= 0.5 \\ &= 50\% \end{aligned}$$

(5) Utilization Factor

$$\rho = \frac{\lambda}{\mu} = \frac{10}{20} = 0.5$$

(6) when $n = 2$

$$\begin{aligned} P_n &= \left(\frac{\lambda}{\mu}\right)^n \left(1 - \frac{\lambda}{\mu}\right) \\ &= \left(\frac{10}{20}\right)^2 \left(1 - \frac{10}{20}\right) \\ &= 0.125 \end{aligned}$$

$$P_n = 12.5\%$$

A4 Long Book

A car arrival at a petrol pump is poisson distribution with $\lambda = 10$ unit/hr. The attendant takes avg. of 5 min/car to supply the petrol. Determine (1) Avg. no. of car in queue. (2) Probability that the pump attendant is idle. (3) Average no. of car in the system.

Arrival Rate,

$$\lambda = 10 \text{ unit/hr}$$

Service time = 5 min/car.

$$\therefore 1 \text{ car} \rightarrow 5 \text{ min.}$$

$$\therefore 12 \text{ cars} \rightarrow 60 \text{ min}$$

Service Rate,

$$\therefore \mu = \frac{60}{5} = 12 \text{ unit/hr}$$

(1) Avg. no. of car in queue

$$= \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{12(12-10)} = 4.167$$

(2) Probability that the pump attendant is idle.

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{10}{12}$$

$$= 0.167$$

$$P_0 = 16\%$$

(3) Avg. no. of cars in the system

$$\bar{n} = \frac{\lambda}{\mu - \lambda}$$

$$= \frac{10}{12-10}$$

$$\bar{n} = 5$$

Q.3/

Arrival at telephone booth are consider to be with an avg. of 10 min betw arrival and next the length of the phone call is 3 min.

Determine (1) what is the probability that a person arriving at the booth they have to wait. (2) What is the avg. length of queue.

(3) If customer has to wait for more than 3 min then second booth can be.

How much flow of arrival be increase to have second booth.

Arrival time λ

$$10 \text{ min}$$

$$1 \text{ person} \rightarrow 10 \text{ min.}$$

$$\therefore \lambda \text{ person} \rightarrow 60 \text{ min} \quad \therefore \lambda = 6 \text{ unit/hr}$$

$$\therefore \text{Service time} = 3 \text{ min.}$$

$$\lambda = 6 \text{ unit/hr}$$

$$\therefore 1 \text{ person} = 3 \text{ min}$$

$$\therefore 12 \text{ person} = 60 \text{ min.}$$

$$\mu \times 3 = 60 \times 1$$

$$\mu = 20$$

$$3$$

A4 Long Book

$$\mu = 20 \text{ unit/hr.}$$

DATE: / /

PAGE NO:

$$\begin{aligned} \textcircled{1} 1 - P_0 &= \frac{\lambda}{\mu} \\ &= \frac{6}{20} \\ &= 0.3 \\ &= 30\% \end{aligned}$$

$$\begin{aligned} \textcircled{2} \text{ What is the avg. length of queue} \\ &= \frac{\lambda^2}{\mu(\mu - \lambda)} \\ &= \frac{6^2}{20(20 - 6)} \\ &= 0.128 \text{ units} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \bar{t}_w = \text{waiting time} &= 3 \text{ min} \times \frac{1}{60} = \frac{3}{60} \\ \therefore \bar{t}_w &= \frac{\lambda}{\mu(\mu - \lambda)} \\ \frac{3}{60} &= \frac{6}{20(20 - \lambda)} \\ \therefore \lambda &= 10 \text{ units/min} \end{aligned}$$

A4 Long Book

DATE: / /

PAGE NO:

- Q.4) The typist receive in the work according to poissions with an avg. rate of 4 job/hr. They are type on FIF with an Avg. job requiring 12 min of the typist time. The arrival time per job exponentially distributed. Determine
- ① Probability that an arriving job will have to wait.
 - ② Probability that the typist is waiting for job.
 - ③ The probability that the two jobs are already in a queue.
 - ④ Avg. waiting time.
 - ⑤ Avg. Queue time length.

Soln.

Data:- Arrival Rate,

 $\lambda = 4 \text{ job/hr}$

Service Time = 12 min

 $\therefore 1 \text{ typist} \rightarrow 12 \text{ min}$ $\mu \text{ typist} \rightarrow 60 \text{ min}$ $\mu \times 12 \rightarrow 60 \times 1$

$$\mu = \frac{60}{12}$$

$$\mu = 5$$

- ① Probability that an arriving job will have to wait.

$$1 - P_0 = \left(\frac{\lambda}{\mu} \right)$$

$$1 - P_0 = \frac{4}{5}$$

A4 Long Book

$$1 - \frac{4}{5} = 0.2$$

DATE: / /

PROB. NO.:

Q. 1) Probability that the system is waiting for 1st b.

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{4}{5}$$

$$= 0.2 = 20\%$$

Q. 2) The probability that the two cars are already in the queue when $n=2$.

$$P_n = \frac{(\lambda/\mu)^n}{n!} \left(1 - \frac{\lambda}{\mu} \right)$$

$$= \left(\frac{4}{5} \right)^2 \left(1 - \frac{4}{5} \right)$$

$$= 0.128 = 12.8\%$$

Q. 3) Avg. waiting time

$$T_w = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$= \frac{4}{5(5-4)}$$

$$= 0.8$$

Q. 4) Avg. queue length

$$P_n = \frac{(\lambda/\mu)^n}{n!} \left(1 - \frac{\lambda}{\mu} \right)$$

$$= \frac{4^n}{5^n} \left(1 - \frac{4}{5} \right)$$

$$= \frac{4^n}{5^n} \cdot \frac{1}{5}$$

$$= \frac{4^n}{5^{n+1}}$$

DATE: / /

PROB. NO.:

Q. 1) A departmental store had a single cashier. Customers arriving at the rate of 20/hr. No. of customers that can be processed by the cashier is 24/hr. What is the

Q. 2) Probability that cashier is idle.

Q. 3) Avg. no. of customers in queue.

Q. 4) Avg. time customer spent in the queue waiting for service.

Soln. Data:

$$\lambda = 20/\text{hr.}$$

$$\mu = 24/\text{hr.}$$

Q. 1) Probability that cashier is idle.

$$P_0 = 1 - \frac{\lambda}{\mu}$$

$$= 1 - \frac{20}{24}$$

$$= \frac{4}{6}$$

$$= 0.667$$

$$P_0 = 66.7\%$$

Q. 2) Avg. no. of customers in queue

$$P_n = \frac{(\lambda/\mu)^n}{n!} \left(1 - \frac{\lambda}{\mu} \right)$$

$$= \frac{20^n}{24^n} \left(1 - \frac{20}{24} \right)$$

$$= \frac{20^n}{24^n} \cdot \frac{4}{6}$$

$$= \frac{20^n}{24^n} \cdot \frac{1}{3}$$

Q. 3) Avg. time customer spent in the queue waiting for service.

$$T_w = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$= \frac{20}{24(24-20)}$$

$$= \frac{20}{24 \cdot 4}$$

$$= \frac{20}{96}$$

$$= 0.208 \text{ hr.}$$

DATE: / /

PAGE NO.:

- Q.6) In a railway marshalling yard goods arrive ~~30 train/day~~ 30 train/day. Assuming that the inter arrival time follows an exponential distribution with an average time of 36 min. Calculate ① expected queue length
 ② Probability when the queue size exceeds to 10.
 ③ Input of material Avg. of 33 per day what will be the change in 1st and 2nd.

soln.

Arrival Rate,

$$\lambda = 30 \text{ train/day}$$

$$\lambda = \frac{30}{24} \text{ train/hr.}$$

$$\therefore \text{Service time} = 36 \text{ min}$$

$$\mu = \frac{60}{36} \text{ min}$$

$$\mu = \frac{60}{36} \text{ min}$$

$$\mu \times 36 = 60 \times 1$$

$$\mu = \frac{60}{36} \text{ train/hr.}$$

$$= 1.67$$

① expected queue length

$$\text{Length} = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$= \frac{(30/24)^2}{1.67(1.67 - 30/24)}$$

$$= \left(\frac{60}{36}\right) \left[\left(\frac{60}{36}\right) - \left(\frac{30}{24}\right) \right]$$

$$= 2.25$$

DATE: / /

PAGE NO.:

- prob. when queue size = 10
 ② when $n = 10$

$$P_n = \left(\frac{\lambda}{\mu}\right)^n (1 - \frac{\lambda}{\mu})$$

$$= \left(\frac{1}{1.67}\right)^{10} \left[1 - \frac{30}{60}\right]$$

$$= 0.014110 = 1.41\%$$

- ③ input of train increase to avg. of 33/day
 for $\lambda = 33 \text{ train/day}$
 $= \frac{33}{24} \text{ train/hr.}$

$$\text{① } Q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$= \frac{(33/24)^2}{1.67(1.67 - 33/24)}$$

$$= \frac{60}{36} \left[\left(\frac{60}{36}\right) - \left(\frac{33}{24}\right) \right]$$

$$= 3.99$$

when $n = 10$

$$= \left(\frac{1}{1.67}\right)^{10} (1 - \frac{33}{60}) = 0.025 = 2.5\%$$

The change in Q size

$$\text{② } P_{10} = 3.99 - 2.25 = 1.74$$

$$\text{③ } \text{prob. Q size} = 0.025 - 0.014 = 2.5 - 1.4$$

$$= 0.011 = 1.1\%$$

A4 Long Book

Ques • Simulation •

It is basically a data generating technique some where it is risky, risky, but or those consuming to conduct real study or experiment to know more about situation and/or problem.

The available analytical methods can not be used in all situation due to large number variable or large number of interrelationship among the variables and complex nature of relationship. It is not possible to develop an analytic model representing the real situation.

It should be noted that the simulation does not solve the problem by itself but it only generate the required information or data needed for the decision making.

The dentist scheduled all his patient 30 min appointment. Some of the patient take more or less than 30 min depending on the time of dental work to be done. Following summary shows various category of work, their probability and time needed to complete full work.

category of service	Time (min) Required	Prob. of category
Filling	45	0.4
Crown	60	0.15
Cleaning	15	0.15
Extraction	45	0.1
Check up	15	0.2

Simulate the dentist's cleaning for 4 hours and determine the average waiting time for the patient as well as the utilization of the doctor. Assume that all the patient shows at the cleaning clinic exactly at their schedule, the arrival time starts.

Step I To find Random No. Interval

As given, use the following Random Number 40, 82, 11, 34, 25, 66, 17, 79

Step II To find Random No. Interval

category of service	Time (min) Req.	Probability of category	Cumulative Probability	Random No. Interval
Filling	45	0.4	0.4	00 to 39
Crown	60	0.15	0.55	40 to 54
Cleaning	15	0.15	0.70	55 to 69
Extraction	45	0.1	0.80	70 to 79
check up	15	0.2	1.00	80 to 100

11

DATE: / /

PAGE NO.:

Step 2 To assign the Random No. to customer (patients) & category of service.

The various parameters of queue system such as arrival pattern of customer, service time and waiting time in the contents as shown below.

$$\therefore \text{No. of Random Number} = 4 \times 60 = 240$$

Patient	Schedule Arrival	Random Numbers	Category of service	Service Time(min)
1	8:00	40	Crown	60
2	8:30	82	checkup	15
3	9:00	112	Filling	45
4	9:30	39	Filling	45
5	10:00	25	Filling	45
6	10:30	66	Filling	45
7	11:00	17	Filling	45
8	11:30	79	Extraction	45

Step 3 Assign the service time to patient (customer)



DATE: / /

PAGE NO.:

(1)		(2)			
Patient Number	Schedule Arrival	Service starts at	Service time (min)	Service ends at	Waiting time (min)
1	8:00	8:00	60	9:00	0
2	8:30	9:00	15	9:15	30
3	9:00	9:15	45	10:00	15
4	9:30	10:00	45	10:45	30
5	10:00	10:45	45	11:30	45
6	10:30	11:30	15	11:45	60
7	11:00	11:45	45	12:30	45
8	11:30	12:30	45	1:15	60
					$\Sigma = 285 \text{ min}$

$$\therefore \text{Average waiting time} = \frac{285 \text{ min}}{8} = 35.625 \text{ min}$$

$\therefore$ No. Idle time for a doctor;

$\therefore$ there is service end time & service start time is same.

2) The service station schedule all its: a customer of 30 min appointment. Some of the customer may more than 30 min depending upon the type of work. Following information show various work and there is prob. The time needed to complete the work is also given.

Type of schedule	Time (min) Required	Probability of schedule
A	60	0.15
B	45	0.4
C	45	0.15
D	15	0.2
E	15	0.1

Simulate the problem and find the individual waiting time and average waiting time. Assume the customer comes at these schedule time. The service starts at 12 noon. Give following Random number for handling above problem. 35, 82, 11, 39, 25, 66, 16, 59.

soln: step 1 To find Random Number Interval.

Type of schedule	Time (min) Required	Probability	Cumulative prob.	Random No. Interval
A	60	0.15	0.15	00 to 14
B	45	0.4	0.55	15 to 54
C	45	0.15	0.70	55 to 69
D	15	0.2	0.9	70 to 89
E	15	0.1	1.00	90 to 100

step 2 To assign the Random No. to customer of type of schedule.

Customer Number	Schedule Arrival	Random No.	Type of schedule	Service time (min)
1	12:00	35	B	45
2	12:30	82	D	15
3	1:00	11	A	60
4	1:30	39	B	45
5	2:00	25	B	45
6	2:30	66	C	45
7	3:00	16	B	45
8	3:30	59	C	45

↓

DATE: / /

PAGE NO: 10

Step 3 Assign the service time to patients (customer)

Customer Number	Schedule Arrival	Service starts at	Service time/min	Service ends at	Waiting time/min
1	12:00	12:00	45	12:45	0
2	12:30	12:45	15	1:00	15
3	1:00	1:00	60	2:00	0
4	1:30	2:00	45	2:45	30
5	2:00	2:45	45	3:30	45
6	2:30	3:30	45	4:15	60
7	3:00	4:15	45	5:00	75
8	3:30	5:00	45	5:45	90
					$\Sigma = 315$

Avg. waiting time = $\frac{315}{8} = 39.375$ mins

DATE: / /

PAGE NO: 10

Q.3) A service station has a single channel service with a following details. Customer comes to the service centre starting after 20 min span.

Category of service	Time Req. (min)	Probability of category
1	55	0.2
2	45	0.15
3	05	0.15
4	55	0.2
5	15	0.1
6	20	0.2

simulate the above problem for 5 hours and determine the average waiting time for customers assume the following random numbers, 4, 94, 71, 850, 52, 24, 57, 15, 66, 25, 70, 22, 26, 99, 97.

Step 1 To find Random no. Interval.

Category of service	Time (min) Required	Prob	Cumulative Prob	Random No. Interval
1	55	0.2	0.2	00 to 19
2	45	0.15	0.35	20 to 34
3	05	0.15	0.50	35 to 49
4	55	0.2	0.70	50 to 69
5	15	0.1	0.80	70 to 79
6	20	0.2	1.00	80 to 99

A4 Long Book

STEP 2 To assign the Random No's to the customer & assign the category of service

$$\text{No. of Random Number} = 5 \times 60 = 15$$

Assuming Schedule arrival starts at 9:00 Am

Customer Number	Schedule Arrival	Random Number	category of service	Service time (min)
1	9:00	4	1	55
2	9:20	94	6	20
3	9:40	71	5	15
4	10:00	50	4	55
5	10:20	52	4	45
6	10:40	24	2	25
7	11:00	57	4	55
8	11:20	15	1	55
9	11:40	66	4	55
10	12:00	25	2	45
11	12:20	70	5	45
12	12:40	22	2	45
13	1:00	77	2	20
14	1:20	77	6	20
15	1:40	77	6	20

DATE: / /

PAGE NO: /

STEP 3 Assign the service time to the customer

customer No.	Schedule Arrival	Service Start at	Service time (min)	Service ends at	Waiting time (min)
1	9:00	9:00	55	9:55	0
2	9:20	9:55	20	10:15	35
3	9:40	10:15	15	10:30	35
4	10:00	10:30	55	11:25	80
5	10:20	11:25	55	12:20	65
6	10:40	12:20	45	1:05	100
7	11:00	1:05	55	2:00	125
8	11:20	2:00	55	2:55	160
9	11:40	2:55	55	3:50	195
10	12:00	3:50	45	4:35	230
11	12:20	4:35	15	4:50	255
12	12:40	4:50	45	5:35	250
13	1:00	5:35	45	6:20	275
14	1:20	6:20	20	6:40	300
15	1:40	6:40	20	7:00	300
					82355

$$\text{Avg. waiting time} = \frac{82355}{15}$$

$$= 157 \text{ min}$$

A4 Long Book

A4 Long Book

DATE: / /

PAGE NO.:

Q.4) An automobile service station centre as a single channel service station for comes to the service station at 30 min interval, simulate the problem for 6 hours and determine average waiting for cars. The service starts at 8:00 am. The details of service station is as follows.

Category of service	Time Req. (min)	Probability of category
washing	45	0.20
check up	55	0.15
Repair	29	0.15
Maintenance	24	0.30
Oiling	45	0.20

Assume random numbers 37, 29, 72, 85, 19, 97, 81, 75, 64, 15, 09, 18, 83, 79, 23, 33, 25

Soln: Step 1 To find Random no. Interval

Category of service	Time (min) Req.	Prob. of Category	Cumulative Prob.	Random no. Interval
washing	45	0.20	0.20	0 to 19
check up	55	0.15	0.35	20 to 34
Repair	29	0.15	0.50	35 to 49
Maintenance	24	0.30	0.80	50 to 79
Oiling	45	0.20	1.00	80 to 99

DATE: / /

PAGE NO.:

Step 2 To assign the Random no's to the customers & assign the category of service

$$\text{No. of Random Number} = \frac{6 \times 60}{30} = 12$$

We have to use only 12 random no's from the given random numbers

Customer Number	Schedule Arrival	Random Number	Category of service	Service Time (min)
1	8:00	37	Repair	29
2	8:30	29	check up	55
3	9:00	72	Maintenance	24
4	9:30	05	washing	45
5	10:00	13	washing	45
6	10:30	99	Oiling	45
7	11:00	81	Oiling	45
8	11:30	75	Maintenance	24
9	12:00	64	Maintenance	24
10	12:30	15	washing	45
11	1:00	03	washing	45
12	1:30	18	washing	45

Step 3 Assign the service time to the customers

customer number	schedule arrives	service start at	service time	service ends at	waiting time
1	8:00	8:00	23	8:23	0
2	8:20	8:20	53	9:13	0
3	9:00	9:25	24	9:49	25
4	9:30	9:49	45	10:34	19
5	10:00	10:34	45	11:19	34
6	10:30	11:19	45	12:04	49
7	11:00	12:04	45	12:49	64
8	11:30	12:49	24	01:13	73
9	12:00	01:13	24	1:37	73
10	12:30	1:37	45	2:22	67
11	1:00	2:22	45	3:07	82
12	1:30	3:07	45	3:52	87
					2:589

Average waiting time = $\frac{589}{12}$
= 49.08 min

end

Q.5 - Part A) Game Theory

Q.6 - Part B) Inventory control

"GAME" Theory

The competition is the unpleasant competition life. We can say that the competitive situation exist when two or more individuals make the decision in situation that involves conflicting interest and in which each is control by the reaction of all individual parties. These competitive situation is called as game.

The term game represent the conflict between two or more parties.

A situation term as game when it poses the following property.

- 1) Number of competitors is finite.
- 2) There is conflict in interest between the participants.
- 3) Which of the participants has finite set of possible process of action.

Strategy:-

Strategy is defined as the complete set of plan of action, specify a size what the players are do under every possible future concerning that might occur during play of game. They are of 2 types.

DATE: / /

PAGE NO.:

- 1) pure strategy
- 2) Mix strategy

1) Pure strategy :-

It is a decision in advance of all plays, always to choose a particular course of action. In other words it is the best strategy for each player to play one particular strategy through out the game is called as pure strategy.

2) Mix strategy :-

It is decision in advance to all place to choose a course of action for each play in accordance with some other distribution.

• Pay-off Matrix :-

A pay-off matrix is the outcome of playing the game. A pay-off matrix is the table showing the amount received by the players.

		Player 'B'			
		1	2	3	4...n
Player A	1	a_{11}	a_{12}	...	a_{1n}
	2	a_{21}	a_{22}	...	a_{2n}
	3	...	...	...	...
	n	...	...	...	a_{nn}

Pay-off matrix

- The value of game, 'V'

A4 Long Book

DATE: / /

PAGE NO.:

• The maximin and Minimax principle

The principle use for the selection of optimal strategy by two player.

Consider to player A & B.

A is the player which to maximize its gain
B wishes to

'A' would like to maximize its minimum gain, we obtain for player A the value is called as maximum value and corresponding strategy is called maximin strategy.

It is given by 'i'

ON other hand player 'B' wish to minimize his loss these value is called as minimax value which is minimum of maximum loss and the corresponding strategy is called minimax strategy. It is given by 'j'

When these two are equal i.e. maximin value equal to the minimax value the corresponding optimum strategy and game is said to have saddle point and the value of game is given by saddle point.

- Saddle point It is the position in the pay-off matrix where the maximum of row minimum coincide with the minimum of column maximum. The pay-off matrix at the saddle point is called as Long Book and its value of game, and it is given by 'V'

DATE: / /

PAGE NO.:

The types of games are

- 1) Two person game and 'n' person game
- 2) Zero ~~sum~~ ^{sum} game
- 3) Two person zero sum game

Note:-

- i) A game is said to be fair, if maximin value is equal to the minimax value.

i.e. $\text{maximin} = \text{minimax} = 0$ to zero.

i.e. $\text{maximin} = \text{minimax} = 0$.

i.e. $\bar{x} = \bar{y} = 0$

A game is said to be strictly determinable.

i.e. maximin value is equal to minimax value.

i.e. $\text{maximin} = \text{minimax} \neq 0$

i.e. $\bar{x} = \bar{y} \neq 0$

i.e. $\bar{x} = \bar{y} \neq 0$

DATE: / /

PAGE NO.:

Part A

Game with saddle point

Generally used for 3 by 3 matrix.

- Q.1) Solve the game whose pay-off matrix is given by

Player A

	Player B		
	1	3	1
	0	-4	-3
	1	5	-1

Soln:-

	B ₁	B ₂	B ₃	Row minimum
A ₁	1	3	1	1
A ₂	0	-4	-3	-4
A ₃	1	5	-1	-1
Column maximum	1	5	1	

Now, by applying principle of maximin & minimax principle,

$$\bar{x} = \text{max}(\text{minimum}) = \text{max}(1, -4, -1) = 1$$

$$\bar{y} = \text{min}(\text{maximum}) = \text{min}(1, 5, 1) = 1$$

Here, $\text{maximin value} = \text{minimax value} = 1$

∴ saddle point exist

If the value of game is given by saddle points & the value of game is 1.

The optimal strategy is the position of saddle point.

∴ The optimal strategies are A₁ & B₁.

Matrix is said to be strictly determinable

DATE: / /

PAGE NO.:

For the value of λ in the game with following matrix is determinable to strictly or not:

$$\text{Player A} \begin{bmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{bmatrix}$$

Ignoring the value of ' λ ' for the above matrix

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{Row minimum} \\ \hline A_1 & \lambda & 6 & 2 & 2 \\ A_2 & -1 & \lambda & -7 & -7 \\ A_3 & -2 & 4 & \lambda & -2 \end{array}$$

Column maximum

Apply maximum & minimum principle

$$\bar{x} = \max \min = \max(\text{minimum}) = \max(2, -7, -2) = 2$$

$$\bar{y} = \min \max = \min(\text{maximum}) = \min(-1, 6, 2) = -1$$

$$\text{Here, } \bar{x} = \bar{y} \neq 0$$

The game is strictly determinable. Therefore the ' λ ' value is

$$-1 \leq \lambda \leq 2$$

DATE: / /

PAGE NO.:

Q.3)

Determine which of the following zero person zero sum game are strictly determinable for each player in case of strictly determinable games.

$$(i) \text{ player A} \begin{bmatrix} 1 & 1 \\ -5 & 2 \\ -7 & 4 \end{bmatrix} \quad (ii) \text{ player A} \begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix}$$

Soln:-

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{Row minimum} \\ \hline A_1 & 1 & 1 & 1 \\ A_2 & -5 & 2 & -5 \\ A_3 & -7 & 4 & -7 \end{array}$$

Column maximum

Apply principle & maximum & minimum principle

$$\bar{x} = \max(\text{minimum}) = \max(-5, -7) = -5$$

$$\bar{y} = \min(\text{maximum}) = \min(-5, 4) = -5$$

$$\text{Here, } \bar{x} = \bar{y} \neq 0$$

The game is strictly determinable & there exist a saddle point

The value of game is -5. Then the optimal strategy is at A₂.

DATE: / /

PAGE NO.:

(ii)

	B ₁	B ₂	Row minimum
A ₁	1	1	1
A ₂	4	-3	-3
Column maximum	4	1	

Apply principle of maximin & minimax,

$$\bar{x} = \max(\text{minimum}) = \max(1, -3) = 1$$

$$\bar{y} = \min(\text{maximum}) = \min(4, 1) = 1$$

$$\text{Here, } \bar{x} = \bar{y} \neq 0$$

∴ The game is strictly determinable & there exist a saddle point.

∴ The value of game is 1
& The optimal strategy is A₁B₂

A4 Long Book

DATE: / /

PAGE NO.:

Q.4

Solve the game whose pay-off matrix is

Player A	-2	0	0	5	3
	3	2	1	2	2
	-4	-3	0	-2	6
	5	3	-4	2	-6

Soln

	B ₁	B ₂	B ₃	B ₄	B ₅	Row minimum
A ₁	-2	0	0	5	3	-2
A ₂	3	2	1	2	2	1
A ₃	-4	-3	0	-2	6	-4
A ₄	5	3	-4	2	-6	-6
Column maximum	5	3	1	5	6	

Apply principle of maximin & minimax

$$\bar{x} = \max(\text{minimum}) = \max(-2, 1, -4, -6) = 1$$

$$\bar{y} = \min(\text{maximum}) = \min(5, 3, 1, 5, 6) = 1$$

$$\text{Here, } \bar{x} = \bar{y} = 1 \neq 0$$

∴ The game is strictly determinable

∴ The value of game is 1

& optimal strategy is A₂B₃

A4 Long Book

Type-2

DATE: / / Name without saddle point

PAGE NO.:

Q.5) Solve the pay-off matrix. Also determine the optimal solution strategy and the value of game.

Player A

	Player B
	B ₁ B ₂
A ₁	5 1
A ₂	8 4

Soln.

Row difference

	B ₁ B ₂
A ₁	5 1
A ₂	8 4

Interchange Ratio

	B ₁ B ₂	Row difference	Ratio
A ₁	5 1	4	$\frac{1}{4}$
A ₂	8 4	1	$\frac{4}{1}$

Colm difference

Ratio

	B ₁ B ₂
A ₁	5 1
A ₂	8 4

The value of game is (V).

$$V = 5 \times \frac{1}{5} + 8 \times \frac{4}{5} = \frac{33}{5}$$

∴ The optimal strategy of A ($\frac{1}{5}, \frac{4}{5}$)

$$B = (\frac{3}{5}, \frac{2}{5})$$

Q.6

In a game of counting coin with two players suppose A wins 1 unit of value when there are 2 heads & wins nothing when there are 2 tails and loss half unit of value when there are 1 head and 1 tail. Determine the pay-off matrix and the strategy for each player and the value of game to A.

Soln.

Let two players are A & B.

Player B

	B ₁ B ₂	Row diff.	Ratio
A ₁	1 $-\frac{1}{2}$	$\frac{3}{2}$	4
A ₂	$-\frac{1}{2}$ 0	1	4

Colm diff.

Ratio

	B ₁ B ₂
A ₁	1 $-\frac{1}{2}$
A ₂	$-\frac{1}{2}$ 0

Interchange Ratio

The value of game V.

$$V = 1 \times \frac{1}{4} + (-\frac{1}{2}) \times \frac{3}{4}$$

$$V = -\frac{1}{8}$$

∴ The optimal strategy of A ($\frac{1}{4}, \frac{3}{4}$)

$$B = (\frac{3}{4}, \frac{1}{4})$$

A4 Long Book

Type: 9.

DATE: / /

PAGE NO.:

Dominance property of principleDominance:-

Some time it is observed that pure strategy either pair is always inferior to at least one of the remaining. The superior strategy are said to be inferior one in such case of dominance we reduced the size of pay-off matrix by deleting those strategy which dominated by other.

The general rules dominance are

- 1) If all the element of k^{th} row are less than or equal to the corresponding element of any other row say R^{th} row then k^{th} row is dominated by R^{th} row.
- 2) If all the element of k^{th} column are greater than or equal to the corresponding element of any other R^{th} column, then k^{th} column is dominated by R^{th} column.
- 3) The dominated Rows and columns may be deleted to reduce the size of pay-off matrix. As the optimal strategy remains unaffected.
- 4) In the pay-off matrix of player A, if each of the same sum of some of the entries of any two row are greater than are equal to corresponding entries of another

A4 Long Book

PAGE NO.:

DATE: / /

row then that row is dominated by the above two row, under such situation the first row of matrix can be deleted.

Q.7) Using principle of dominance solve the problem

$$B$$

$$A \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

Soln Since all element in the third row less than or equal to the corresponding element of the second row. $\therefore$ The third row is dominated by second row.

$\therefore$ Delete the dominated Row and then the pay-off matrix is

$$B$$

$$A \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \end{bmatrix}$$

The element of the 3rd column are greater than or equal to the corresponding element of the 1st column.

$\therefore$ The 3rd column is dominated by 1st column. $\therefore$ The dominated column can be reduced. The reduced matrix is

A4 Long Book