Unit 2 — Cam Dynamics

Prob. (5:09, 13 marks) :- The following data refers to the radial reciprocating cam. The cam is dwell rise & dwell return cam. The total rise of follower is 40 mm.

Cam angle rise = 120°

Cam angle return = 80°

Mass of follower = 1.5 kg

Process resistance = 40 N

Spring stiffness = 15 N/cm

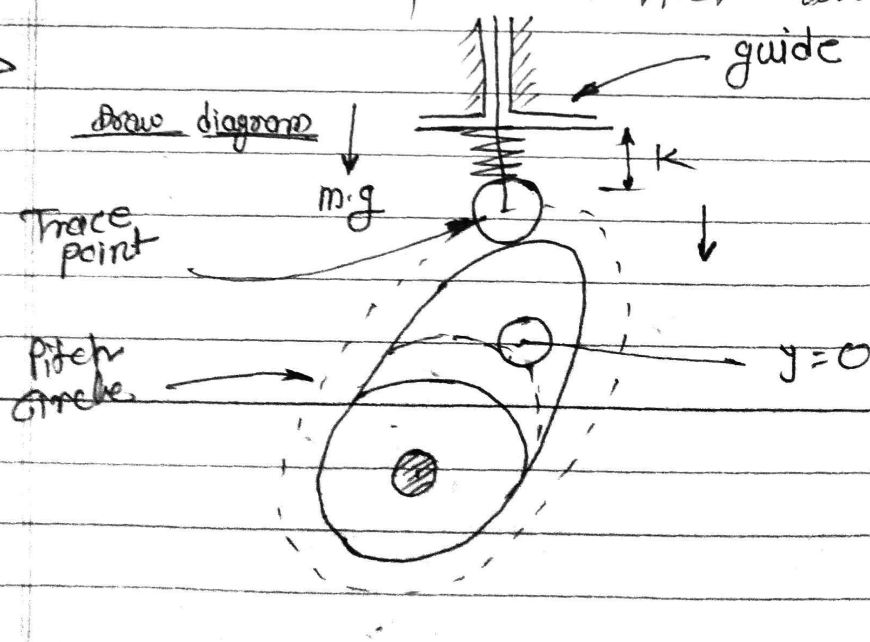
Follower rise with S.H.M. & return with parabolic motion.

Initial spring compression is 1 cm

Determine jumps speed of the cam.

Sketch the displacement & Accl diagram.

Draw diagram



$F_P = 40$ N

$S = 40$ mm , $\delta = 1$ cm

Given : $m = 1.5$ kg

wt. of follower = $W = m \times g$

$= 1.5 \times 9.81$

$W = 14.715$ kg

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$K = 15\,N/cm$
$= 15 \times 10^{2}$
$= \sout{1500\,N/m}$ ~~15 N/mm~~
$= 1500\,N/m$

~~Sp~~ Rise of follower = $\frac{stroke}{2}$

$y = \frac{40}{2}$

$y = 20\,mm$

Spring force = ~~15 × 20~~ $K \times Y$
$=$ ~~300~~ 1500×20
$=$ ~~30000 N~~ $30\,N$

out stroke = S.H.M.
return stroke = Parabolic. motion

$\theta_o = 120°c = 120 \times \frac{\pi}{180} = 2.094\,rad$

$\theta_r = 80°c = 80 \times \frac{\pi}{180} = 1.396\,rad$

A → Acceleration = $a = \ddot{y} = \frac{-\pi^2 \omega^2 S}{2\theta_o^2}$
(S.H.M.)

$\omega = 2\pi N$

for Jump of Phenomenon = $F_c = 0$

F_p

$$mg + m\ddot{y} + K(y+\delta) + 40 = 0$$

$$14.715 + 1.5 \times \ddot{y} + 15\times10^{2}\left(20\times10^{-3} + 1\times10^{-2}\right) + 40 = 0$$

$$\therefore Accl^{n} = a = \ddot{y} = -66.47 \ m/sec^{2}$$

Speed for outstroke (SHM)

$$\ddot{y} = \frac{-\pi^{2}\omega^{2}S}{2\theta_o^{2}}$$

$$-66.47 = \frac{-\pi^{2}\omega^{2}40\times10^{-3}}{2\times(2.094)^{2}}$$

$$\therefore \boxed{\omega = 38.35 \ rad/s}$$

$$\therefore \omega = \frac{2\pi N}{60}$$

$$\therefore \boxed{N = 366 \ rpm}$$

Jump speed for return stroke (parabolic)

$$\ddot{y} = \frac{-4\omega^{2}S}{\theta_r^{2}}$$

$$-66.47 = \frac{-4\times\omega^{2}\times40}{(1.396)^{2}}$$

$$\therefore \boxed{\omega = 28.45}$$

$$\therefore \omega = \frac{2\pi N}{60}$$

$$-0.89 = 2\pi N$$

$$\boxed{N = 270 \ rpm}$$

---

for SHM

$$y'' = \frac{-\pi^{2}\omega^{2}S}{2\theta_o^{2}}$$

for cycloidal

$$\ddot{y} = \frac{-2\pi\omega^{2}S}{\theta_o^{2}}$$

for Parabolic

$$y'' = \frac{-4\omega^{2}S}{\theta_r^{2}}$$

$$mg + my'' + K(y+\delta) + F_p = 0$$

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Prb. (7) following particulars for the Cam mechanism

Cam angle for outstroke = 120°

Cam angle for return = 90°

Cam angle for dwell at the end = 10°

Cam shaft speed = 650 rpm (Counter Clockwise)

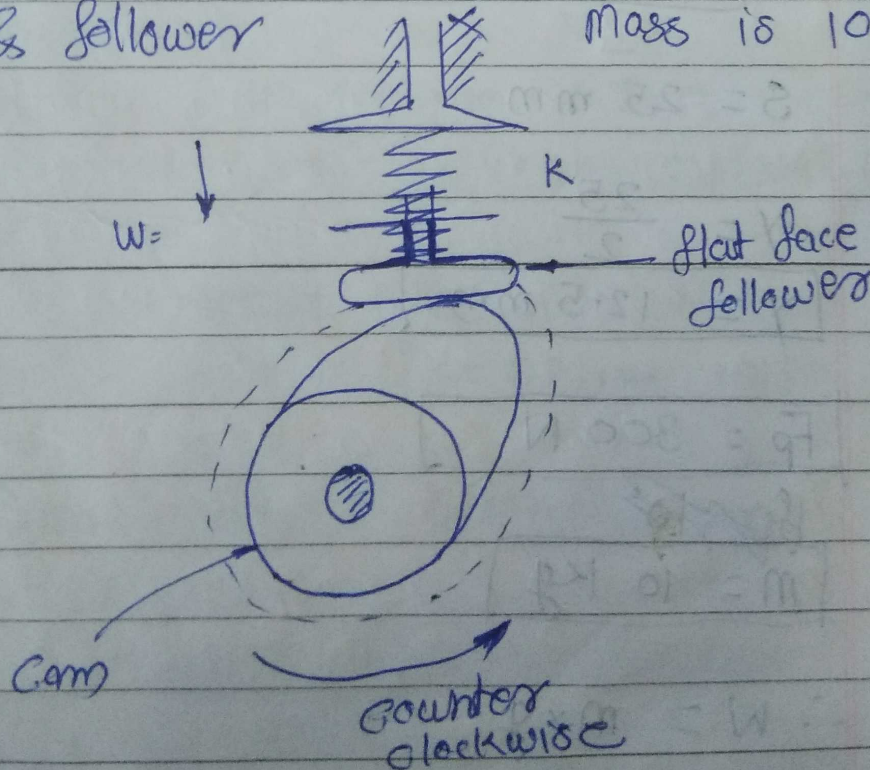
follower is the radial reciprocating flate face type having width 10 cm.

follower stroke 25 mm

follower motion is Cycloidal during outstroke & return stroke.

follower process resistance is 300 N

Find the neccessary spring stiffness To ensure the Contact betn cam & follower mass is 10 kg.



Spring stiffness = k = ?

Given:

Outstroke & return stroke are Cycloidal

$\theta_o = 130 \times \frac{\pi}{180} = 2.26$ rad

$\theta_r = 90 \times \frac{\pi}{180} = 1.57$ rad

N = 650 rpm.

$\therefore \omega = \frac{2\pi N}{60}$

$= \frac{2 \times \pi \times 650}{60}$

$\therefore$ $\boxed{\omega = 68.067 \text{ rad/s}}$

S = 25 mm

$Y = \frac{25}{2}$

$\boxed{Y = 12.5 \text{ mm}}$

$\boxed{F_P = 300 \text{ N}}$

~~~~

$\boxed{m = 10 \text{ Kg}}$

$\therefore W = m \times g$

$= 10 \times 9.81$

$\boxed{W_t = 98.1 \text{ N}}$
~~~~

~~$Mg + My'' + K(y+\delta) + \delta = 0$~~

$Mg + my'' + K(y+\delta) + Fp = 0$

During outstrok

$$y'' = \frac{-2\pi\omega^2 s}{\theta_o^2}$$

$$= \frac{-2\pi \times 68.067^2 \times 25\times10^{-3}}{(2.26)^2}$$

$$\boxed{y'' = -142.48 \ m^2/s}$$

$$Mg + K\times y + K\times\delta + My'' + Fp = 0$$

δ is not given $\therefore \delta = 0$

$$98.1 + K\times 12.5\times10^{-3} + K\times 0 + 10\times(-142.48) + 300 = 0$$

~~82136~~ $\boxed{K = 82136 \ N/m}$ Ans

$\boxed{K = 82.136 \ KN/m}$

During return stroke

$$y'' = \frac{-2\pi\omega^2 s}{\theta_r^2}$$

$$= \frac{-2\pi\times68.067^2\times25\times10^{-3}}{1.57^2}$$

$$y'' = -295.25$$

$$\therefore mg + K \times y + K \times \delta + m \times y'' + F_p = 0$$

$$\delta = 0$$

$$98.1 + K \times 12.5 \times 10^{-3} + 0 + 10 \times (-295.25) + 300 = 0$$

$$\boxed{K = 204352 \ N/m} \text{ Ans}$$

$$\boxed{K = 204.352 \ KN/m}$$

S: 08,09,010

Prob. (13 marks)

Cam & follower mechanism has a Cam machine so that it will raise the follower by 40 mm with parabolic motion with 120° cam rotation & dwell for 30° & return in parabolic motion for the remaining part of the rotation. Spring stiffness = 5000 N/

Mass of follower = 18 kg.

free load = 350 N

Find (i) cam speed when jump of begins

(ii) Sketch the displacement & acceleration diagram.

~~follower stroke~~

⇒

$120 + 30$ $\quad$ $\boxed{\theta_o = 120^\circ}$

$= 150$

$\therefore \theta_r = 360 - 150$

$\boxed{\theta_r = 210^\circ}$

$S = 40\ mm = 0.04\ m$

$\therefore y = \frac{40}{2} = 20\ mm = 0.02\ m$

$\therefore \boxed{y = 0.02\ m}$

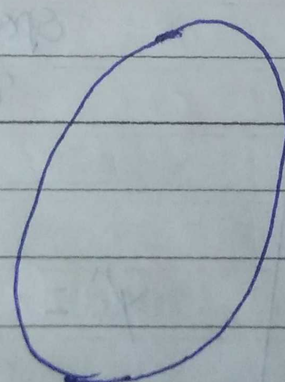
Parabolic

$\theta_o = 120^\circ = 120 \times \frac{\pi}{180} = 2.09$ rad.

Parabolic

$\theta_r = 210^\circ = 210 \times \frac{\pi}{180} = 3.66$ rad.

m



$M = 18\ kg$

$F_p = 350\ N$

$\therefore W = 18 \times 9.81$

$\boxed{W = 176.58\ N}$

Spring ~~stiffness~~ force = K × y

= 5000 × 0.02

Spring ~~stiffness~~ force = 100 N

Inertia force = $m y''$ =

for jump of speed $f_c = 0$

$m \cdot g + m y'' + K(y + \delta) + F_p = 0$

∴ $m y'' + W + K y + F_p = 0$

$18 \times y'' + 176.58 + 100 + 350 = 0$

∴ $y'' = -34.81 \ m^3/s$

for parabolic motion

Speed during outstroke (Parabolic)	Speed during return stroke (Parabolic)
$y'' = \frac{-4 \omega^2 S}{\theta_o^2}$	$y'' = \frac{-4 \omega^2 S}{\theta_r^2}$
$-34.81 = \frac{-4 \times \omega^2 \times 0.04}{(2.09)^2}$	$-34.81 = \frac{-4 \times \omega^2 \times 0.04}{(3.66)^2}$
$\omega = 30.82$	∴ $\omega = 53.98$
∴ $\omega = \frac{2\pi N}{60}$	$\omega = \frac{2\pi N}{60}$
$30.82 = \frac{2\pi N}{60}$ ∴ $N = 294.30$ rpm	∴ $N = 515.47$ rpm

Scale

X-axis 1 cm = 30°

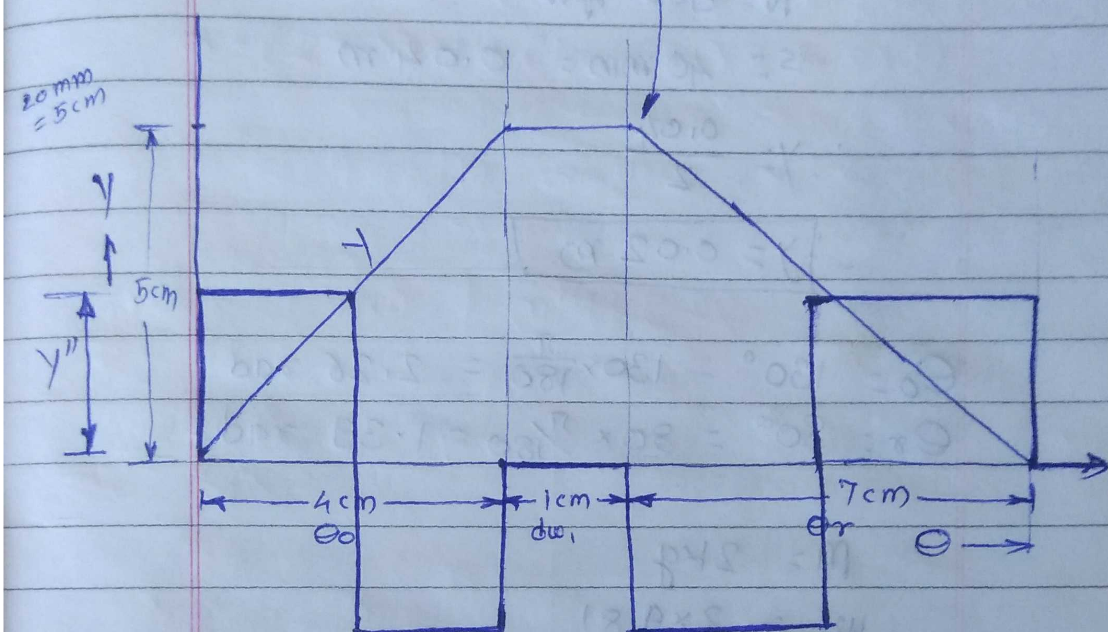
Displacement diagram

$\theta_o = 120° = 4$ cm

$\delta_{w_1} = 30° = 1$ cm

$\theta_r = 210 = 7$ cm

for uniform ~~acceleration~~ velocity
or
for parabolic



Acceleration diagram

y → displacement Curve

y'' → Acceleration Curve

② Cam Mechanism has a following particulars

Cam speed = 900 rpm.

follower stroke = 40 mm

Cam angle for outstroke = 130°

Cam angle for return stroke = 80°

follower motion is uniform acceleration & retardation during outstroke & S.H.M. during return stroke.

Mass of follower = 2 kg.

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Find the necessary spring stiffness to ensure the contact of follower with cam throughout the ~~follower~~ cycle.

⇒ Given;

$N = 900$ rpm

$s = 40$ mm $= 0.04$ m

$\therefore y = \frac{0.04}{2}$

$\therefore y = 0.02$ m

$\theta_o = 130° = 130 \times \frac{\pi}{180} = 2.26$ rad

$\theta_r = 80° = 80 \times \pi/180 = 1.39$ rad.

$M = 2$ kg

wt. $= 2 \times 9.81$

$W = 19.62$ N

Assume if not given $F_p = 0$

$\therefore My'' + W = 0$

$2 \times y'' + 19.62 = 0$

$y'' = -9.81$ m/s²

$\omega = \frac{2\pi N}{60}$

$= \frac{2\pi \times 900}{60}$

$\therefore \omega = 94.24$ rad/s

during outstroke
(uniform acceleration & retardation)

$$y'' = \frac{-4\omega^2 S}{\theta_0^2}$$

~~$-9.81 = \frac{-4\omega^2 \times 0.09}{(2.26)^2}$~~

~~$\therefore \omega = -17.69$~~

$$y'' = \frac{-4(94.24)^2 \times 0.04}{(2.26)^2}$$

$$\boxed{y'' = -278.21 \ m^3/s}$$

~~during return stroke~~ during return strok

for SHM

$$y'' = \frac{-\pi^2\omega^2 S}{2\theta_r^2 c}$$

$$= \frac{-\pi \times 94.24^2 \times 0.04}{2 \times (1.39)^2}$$

$$\boxed{y'' = -~~288.81~~ \ ~~m^3/s~~}$$
907 m/s2

for outstroke

$$F_c = Mg + My'' + K(\delta + y) ~~+ cF_p~~ = 0$$

$0 = 19.62 + 2\times(-278.21) + K(0 + y) + 0 = 0$

$19.62 + 2(-278.21) + K(0.02) ~~+0~~ = 0$

$\therefore K = 26840 \ N/m$

∴ $\boxed{K = 26.84 \text{ KN/m}}$ during outstroke

For Return Stroke

$$Mg + My'' + K(y + \delta) + F_p = 0$$

(δ → 0, F_p → 0)

$$\therefore Mg + My'' + K(y) = 0$$

$$19.62 + 2(-288.81) + K(0.02) = 0$$

66/133

2nd 47.62 %   42.63 %

∴ K = ~~27900~~ N/m

3rd 55/129

∴ $\boxed{K = 27.9 \text{ KN/m}}$ during return stroke

4th 28/116 = 24.13 %

~~k = 8.9 KN/m~~

S.09 Following data refers to the radial reciprocating follower & cam system.

The Cam is dwell - rise - ~~dwell~~ dwell - return cam

Total rise of follower = 40 mm

Cam angle of the rise = 120°

Cam angle of the return = 80°

Mass of follower = 1.5 kg.

Process resistance = 40 N

Spring stiffness = 15 N/cm

Follower rise with S.H.M. & return with Parabolic motion.

Initial Compression = 1 cm
Determine Jump off Speed for Horizontal Cam & follower System.

⇒ Given :-
It is the Horizontal Cam & follower.
So weight of follower = W = mg = 0

$S = 40\ mm = 0.04\ m$

$\therefore y = \frac{S}{2} = \frac{0.04}{2} = 0.02\ m$

$\theta_o = 120° = 120 \times \frac{\pi}{180} = 2.09\ rad$

$\theta_r = 80° = 80 \times \pi/180 = 1.39\ rad.$

$m = 1.5\ kg.$

$K = 15\ N/cm$
$= 15 \times 10^2$
$\boxed{K = 1500\ N/m}$

$F_p = 40\ N$

outstroke ⟶ S.H.M.
Return stroke → Parabolic motion

$\delta = 1\ cm = 0.01\ m$

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$\therefore \cancel{w} + my'' + k(\delta + y) + F_p = 0$ (w → 0)

$0 + 1.5\,y'' + 1500\,(0.01 + 0.02) + 40 = 0$

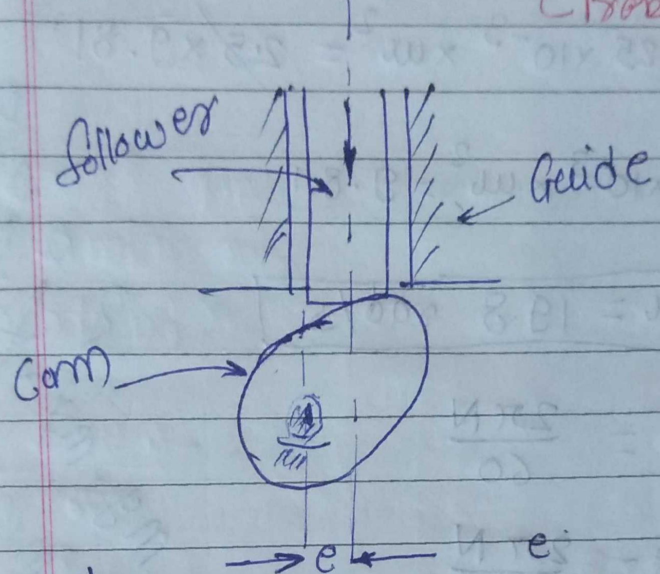
$\boxed{y'' = -56.66 \ m^3/s}$

Jump Speed for outstroke (S.H.M.)	Jump Speed for return stroke (Parabolic)
$y'' = \dfrac{-\pi^2 \omega^2 S}{2\theta_o^2}$	$y'' = \dfrac{-4\omega^2 S}{\theta_r^2}$
$-56.66 = \dfrac{-\pi^2 \times \omega^2 \times 0.04}{2 \times (2.09)^2}$	$-56.66 = \dfrac{-4 \times \omega^2 \times 0.04}{(1.39)^2}$
$\boxed{\omega = ~~62.76~~ \ 35.40 \ rad/s}$	$\boxed{\omega = 26.15 \ rad/s}$
$\therefore \omega = \dfrac{2\pi N}{60}$	$\omega = \dfrac{2\pi N}{60}$
35.40 ~~62.76~~ $= \dfrac{2\pi N}{60}$	$26.15 = \dfrac{2\pi N}{60}$
$\therefore$ ~~N = 599~~ $\boxed{N = 338.04 \ rpm}$	$\boxed{N = 249.78 \ rpm}$

$mg + my'' + K(y+\delta) + F_p = 0$

Prob. ① Cam & follower mechanism is shown in fig. is constrained to move in a vertical direction. The mass of follower is 2.5 kg.

(Problems on Eccentric Cam)



The cam is having eccentricity is 25 mm. Determine the jump speed.

⟹ Given:

$m = 2.5 \text{ kg}$

$W = 2.5 \times 9.81$

$\boxed{W = 24.52 \text{ N}}$

$e = 25 \times 10^{-3} \text{ m}$

To avoid jumping (contact force) $F_c = 0$

Inertia force + wt. of follower + Spring force (→0) + follower resistance + force due to initial compr. = 0

$\ddot{y} = -e\omega^2$

$my'' + w = 0$

$m \times (-$ $\quad$ $[m \times e \times \omega^2 = mg]$

~~$2.5 \times y'' + 2.5 \times 9.81$~~

$2.5 \times (-e\omega^2) + 2.5 \times 9.81 = 0$

$2.5 \times 25 \times 10^{-3} \times \omega^2 = 2.5 \times 9.81$

$\therefore 25 \times 10^{-3} \times \omega^2 = 9.81$

$\boxed{\omega = 19.8 \text{ rad/s}}$

$\therefore \omega = \frac{2\pi N}{60}$

$19.8 = \frac{2\pi N}{60}$

$\boxed{\therefore N = 189 \text{ rpm}}$ Ans

$mg = m \cdot e \times \omega^2$

Pb ② In a Cam & follower mechanism, follower moves to the right through the distance of 50 mm with parabolic motion by 100° cam rotation in a counter clockwise direction. The dwell is 50° & return in a parabolic motion to the starting position in a remaining cam angle. The spring rate is 5 KN/m & the mechanism is assemble with the 40 N free load (Fp).

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The mass of follower is 18 kg.
Calculate (i) Jump speed
(ii) what should be the speed to avoid jumping.
Assume mechanism is Horizontal.

⟹ Given :-

$S = 50\ mm = 0.05\ m$

$y = \frac{S}{2} = \frac{0.05}{2}$

$\therefore \boxed{y = 0.025\ m}$

$\theta_o = 100°$

$\delta_{w1} = 50°$

$\therefore \theta_r =$ ~~100+50~~ $360 - 150$

$\theta_r = 210°$

$\therefore \theta_o = 100 \times \frac{\pi}{180} = 1.74\ rad$

$\theta_r = 210 \times \pi/180 = 3.66\ rad.$

outstroke & return stroke are in Parabolic motion.

$K = 5\ kN/m$

$\boxed{K = 5000\ N/m}$

$F_P = 40\ N$

$M = 18\ kg.$

$W = 0$ --- Horizontal Mechanism

As mechanism is Horizontal
wt. of follower $W = 0$

$\therefore W + my'' + K(\delta + y) + F_p = 0$

$0 + 18 \times y'' + 5000(0 + 0.025) + 40 = 0$

$\therefore \boxed{y'' = -9.16 \ m/s^2}$

Jump Speed for Outstroke (Parabolic)

$y'' = \dfrac{-4\omega^2 S}{\theta_o^2}$

$-9.16 = \dfrac{-4\omega^2 \times 0.05}{1.74^2}$

$\boxed{\omega = 11.77 \text{ rad/s}}$

$\therefore \omega = \dfrac{2\pi N}{60}$

$11.77 = \dfrac{2\pi N}{60}$

$\therefore \boxed{N = 112.39 \text{ rpm}}$

Jump Speed for return Stroke (Parabolic)

$y'' = \dfrac{-4\omega^2 S}{\theta_r^2}$

$-9.16 = \dfrac{-4\omega^2 \times 0.05}{3.66^2}$

$\boxed{\omega = 24.76 \text{ rad/s}}$

$\therefore \omega = \dfrac{2\pi N}{60}$

$24.76 = \dfrac{2\pi N}{60}$

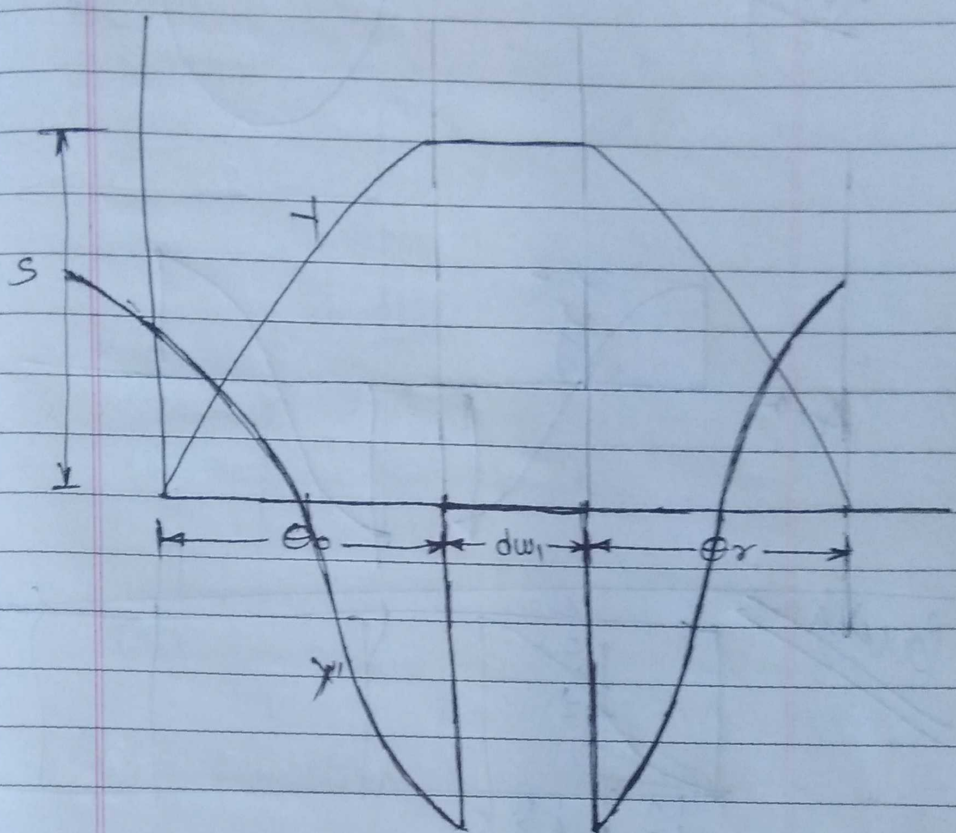
$\therefore \boxed{N = 236.52 \text{ rpm}}$

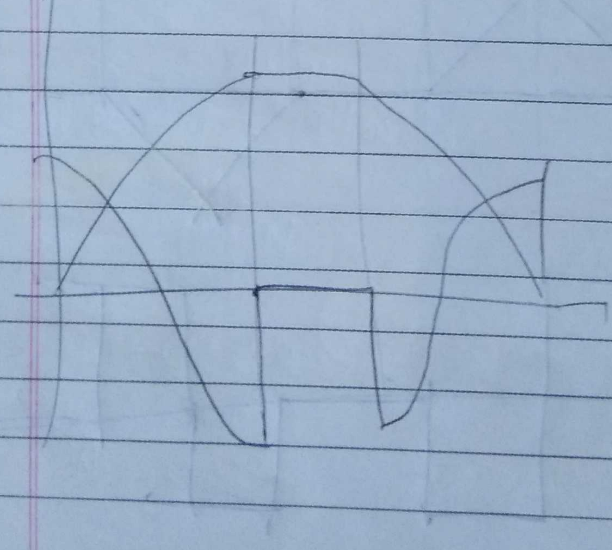
$\therefore$ Ans (i)

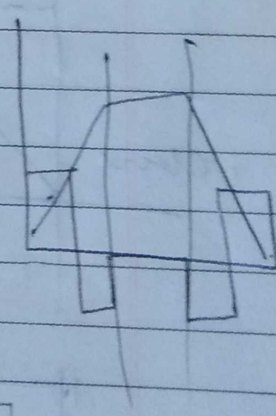
Ans (i) To avoid jumping, the speed of the cam should be less than 112.39 rpm.

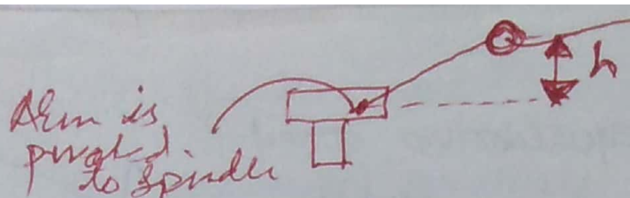
xox

Displacement & Acceleration diagrams for S.H.M.

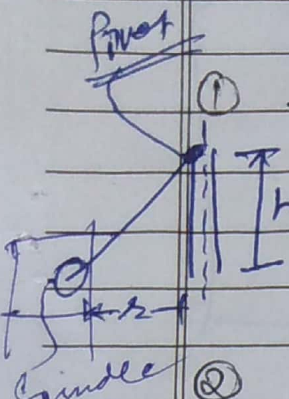








* Terms used in Governors



① **Height of the governor** :→ It is the vertical distance from the centre of the ball to a point where the arm is pivoted to the spindle of the governor.

② **Equilibrium speed** :→ It is the speed at which the governor balls, arms are in complete equilibrium, and sleeve does not tend to move upwards or downwards.

③ **Mean equilibrium speed** :→ It is the speed at the mean position of the balls or the sleeve.

④ **Sleeve lift** :- It is the vertical distance which the sleeve travels due to change in equilibrium speed.

⑤ **Sensitiveness of Governor** :- If two governors A & B running at the same speed, when this speed increases and decreases by a certain amount, the lift of the sleeve of a governor A is greater than B. It is said that governor A is more sensitive than governor B.

It is defined as, ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed. i.e $\frac{N_2 - N_1}{N}$

Let N_1 = min. equilibrium speed,

N_2 = maximum equilibrium speed.

N = mean equi. speed = $\frac{N_1+N_2}{2}$,

∴ Sensitivityness of the Governor $= \frac{N_2-N_1}{N}$

$= \frac{2(N_2-N_1)}{(N_1+N_2)}$

⑥ Stability of Governors: A Governor is said to be a stable when for every speed there is definite configuration i.e. there should be only one radius of rotation of the governor balls at which the governor is in equilibrium.

For a stable governor, if the equilibrium speed increases, the radius of governor ball must also increase.

A governor said to be unstable, if the radius of rotation decreases as the speed increases.

⑦ Isochronous Governors: → A governor is said to be isochronous when the equilibrium speed is constant (i.e., range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction.

The ~~Isochronous~~ Isochronism, is the stage of infinite sensitivity

Scanned by CamScanner

* Types of Governors :

(i) Centrifugal governors : This is common type of governors. Its action depends on the change of speed. It has a pair of masses, known as governor balls, which rotates at a distance from the axis of rotation.

The action depends upon the centrifugal effect produced by the masses. With the increase in speed, the balls tends to rotate at a greater radius from the axis and through suitable linkages the throttle valve is made to close due to lift of ↑sleeve to the required extents. When the speed decreases, the balls rotate at a smaller radius and valve opens according to the requirements.

Speed ↑ valve closed. Throttle
Speed ↓ valve open

(ii) Inertia governor : (inertia force)

In this type, the positions of balls are affected by a forces set up by an angular acceleration or deceleration of the given spindle in addition to centrifugal forces on the balls. Using suitable linkages and springs, the change in position of the balls is made to open or close

Inertia force due to acceleration or deceleration of spindle

* Centrifugal force on balls

* Difference between Flywheel and Governor

Flywheel	Governor
i] The function of flywheel is to control the speed variations caused by the fluctuations of turning moment during a cycle.	i] The function of governor is to keep the variation in mean speed of the engine within prescribed limits due to the fluctuation in load over a period of time.
ii] A flywheel stores a energy and gives up the energy whenever required during a cycle.	ii] A governor regulates a speed by regulating the quantity of charge of the prime mover.
iii] It regulates the speed during one cycle only.	iii] It regulates the speed over a period of time.
iv] A flywheel has no control over the quantity of charge.	iv] A governor takes care of quantity of working fluid.
v] A flywheel is not an essential element for every prime mover. only utilize when there is a fluctuation of energy.	v] It is an essential element of a prime mover. It adjust supply of charge according to the load on the prime mover.

* Introduction Regarding Governors :

* The flywheel controls the cyclic fluctuations in speed due to fluctuation of energy.

* The I.C. engine, come across the change in speed due to change in load which can not be controlled by the flywheel.

* When the load on the engine decreases the speed of the engine increases. Similarly when the load on the engine increases, the speed of the engine decreases.

* The variation in speed occurring due to variation in load is controlled by ~~a~~ making variation in fuel supply.

  This function ~~is of~~ is achieved by a mechanical device called as Governor.

* Thus, the function of governor is to automatically maintain the speed of an engine within the pres-cribed limit for varying load conditions.

* Types of Governors *

The governors are broadly classified as follows.

- Governors
  - Centrifugal governor
    - Pendulum type simple (Watt governor)
    - Loaded type governors
      - Dead wt. type
        - Porter governor
        - Proell governor
      - Spring controlled governor (Hartnell govern.)
        - Hartnell governor
        - Wilson Hartnell governor
        - Hartung governors
        - Spring controlled gravity governor
        - Pickering governor
  - Inertia and flywheel governor

→ Sensitivity, hunting, stability 6m

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~~Def E₁ = Energy req to push a hole~~

⑧ **Hunting:** A governor is said to be hunt, if speed of the engine fluctuates continuously above and below the mean speed.

This is cause due to the too sensitive governor, which changes the fuel supply by a large amount when a small change in the speed of rotation takes place.

For ex. when load on engine increases, the engine speed decreases and if the governor is very sensitive, the sleeve of governor immediately falls to its lowest position, resulting in opening of fuel valve to engine in excess, so speed rapidly increases again and governor ~~speed~~ sleeve rises to a highest position. Due to this the control valve cut off the fuel supply to the engine and engine speed decreases.

This cycle is repeated continuously.

This type of governor may admit either max. or min. amount of fuel. This effect will cause wide fluctuation in engine speed, or we can say, the engine will hunting

(Force)

⑨ The Effort of a governor: It is the mean force exerted at the sleeve for a given % change of

3) Rotor Governor:

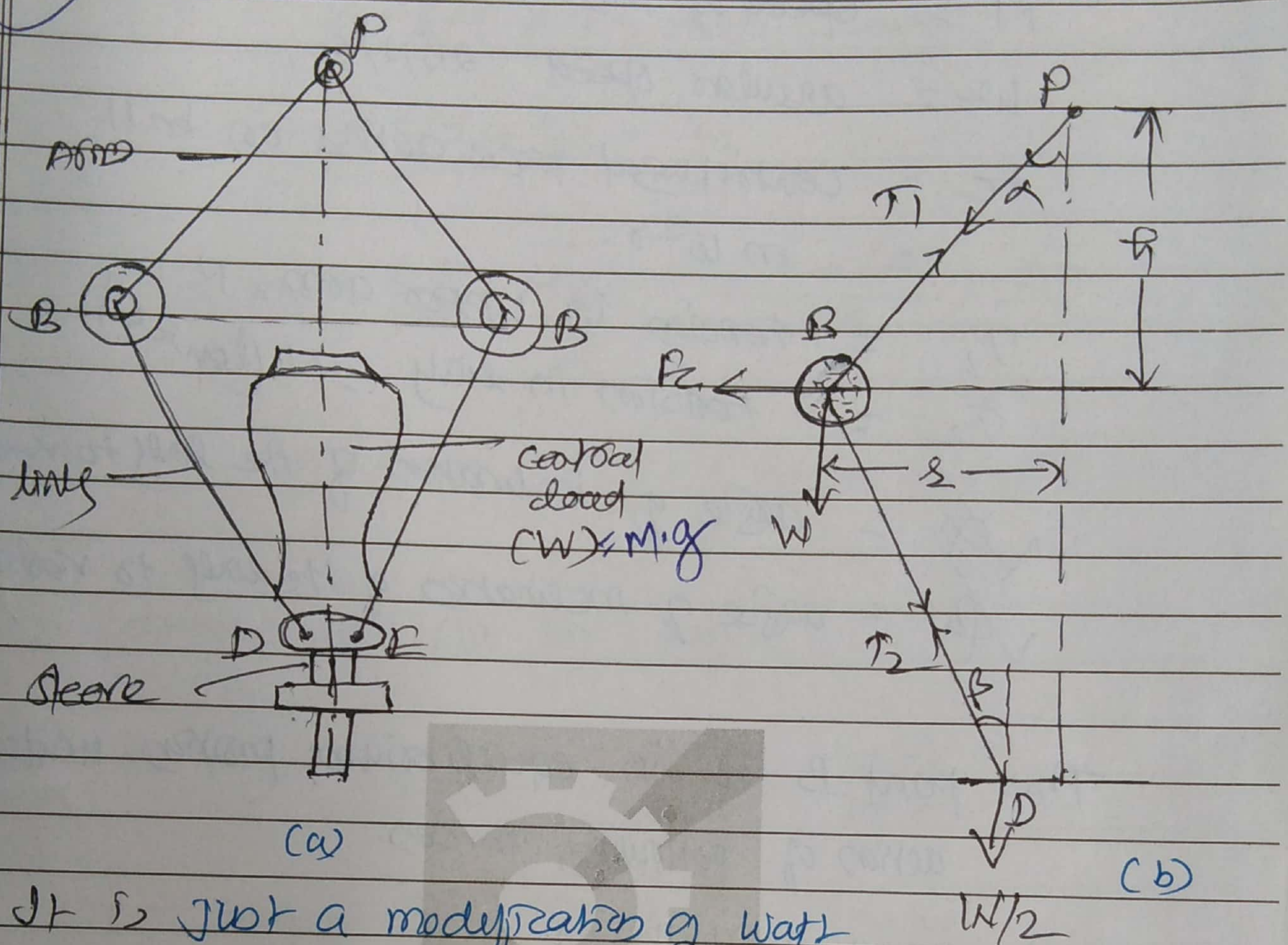
When the governor is running steadily there is no force at the sleeve.

But when the speed changes, there is a resistance at the sleeve which opposes its motion.

Power of Governor: It is the work done at the sleeve for a given % change of speed. It is the product of the mean value of the effort and distance through which the sleeve moves.

i.e., power = mean effort × lift of sleeve. →

2) Porter Governor:



It is just a modification of watt governor with a central load attached to the sleeve as shown in figure.

The load moves up and down at the central spindle.

This additional downward force increases the speed of revolution required to provide the balls to rise to any predetermined level.

Let

m = mass of each the balls, kg

w = wt of each ball, N

M = mass of central load, kg

W = wt of central load, N

r = radius of rotation, m

h = ht. of governor, m

N = speed of ball, rpm

ω = angular speed $2\pi N/60$,

F_C = centrifugal force acting on ball

= $m\omega^2 r$.

T_1 = tension in upper arm, N

T_2 = tension in link, N (lower)

α = angle of inclination of the link to vertical

β = angle of inclination of the link to vertical.

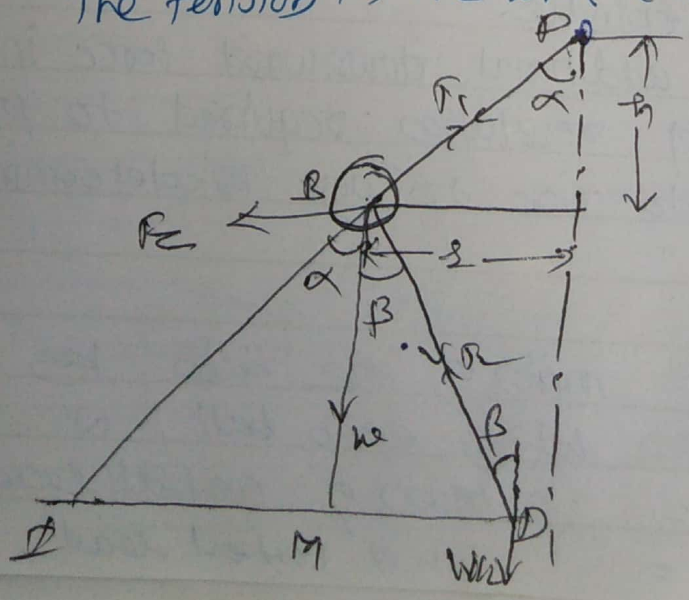
The point B is in equilibrium position under the action of following forces.

(i) Wt of the ball, w

(ii) The centrifugal force (F_C)

(iii) The tension in the arm (T_1)

(iv) The tension in the link (T_2).



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DATE (i)

taking moment about point I.

$F_C \times BM = W \times IM + W/2 \times ID$

$\therefore F_C = mg \times \frac{IM}{BM} + W/2 \times \frac{ID}{BM}$

$= mg \times \frac{IM}{BM} + \frac{Mg}{2} \times \left(\frac{IM + DM}{BM}\right)$

$F_C = mg \times \tan\alpha + \frac{M.g}{2} \times (\tan\alpha + \tan\beta)$ — (ii)

divide eqn (ii) by $\tan\alpha$.

$\frac{F_C}{\tan\alpha} = mg + \frac{Mg}{2}\left(\frac{\tan\alpha}{\tan\alpha} + \frac{\tan\beta}{\tan\alpha}\right)$

$= m.g + \frac{M.g}{2}(1 + q)$

Assume
$q = \frac{\tan\beta}{\tan\alpha}$
$\tan\alpha = \frac{r}{h}$
$F_C = m.\omega^2 r$

$\therefore m.\omega^2 r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$

$m.\omega^2.h = mg + \frac{M.g}{2}(1+q)$

or $h = \frac{mg + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2}$

take g common

$h = \left[\frac{m + \frac{M}{2}(1+q)}{m}\right] \times g/\omega^2$

or $$h = \left[\frac{m + M/2 (1+q)}{m} \right] \times \frac{g}{\omega^2}$$

* When $\tan\alpha = \tan\beta$ or $q = 1$ then

$$h = \frac{m+M}{m} \times \frac{g}{\omega^2}$$

$\omega = \frac{2\pi N}{60}$ put

* $$N^2 = \frac{m+M}{m} \times \frac{895}{h}$$

Corollary: If frictional force = F acting on the sleeve moves up and down the spindle, the force acts in a direction opposite to that of the sleeve.
(↓ motion of a.)

$q = $ true

$$\therefore N^2 = \left[\frac{mg + \left(\frac{Mg \pm F}{2} \right)(1+q)}{mg} \right] \times \frac{895}{h}$$

When $q = 1$

$$N^2 = \left[\frac{mg + (Mg \pm F)}{mg} \right] \times \frac{895}{h}$$

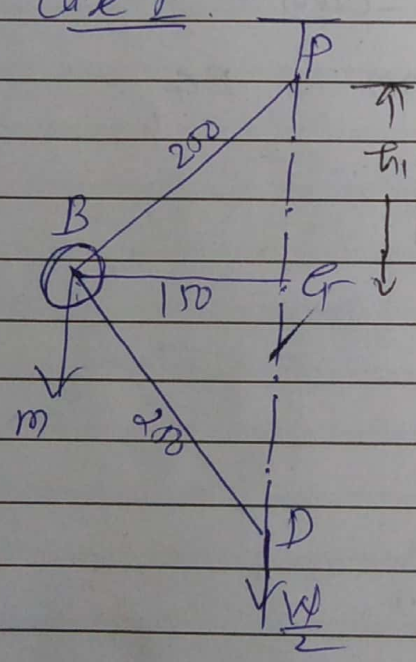
When
+ When sleeve moves upward or governor speed increases
− When sleeve moves downward or governor speed decreases.

Example (Porter):

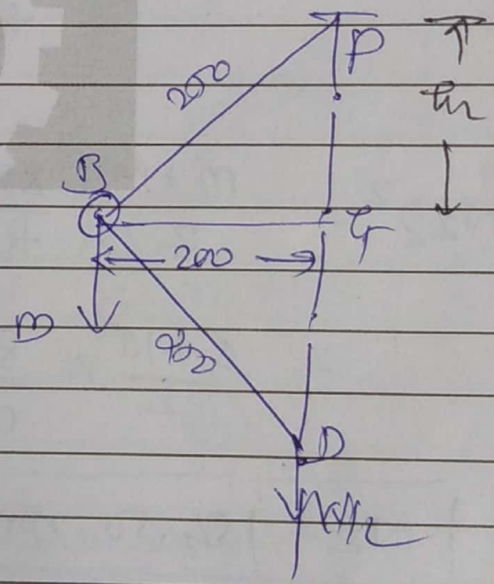
1) A porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and mass of the central load on the sleeve is 15 kg. The radius of rotation of ball is 150 mm, when the governor begin to lift and 200 mm when the governor at maximum speed. Find minimum and maximum speed, Range of speed.

Soln: arm length = 0.25 m.
mass of ball (m) = 5 kg
mass of load (M) = 15 kg.
radius of rotation case I r_1 = 150 mm
case II r_2 = 200 mm

Case I:



Case II



To find: speed range N_1 and N_2 for each case.

Formula: here $\alpha = \beta$, since $\tan\alpha = \tan\beta$

$$N^2 = \frac{m+M}{m} \times \frac{895}{h}$$

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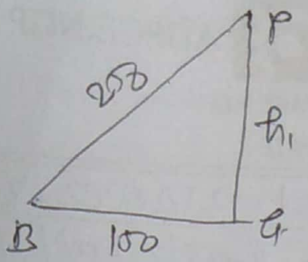


Let N_1 = minimum speed

$h_1 = \sqrt{(250)^2 - (150)^2}$

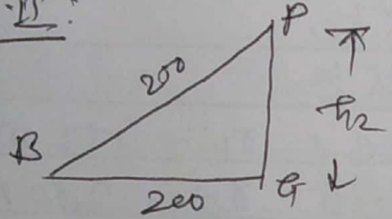
$= 200$ mm. $\boxed{PG = h_1 = 0.2\,m}$

$\therefore N_1^2 = \frac{m+M}{m} \times \frac{895}{h_1}$

$= \frac{5+15}{5} \times \frac{895}{0.2}$

$\boxed{N_1^2 = 17900 \text{ rpm}} \Rightarrow \boxed{N_1 = 133.80 \text{ rpm}}$ Ans

Case II:



minimum speed when $r_2 = 0.2$ m

$h_2 = \sqrt{(250)^2 - (200)^2}$

$h_2 = 150$ mm $= PG$

$(N_2)^2 = \frac{m+M}{2} \times \frac{895}{h_2}$

$= \frac{5+15}{2} \times \frac{895}{0.15}$

$\boxed{N_2 = 154.50 \text{ rpm}}$ Ans

* Range of speed $= N_2 - N_1$

$= 154.50 - 133.80$

$= 20.7$ rpm Ans

Example 2 : (Porter)

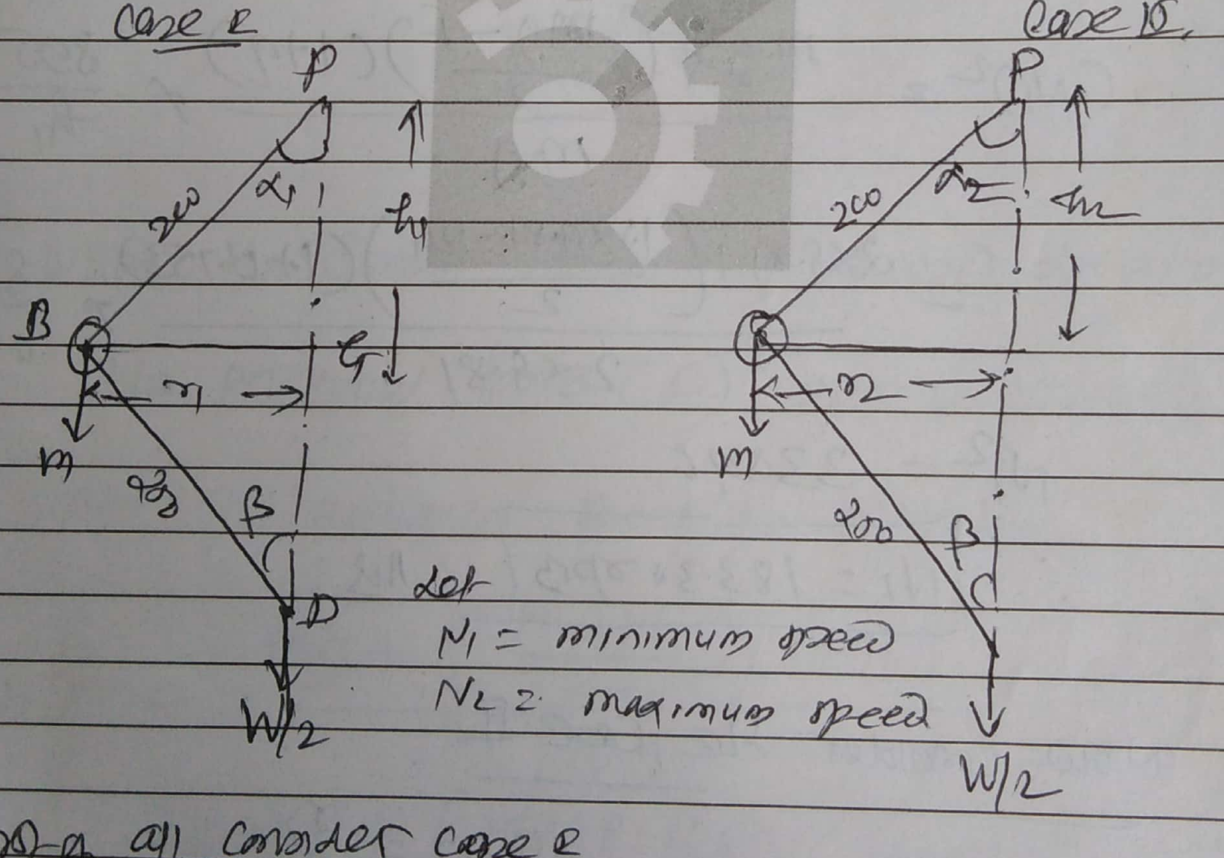
In porter governor, the upper and lower arms are 200 mm and 250 mm resp. and pivoted on the axis of rotation. The mass of the central load is 15 kg. and mass of each ball is 2 kg. and friction of the sleeve together with the resistance of the operating gear is equal to a load of 24 N at the sleeve.

If the limiting inclinations of the upper arms to the vertical are 30° and 40°.

Find, taking friction into account, range of speed of the governor.

Soln: Given data: $m = 2$ kg $M = 15$ kg

$F = 24N$ $\alpha_1 = 30°$ $\alpha_2 = 40°$

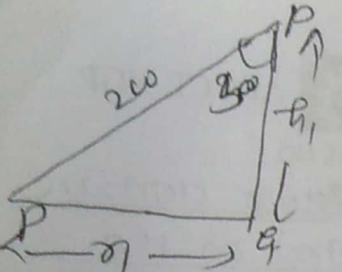


Let
N_1 = minimum speed
N_2 = maximum speed

First of all consider case I

$PB = 0.2m$

$\alpha_1 = 30°$ Calculate the value of r_1 and h_1,



$h_1 = 0.1732\,m$ (ht of governor)

$r_1 = 0.1\,m$ (min radius of rotation)

$\tan\alpha_1 = \tan 30° = 0.5774$

$\tan\beta_1 = \frac{0.1}{0.23} = 0.4348$

$\therefore q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.4348}{0.5774}$

$\boxed{q_1 = 0.753}$

When sleeve moves downwards, the frictional force (F) act upwards, min speed is given by:

$$(N_1)^2 = \frac{m \cdot g + \left(\frac{Mg - F}{2}\right)(1+q)}{m \cdot g} \times \frac{895}{h_1}$$

$$= \frac{2\times9.81 + \left(\frac{15\times9.81 - 24}{2}\right)(1+0.753)}{2\times9.81} \times \frac{895}{0.1732}$$

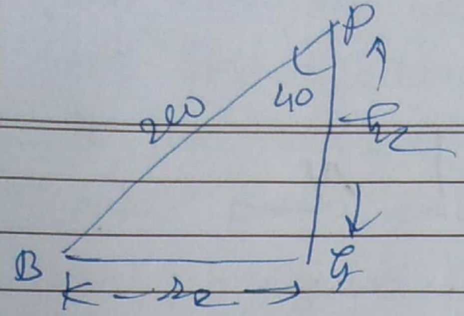
$N_1^2 = 33596$

$\therefore \boxed{N_1 = 183.30\ rpm}$ Ans

Now consider the case II:

$\sin 40 = \frac{r_2}{200}$

$\boxed{r_2 = 127\ mm}$



$r_2 = 200 \times \sin 40 = 127\ mm$

$\tan 40 = \frac{127}{h_2}$

$\therefore \boxed{h_2 = 152\ mm}$

$\tan\beta_2 = \frac{BG}{DG} = 0.59$

$\tan\alpha_2 = 0.839$

$q_2 = \frac{\tan\beta_2}{\tan\alpha_2}$

$= \frac{0.59}{0.839}$

$\boxed{q_2 = .703}$

When the sleeve moves in upward direction, the frictional force (F) act downward and and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + \left(\frac{M.g + F}{2}\right)(1+q_2)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 + 24}{2}\right)(1+0.703)}{2 \times 9.81} \frac{895}{\times 0.152}$$

$N_2^2 = 40236$

$\therefore \boxed{N_2 = 221 \text{ rpm}}$ Ans

$\therefore$ Hence the range of speed

$= N_2 - N_1$

$= 221 - 183$

$\boxed{\text{Range of speed} = 38 \text{ rpm}}$ Ans

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Ex: (porter governor):

(8) In a porter governor, ~~each of the four~~ arms is 400 mm long. The upper arms are pivoted on the axis of the sleeve whereas the lower arms are attached to the sleeve at a distance of 45 mm from the axis. Each ball has a mass of 8 kgs and the load on the sleeve is 60 kgs. What will be the equilibrium speeds for the two extreme radii of 250 mm and 300 mm of rotation of the governor balls?

Soln ⇒ Given data: mass of each ball (m) = 8 kgs

mass of central load (M) = 60 kgs.

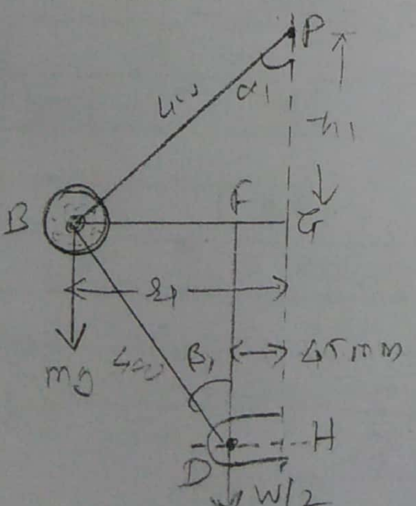
BP = BD = 400 mm

DH = 45 mm

i) r_1 = 250 mm,

ii) r_2 = 300 mm,

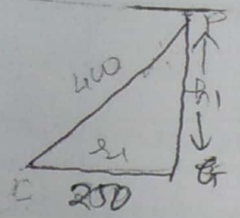
i) minimum position i.e., r_1 = 250 mm



Let N_1 = minimum speed when r_1 = BG = 250 mm.

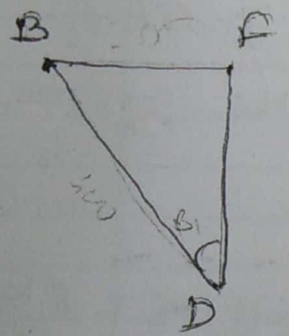
and N_2 = maximum speed when r_2 = BG = 300 mm

First find the ht. of the governor



$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2}$$
$$= \sqrt{(400)^2 - (250)^2}$$
$$= 0.312 \text{ m}$$

$$BF = BG - FG$$
$$= 250 - 45$$
$$= 205 \text{ mm}$$



From figure $DF = \sqrt{BD^2 - BF^2}$
$$= \sqrt{(400)^2 - (205)^2}$$
$$= 343.47 \text{ mm}$$

$$\tan\alpha_1 = \frac{BG}{PG} = \frac{250}{312} = 0.801 \Rightarrow \boxed{\tan\alpha_1 = 0.801}$$

~~$\alpha_1 = \tan^{-1}(0.801) \Rightarrow \alpha_1 =$~~

$$\tan\beta_1 = \frac{205}{343.50} \Rightarrow \boxed{\tan\beta_1 = 0.596}$$

$$\therefore q_1 = \frac{\tan\beta_1}{\tan\alpha_1} = \frac{0.596}{0.801} = 0.744.$$

$$\boxed{q_1 = 0.744}$$

We know the formula

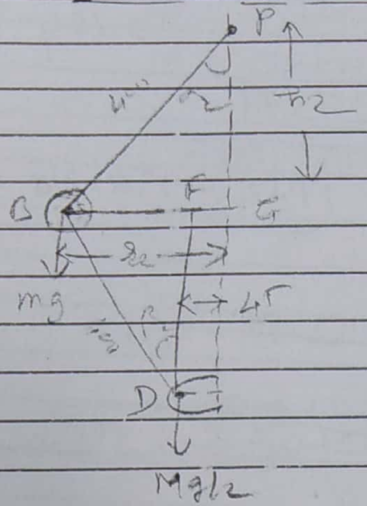
$$(N_1)^2 = \frac{m + M/2(1+q_1)}{m} \times \frac{895}{h_1}$$

$$= \frac{8 + 60/2(1+0.744)}{8} \times \frac{895}{0.312}$$

$$= ~~147~~ \quad 21629$$

$\therefore$ $N_1 = 147$ rpm

(ii) When $r_2 = 300$ mm.



from figure,

$h_2 = PG = \sqrt{(BP)^2 - (BG)^2}$

$= \sqrt{(400)^2 - (300)^2}$

$= 264.57$ mm

$BF = BG - FG$

$= 300 - 45$

$= 255$ mm

$DF = \sqrt{(BD)^2 - (BF)^2}$

$= \sqrt{(400)^2 - (255)^2}$

$= 308.20$ mm

$\tan\alpha_2 = \frac{BG}{PG}$

$= \frac{300}{264.57} \Rightarrow \tan\alpha_2 = 1.133$

$\tan\beta_2 =$ ~~255~~ $\frac{BF}{DF} = \frac{255}{308.20}$ (~~200~~) $=$ ~~0.665~~ $= 0.827$

$\therefore q_2 = \frac{\tan\beta_2}{\tan\alpha_2} = \frac{0.827}{1.133}$ (~~0.665~~) $= 0.587$

~~$q_2 = 0.587$~~

$q_2 = 0.729$

$$\therefore (N_2)^2 = \frac{m + M/2(1+q_2)}{m} \times \frac{895}{h_2}$$

$$= \frac{8 + 60/2(1 + \overset{0.729}{\cancel{0.587}})}{8} \times \frac{895}{0.264}$$

$$= \cancel{23565.80}\ 25371$$

$\therefore$ ~~$N_2 = 153.51$~~ $\boxed{N_2 = 159.28 \text{ rpm}}$

$\therefore$ Range of speed = ~~153.51 − 147~~

= ~~6.51 rpm~~ Ans

= 159.28 − 147

= 12.28 rpm Ans

Problem : CW-2010 : A porter governor has equal arms each 250 mm long and pivoted on the axis of rotation.

Each ball has a mass of 5 kg and the mass of central load on the sleeve is 30 kg. The radius of rotation of the ball is 150 mm when ball begins to lift and 200 mm, when the governor is at maximum speed.

Determine : (i) minimum and maximum speed,

(ii) Range of speed of governors.

Soln ⇒ Given data:

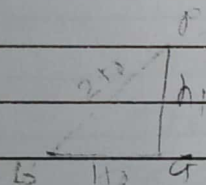
$PB = BD = 0.250\ m.$
$m = 5\ kg$
$M = 30\ kg.$

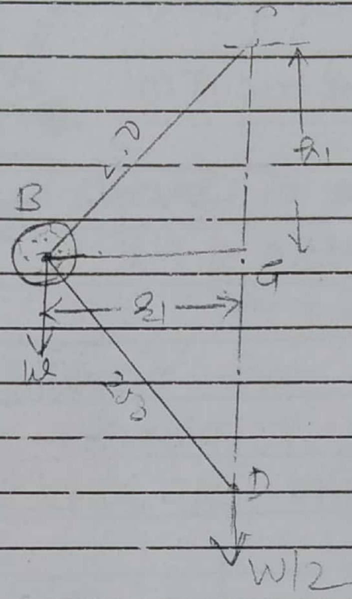
given condition for radius of rotation.
$r_1 = 0.150\ m$

$r_2 = 0.20\ m.$

When $r_1 = 150\ mm$

Minimum speed will be N_1 when at r_1.





from ∠PBG,

$PG = \sqrt{(BP)^2 - (BG)^2}$

$= \sqrt{(250)^2 - (150)^2}$

$PG = 200\ mm$ | $PG = h_1 = 0.2\ m$

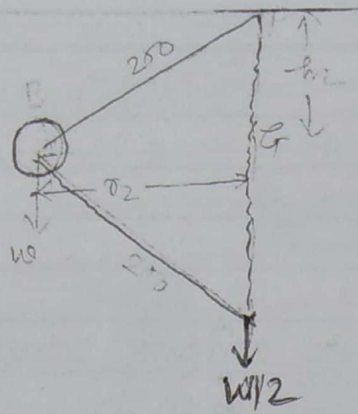
Here $\tan\alpha = \tan\beta \quad \therefore q = 1$

$N_1^2 = \frac{m + M}{m} \times \frac{895}{h_1}$

$= \frac{5 + 30}{5} \times \frac{895}{0.2}$

$N_1 = 177\ rpm$

When $r_2 = 0.2m$ At maximum speed:



$h_2 = PG = \sqrt{(250)^2 - (200)^2}$

$$\boxed{h_2 = PG = 0.15m}$$

Max. speed N_2 = $N_2^2 = \frac{m+M}{m} \times \frac{895}{h_2}$

$= \frac{5+30}{5} \times \frac{895}{0.15}$

$$\boxed{N_2 = 204.42 \text{ rpm}}$$

* Speed range of the governor $= N_2 - N_1$

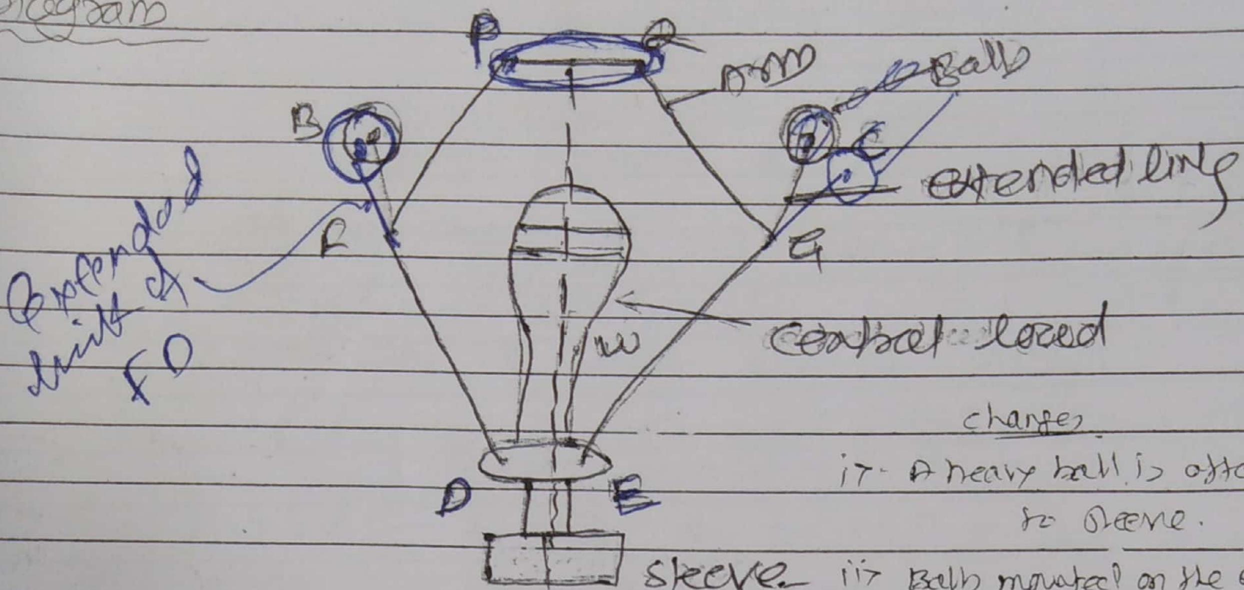
$= 204.42 - 177$

$=$ 27.42 rpm Ans

Porter Governor

Date :

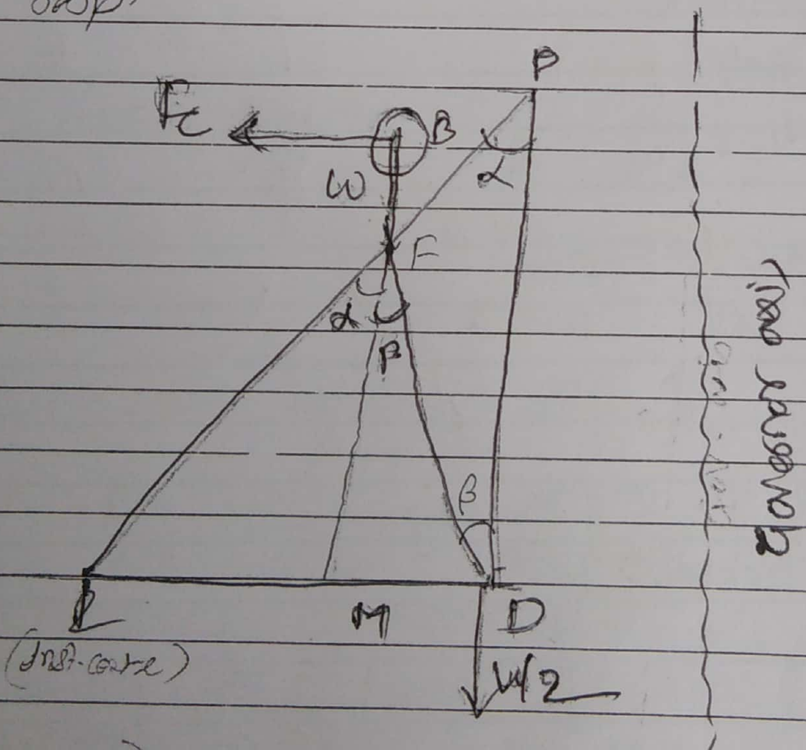
Diagram



changes
i) A heavy ball is attached to sleeve.
ii) Balls mounted on the extension of lower arms.
iii) helps to increase the stability..

Construction

The porell governor has the ball fixed at B and C to the extension of links DF and EG. The arms FP and GQ are pivoted at P and Q resp.



Consider the equilibrium of forces on one-half of the governor as shown.

The instantaneous centre (I) lies on the intersection of the line PF produced and the line from D drawn lar to the spindle axis.

The lar BM is drawn on ID.

Taking moment about I we have,

$$F_c \times BM = w \times IM + \frac{W}{2} \times ID. \quad \text{---- (i)}$$

Here, m = mass of each ball,
w = wt of each ball = mg
M = mass of central load
W = wt of central load
= M.g.

h = ht of governor
N = speed of ball
$\omega = \frac{2\pi N}{60}$ rad/sec

F_c = Centrifugal force acting on the ball = $m.\omega^2.r$

α = angle of inclination of the arm to the vertical,

β = angle of inclination of the lower link to the vertical.

simplify Eqn - (i)

$$F_c \times BM = mg \times IM + \frac{M.g}{2} \times ID \quad \text{---- (ii)}$$

divide by BM

$$f_c = \frac{m.g \times QM}{BM} + \frac{M.g}{2} \times \frac{QD}{BM}$$ here $QD = QM + MD$

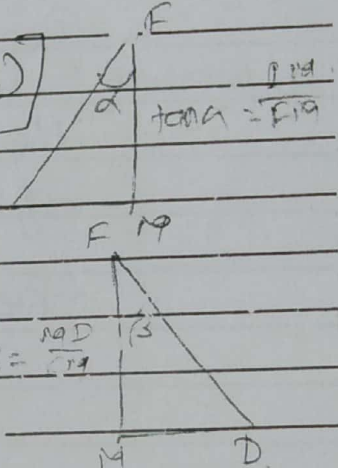
$$= m.g \times \frac{QM}{BM} + \frac{M.g}{2}\left[\frac{QM + MD}{BM}\right]$$

= multiply the above eqn by FM and divide by FM

$$f_c = FM\left[m.g \times \frac{Q.M}{BM} \times \frac{1}{FM}\right] + FM.\frac{M.g}{2}\left[\frac{QM + MD}{BM} \times \frac{1}{FM}\right]$$

$$= \frac{FM}{BM}\left[m.g \times \frac{QM}{FM}\right] + \frac{FM}{BM}\left[\frac{M.g}{2}\left(\frac{QM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$



$$f_c = \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right] \quad \text{--- (i)}$$

$\tan\beta = \frac{MD}{FM}$

here we know,

$$f_c = m.\omega^2.r$$

$$\tan\alpha = r/h \text{ and } q = \frac{\tan\beta}{\tan\alpha}$$

put these values in eqn (i)

$$m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

$$\omega^2 = \frac{FM}{BM} \times \frac{r}{h.m.r}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

$$\omega^2 = \frac{FM}{BM} \times \left[\frac{m.g}{m} + \frac{M.g}{2}(1+q)\right]\frac{1}{h},$$ take 'g' common

$$\frac{\;\;}{BM} \left[\frac{\;\;}{m} \right] \cdot \frac{\;\;}{h}$$

Sub. $w = \frac{2\pi N}{60}$, $g = 9.81$ ⇒

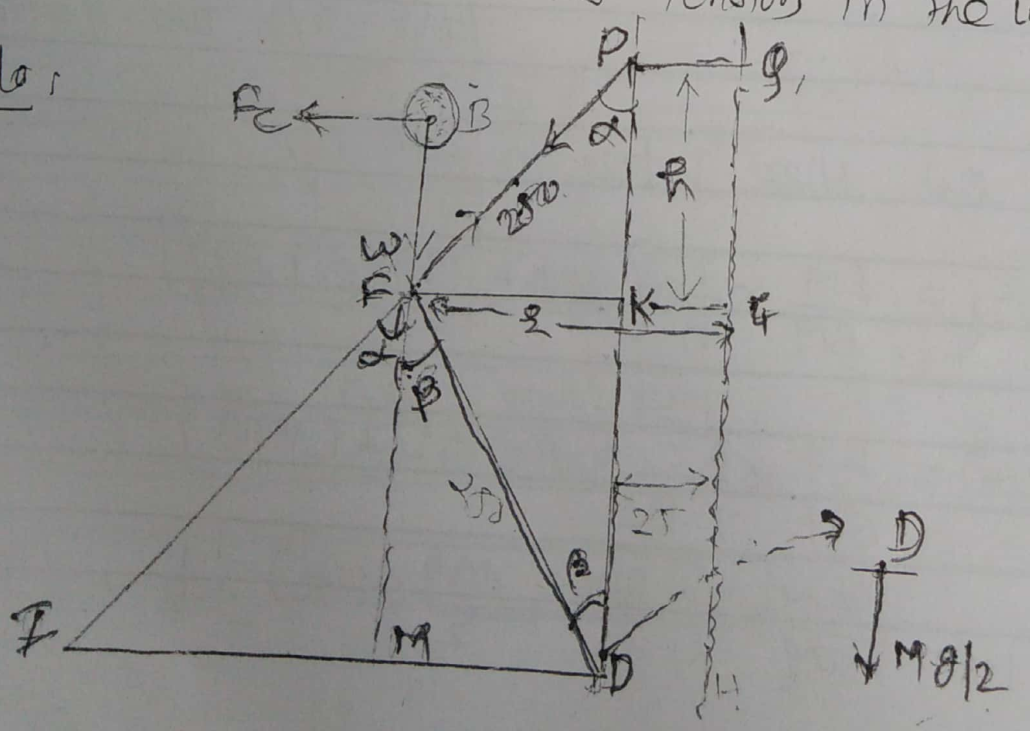
$$N^2 = \frac{FM}{BM} \cdot \left[\frac{m + M/2\,(1+q)}{m} \right] \cdot \frac{895}{h}$$

Example: A governor of the porell type has each arm 250 mm (proell) long. The pivot of the upper and lower arms are 25 mm from axis. The central load acting on the sleeve has a mass of 25 kg and the each rotating ball has a mass of 3.2 kg. When the governor sleeve is in mid-position, the the extension link of the lower arm is vertical and the radius of the path of rotation of the masses is 175 mm. The vertical height of the governor is 200 mm.

If the governor speed is 160 rpm, when in mid-position, find 1) length of the extension.

2) Tension in the upper arm.

data:



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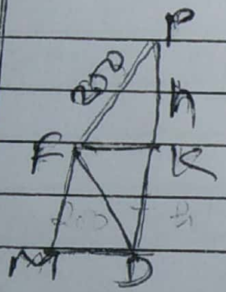
arm length PF = DF = 0.25 m
PG = KG = 25 mm = 0.025 m

central load (M)
mass (M) = 25 kg
ball mass (m) = 3.2 kg.
r = FG = 175 mm = 0.175 m
height (h) = 200 mm = 0.2 m

rotational speed (N) = 160 rpm

To calculate: length of extension i.e BF.

From figure, we know that,



F' FM will be equal to ht of governor

FM = KD = h = 0.2 m.

here if we calculate $\alpha = \beta$

$$\therefore \; q = \frac{\tan\beta}{\tan\alpha} = 1$$

By formula

$$N^2 = \frac{FM}{BM} \cdot \left[\frac{m + M/2\,(1+q)}{m} \right] \cdot \frac{895}{h}$$

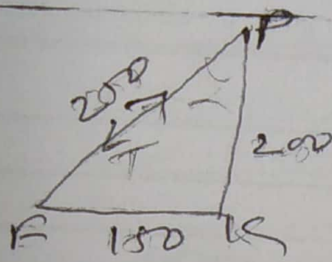
$$(160)^2 = \frac{0.2}{BM} \cdot \left[\frac{3.2 + 25/2\,(2)}{3.2} \right] \cdot \frac{895}{0.2}$$

$\therefore$ BM = 0.308 m

From figure BF = BM − FM
= 0.308 − 0.20 = 0.108 m

$\therefore$ **BF = 108 mm** Ans

2. Tension in the upper arm:



$PK = \sqrt{(PF)^2 - (FK)^2}$, $FK = FG - KG$

$= 175 - 25$

$= 150\ mm$

$= \sqrt{(250)^2 - (150)^2}$

$= 200\ mm$

Tension is given by $T_1 \cos\alpha = mg + Mg/2$

$\cos\alpha = \frac{200}{250}$

$\cos\alpha = 0.8$

$T_1 \times 0.8 = 3.2 \times 9.81 + \frac{25 \times 9.81}{2}$

$\therefore \boxed{T_1 = 192.52\ N}$ Ans

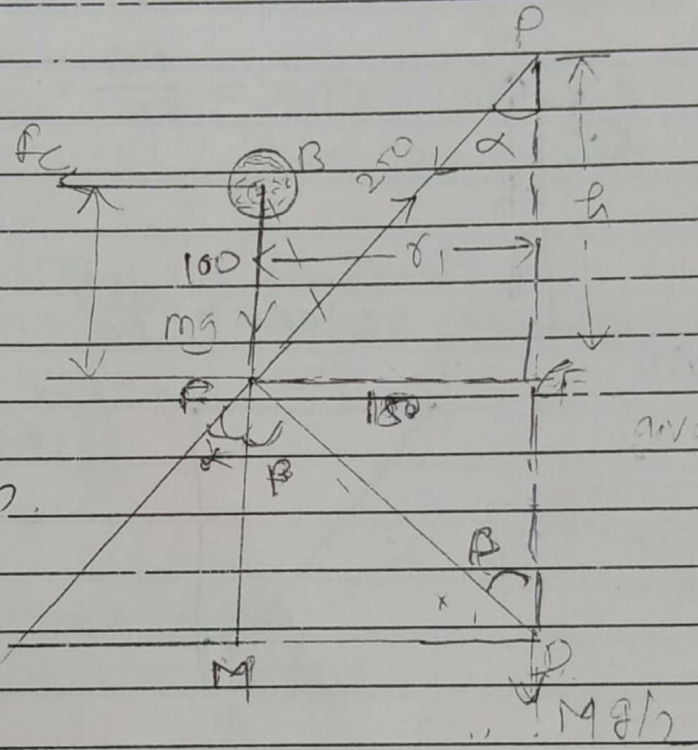
Date:

Ex: A Porter governor has all the four arms of length 250 mm. The upper and lower ends of the arms are pivoted on the axis of rotation of the governor.

The extension arms of the lower links are each 100 mm long are parallel to the axis when the radius of the ball path is 150 mm. The mass of each ball is 4.5 kg and the mass of the central load is 36 kgs.

Determine the equilibrium speed of the governor.

Soln: Given data:



length of arm PF = DF
= 0.25 m

extension of lower arm

BF = ~~150 mm = 0.15 m~~
= 100 mm = 0.1 m.

radius of ball path
r = 150 mm
= 0.15 m.

mass of each ball (m) = 4.5 kgs.

mass of central load (M) = 36 kgs.

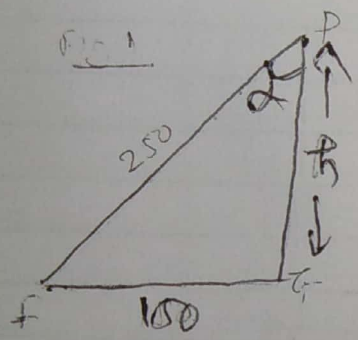
equilibrium speed of the governor: the radius of the governor is the distance of the point of intersection of the

upper and lower arm of from the governor axis.
When the extension of the lower arms are parallel to the governor axis, then the radius of governor (FG) is equal to the radius of rotation (r).

Here $r = FG = 150\,mm$

Let N_1 = Equilibrium speed at minimum radius, i.e $FG = 150\,mm = 0.15\,m$.

From figure.



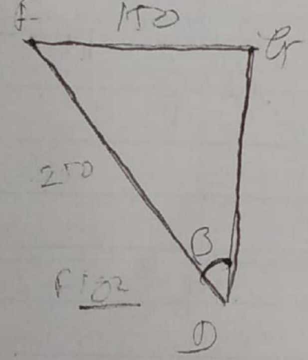
$\sin\alpha = \frac{FG}{PF} = \frac{150}{250} = 0.6$

$\therefore \boxed{\alpha = 36.86°}$

$\tan\alpha = \frac{150}{PG} \Rightarrow \tan 36.86 = \frac{150}{PG}$

$\boxed{PG = h = 200\,mm}$

From figure, $\boxed{PG = 200\,mm}$



$\sin\beta = \frac{150}{250}$

$\therefore \boxed{\beta = 36.87°}$ not necessary to calculate

$q = \frac{\tan\beta}{\tan\alpha} = 1.$ $\boxed{q = 1}$

$\tan\beta = \frac{150}{DG}$

$\therefore DG = \frac{150}{\tan 36.87}$

$\boxed{DG = 200\,mm}$

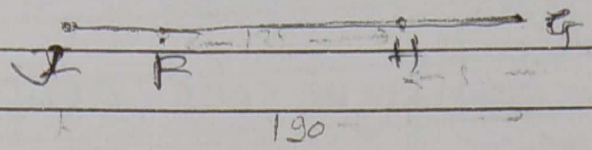
$\therefore BM = BF + FM$
$= 100 + 200 \Rightarrow \boxed{BM = 300\,mm}$

$\therefore \boxed{FM = DG = 200\,mm}$

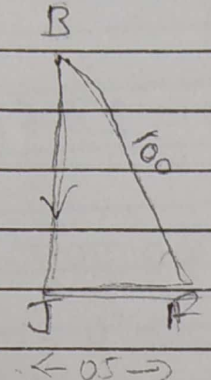
h = 160.87 mm , $\sin 40^\circ = \frac{FH}{210}$

∴ FH = 135 mm

from figure:



JF = JG − FH − HG
= 190 − 135 − 50
= 05 mm, JF = 5 mm



from figure.

$BJ = \sqrt{(BF)^2 - (JF)^2}$

$= \sqrt{(100)^2 - (5)^2}$

= 99.87 mm

Now, BM = BJ + JM | we know
h = PH = HD = JH = 160.87 mm

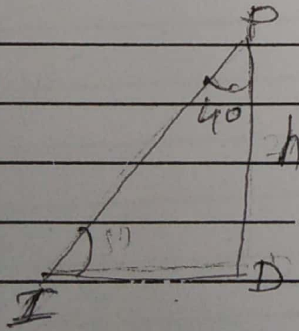
= 99.87 + 160.87

BM = 260.74 mm,

we know,

~~JF = MN = 5 mm~~

~~FH = ND = 135 mm.~~



∴ $\tan 50 = \frac{PD}{ID}$

∴ ID = tan 50° × 321.75

ID = 269.98 mm

IM = ID − MN − ND = 129.98 mm

Now. Taking moment about inst. centre I

$F_c \times BM = m.g \times IM + \frac{M.g}{2} \times ID.$

$F_c \times 260.74 = 7 \times 9.81 \times 129.98 + \frac{140 \times 9.81}{2} \times 269.98$

$\therefore F_c = 745.27\ N$

we know, centrifugal force acting on the ball, is given by

$F_c = m \cdot \omega^2 \cdot r$ | r = radius of rotation = 0.19 m

$745.27\ N = 7 \times \omega^2 \times 0.19$

$\therefore \omega^2 = 560.35 \quad \therefore \omega = 23.67\ r/s$

* $\omega = \frac{2\pi N}{60} \quad \therefore N = \frac{60 \times 23.67}{2 \times 3.14}$

$N = 226.17\ rpm$

Ans

We know that

$$(N_1)^2 = \frac{f_m}{B_m}\left[\frac{m + m/2\,(1+a)}{m}\right]\frac{895}{h}$$

$$= \frac{0.20}{0.30}\left[\frac{4.5 + 36/2\,(1+1)}{4.5}\right]\frac{895}{0.20}$$

$$= 2980.35\,(9) = 26823.15$$

$$\therefore \boxed{N_1 = 163.78 \text{ rpm}}$$ Ans

Problem: (Proell Governor)

The following particulars refer to a proell governor with open arms:

Length of all arms = 210 mm,

Distance of pivot of arms from the axis of rotation = 50 mm

Length of extension of lower arms to which each ball is attached = 100 mm;

mass of each ball = 7 kg;

mass of central load = 140 kg

If radius of rotation of ball is ~~180~~ 190 mm when the arms are inclined at an angle of 40° to the axis of rotation

Find the equilibrium speed for the above configuration.

Solution: Given data:

PF = DF = 210 mm,

DK = PG = 50 mm

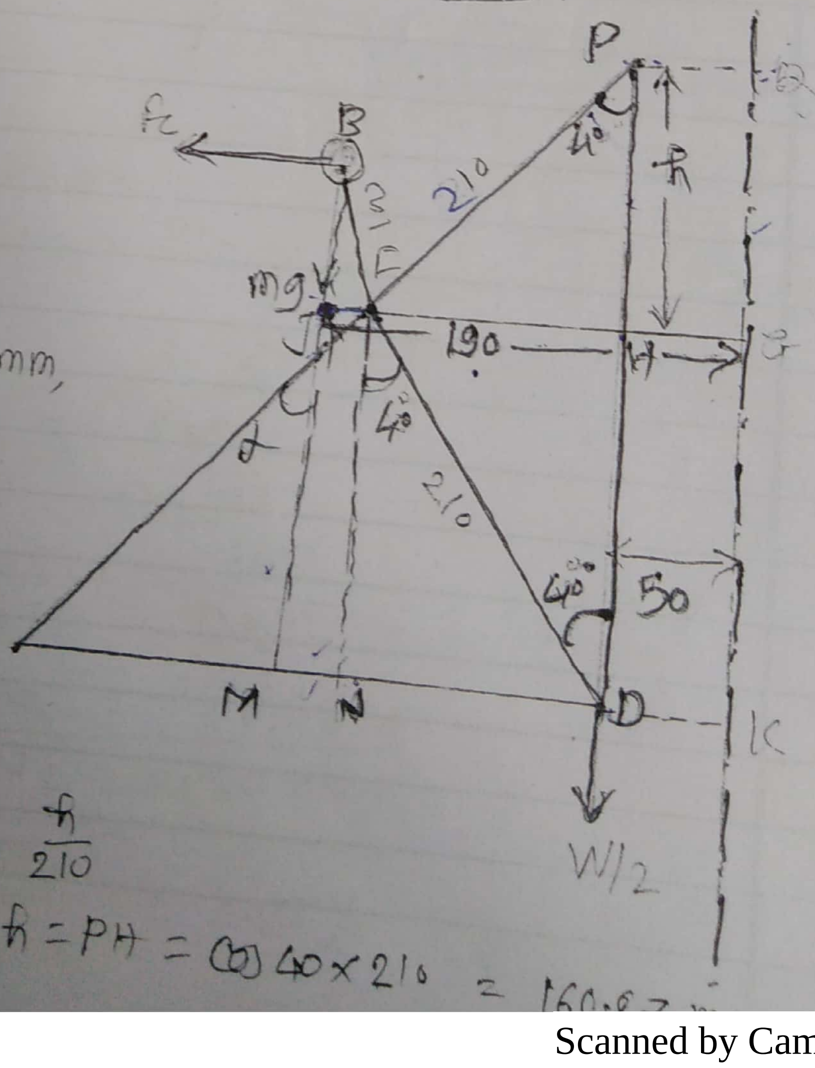
BF = 100 mm,

m = 7 kg

M = 140 kg,

JG = r = ~~180 mm~~ = 190 mm,

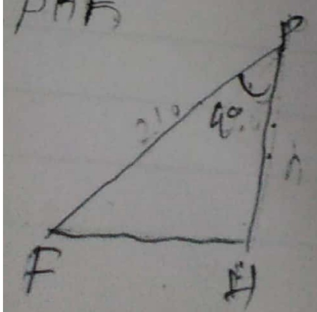
Figure



st of the governor is not-

so consider the

PHF



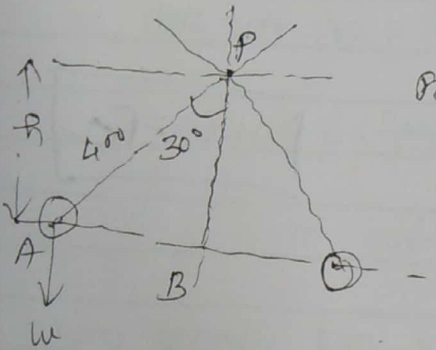
$\cos 40° = \frac{h}{210}$

$\therefore h = PH = \cos 40 \times 210 = 160.87$

Q. (Watt) The length of the upper arm of a ~~watt~~ governor is 400 mm and its inclination to the vertical is 30°. Find the % increase in speed, if its ball rise by 20 mm.

Soln ⇒ Given data:

~~initial ht (h1) = 400 mm~~



from figure

$\sin\alpha = \frac{AB}{AP} \Rightarrow \sin 30° = \frac{AB}{400}$

$\therefore AB = 200\ mm$

$\tan\alpha = \frac{200}{PB}$

$\tan 30° = 200/PB \quad \therefore PB = 346.41\ mm$

Height of gover

$\therefore h_1 = 346.41\ mm$

since ball rise by 20 mm

$\therefore$ New ht $(h_2) = h_1 - 20$

$= 346.41 - 20 = 326.41\ mm$

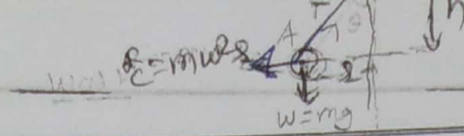
we know $h_1 = \frac{895}{(N_1)^2}$, $h_2 = \frac{895}{(N_2)^2}$

$\therefore N_1^2 = \frac{895}{(.346)^2}$ $\quad 326.41 = \frac{895}{(N_2)^2}$

$N_1 = 50.85\ rpm$ $\quad \therefore N_2^2 = 895/0.326$

$N_2 = 52.40\ rpm$

% change $= \frac{N_2 - N_1}{N_2} = \frac{52.40 - 50.85}{52.40} = 2.95\%$ Ans



$T\sin\theta = F_c$ — (ii)

Let,

m = mass of ball in kg.
w = wt. of the ball in N. | $w = mg$
T = Tension in the arms, N
ω = angular velocity of the arms and balls about the spindle axis, rad/sec.

r = horizontal distance from the centre of the to the spindle axis, m

F_c = centrifugal force acting on the ball, :

$$= \frac{mv^2}{r} = \frac{m(\omega^2 r^2)}{r} = \boxed{m\omega^2 r = F_c}$$

h = ht. of the governor. m.

Assume that the wt. of the arms, link and sleeve are negligible as compared to the wt. of balls

Force acting on the ~~ball~~ system :
1] centrifugal force (F_c) on ball
2] Tension in arm (T)
3] wt. of ball. (W)

take the moment about point O, we have,

$$F_c \times h = W \times r$$
$$\therefore F_c \times h = m \times g \times r$$
$$m\omega^2 r \times h = m g r$$

$\boxed{W \times r = mgr}$
$\boxed{F_c = m\omega^2 r}$

$$\therefore \boxed{h = g/\omega^2}$$

$\omega = \frac{2\pi N}{60}$ rad/sec
$g = 9.81$ m/s²

$$h = 9.81 / \frac{(2\pi N)^2}{(60)^2}$$

$$\therefore \boxed{h = 895.1 / N^2}$$

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1

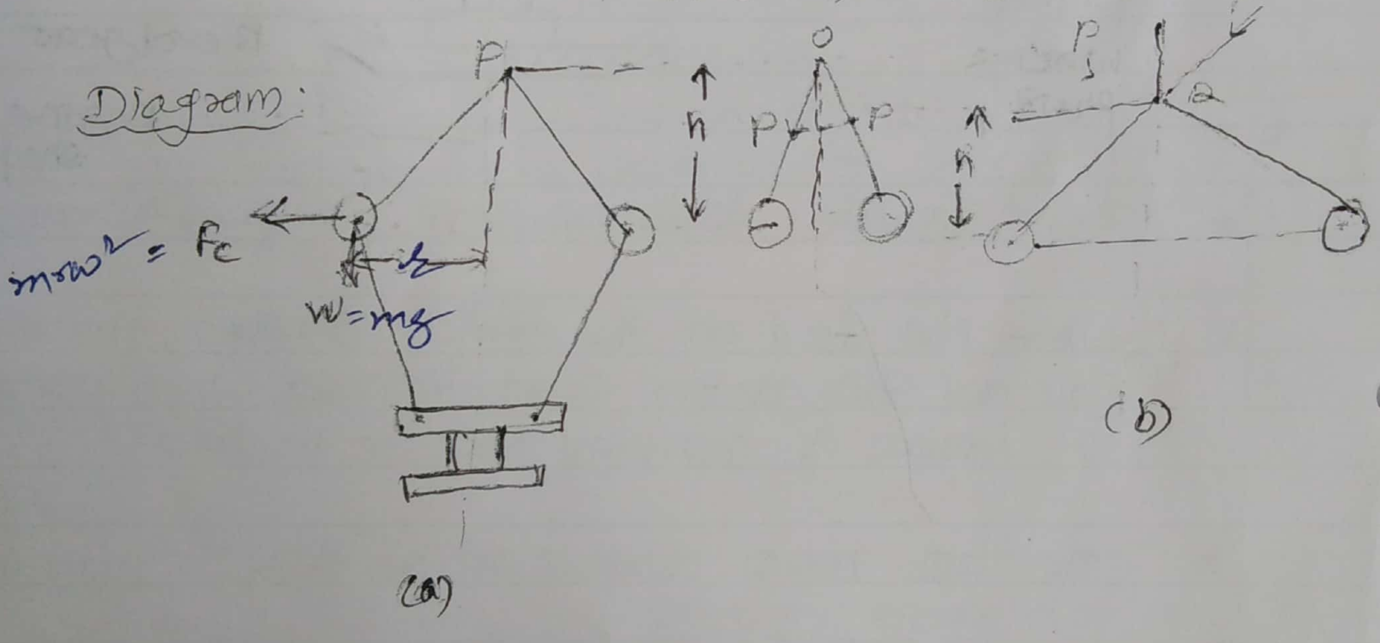
* Watt Governor:

* This is the simplest form of centrifugal governor.
* It is somewhat pendulum type, with ~~sleeve~~ links attached to sleeve of negligible mass.
* The arm of the governor may be connected to the spindle in three ways.

1] The pivot P, may be on the spindle axis as shown in figure (a)

2] The pivot P, may be offset from the spindle axis and the arms when produced intersect at O, shown in figure.

Diagram:



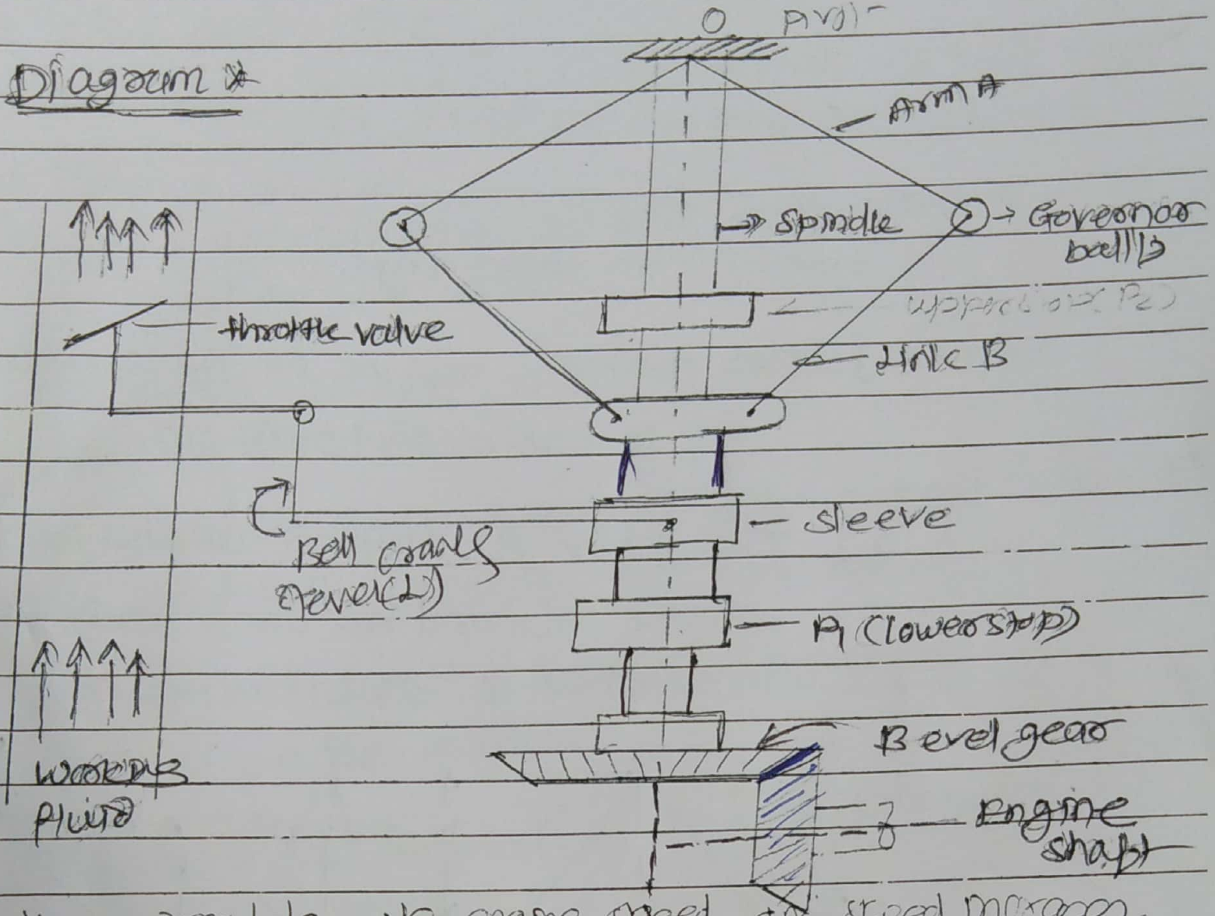
(a)

(b)

3] The pivot ~~as~~ P, may be offset, but the arm cross the axis at O, shown in figure (c).

the throttle valve. Due to heavy weight of governor they are not used in practice.

mechanism. lever to increase the supply of working fluid



* Diagram *

* thus regulate the engine speed and speed increases.

* When the load on the engine decreases, the engine and the governor speed increases, which results in increase of centrifugal force on the balls.

* The balls moves outwards and the sleeve rises upwards

* This upward movement of sleeve reduces the supply of working fluid and hence speed decreased and power output reduced.

* ~~These balls are known as fly balls.~~

* The balls revolve with a spindle, which is driven by the engine through bevel gears.

* Upper arms pivoted to spindle, so that the balls may rise up or fall down as they revolve about vertical axis.

* The arms are connected to link to the sleeve, which is keyed to the spindle.

* The sleeve revolves with the spindle, but only slide up and down.

* The balls, and sleeve rises when spindle speed increases, and falls when the speed decreases.

* For limit of travel of sleeve, two stops S–S provided on the spindle.

* The sleeve is connected by Bell crank lever to the throttle value to control the working fluid supply.

* Working:

* When load on engine increases, engine and governor speed decreases.

Resulting
* Reading in decrease of centrifugal force on the balls.

* Hence balls move inwards, and sleeves moves in downwards, this downward movements operates the throttle values at the other end of ball crank

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Date :

* Governors :→

* The main function of governor is to regulate the mean speed of an engine, when there are variations in the load. i.e, when the load on an engine increases, its speed decreases.

* When the load on the engine decreases, its speed increases and thus less working fluid is required.

* The governor automatically controls i.e supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limit.

* Types of Governors :

It may be classified into two types:

1) Centrifugal governors
2) Inertia governors.

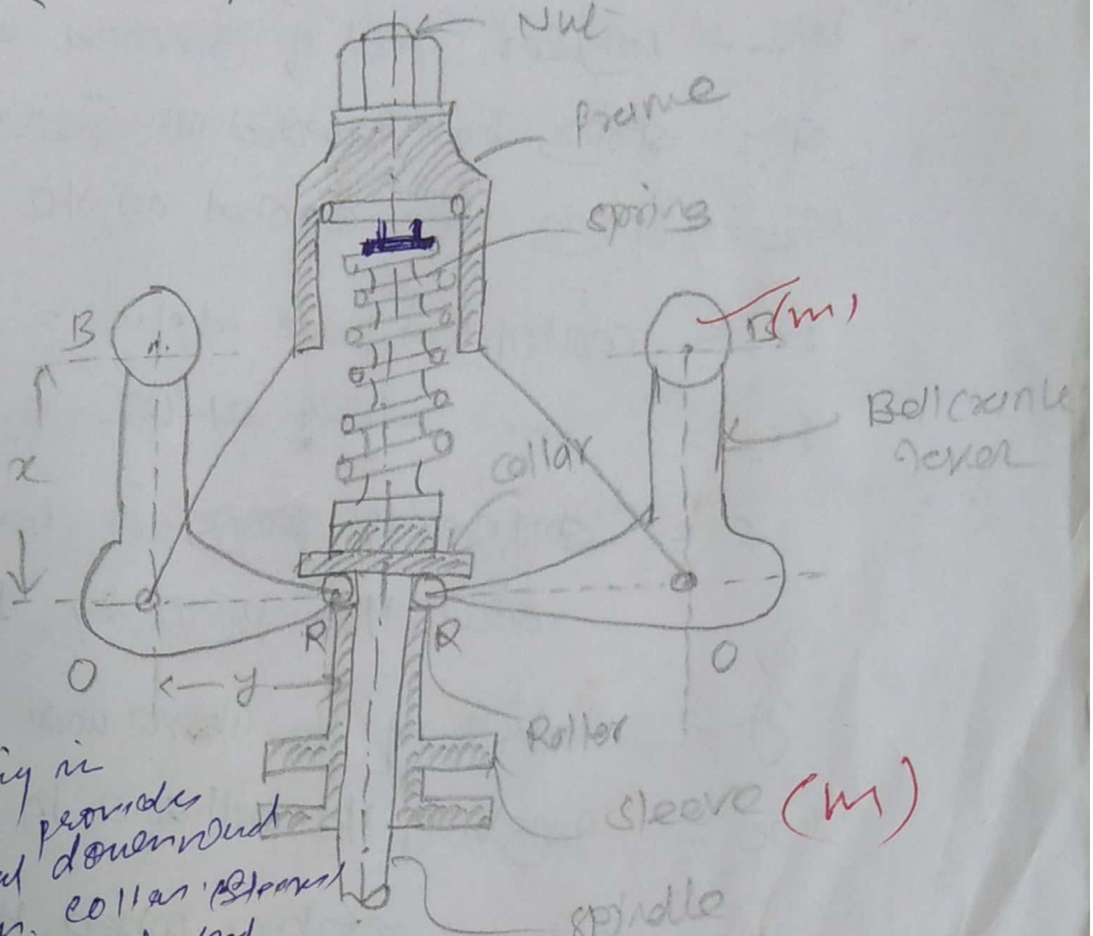
1] Centrifugal Governors :

* It is based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as controlling force.

* consists of two balls of equal mass, attached to arm

Explain Hartnell Governor. Derive Stiffness

(4) * Hartnell Governor: A hartnell governor is a spring loaded governor as shown in figure. It consists of two bell crank lever pivoted at point O O.



500-600 rpm

It consists of two bell crank lever

Helical spring in compression provides two equal downward force on collar (sleeve)

Spring force can be adjusted by screwing nut (on)

Fig: Hartnell governor

The ~~spring~~ frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB. and roller at horizontal arm OR. A helical spring in compression provides downward forces on rollers through a collar on the sleeve. Its speed is 500 / 600 rpm

Let, m = mass of each ball,

M = mass of sleeve

r_1 = minimum radius of rotation

r_2 = maximum radius of rotation.

ω_1 = Angular speed of governor at r_1,

ω_2 = Angular speed of governor at r_2.

S_1 = spring force exerted on sleeve at ω_1

S_2 = spring force exerted on the sleeve at ω_2

F_{C1} = centrifugal force at ω_1 = $m\omega_1^2 r_1$.

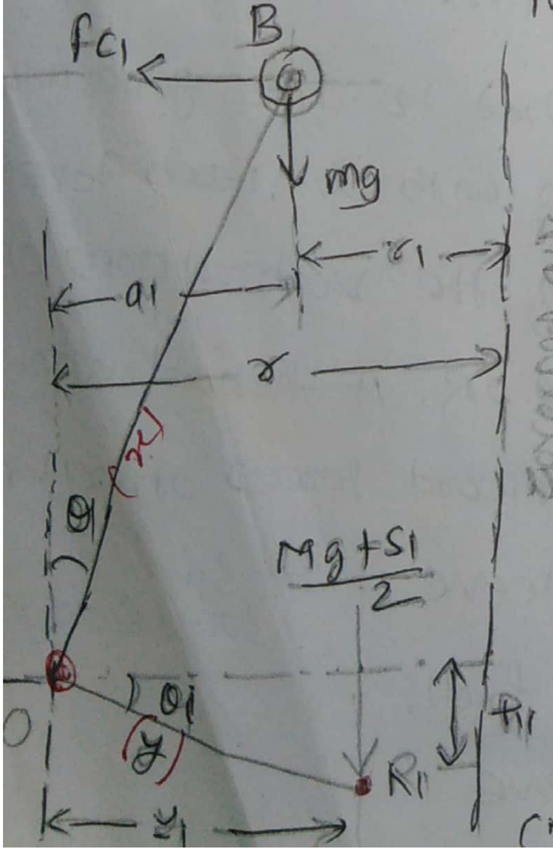
F_{C2} = — " — " at ω_2 = $m\omega_2^2 r_2$.

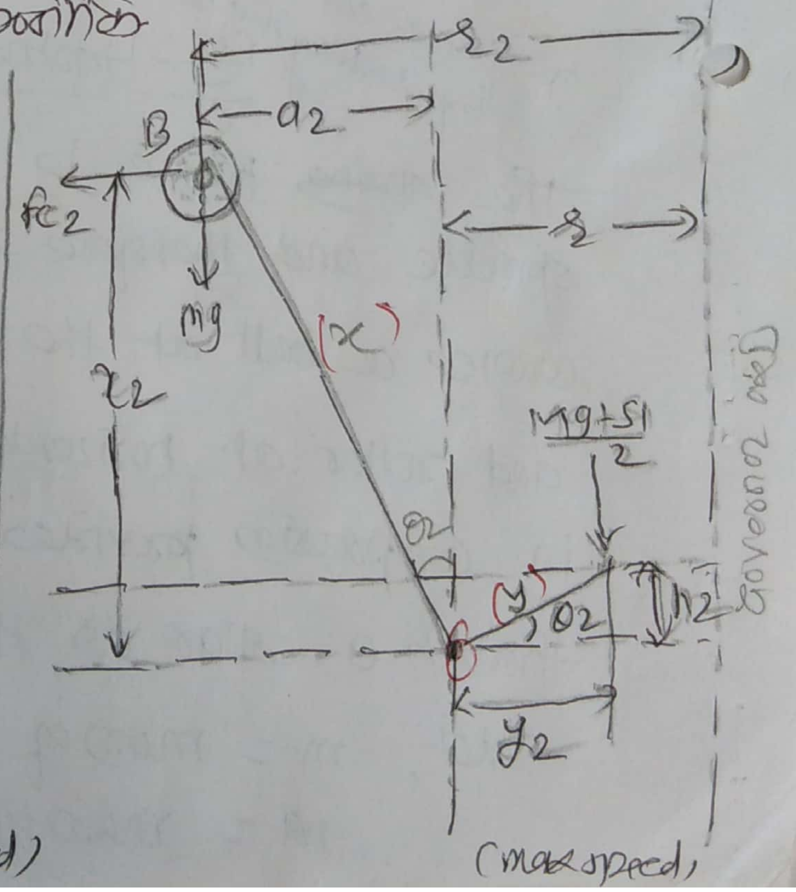
s = stiffness of spring or force req. to compress the spring by 1 mm.

y = length of the sleeve arm in m.

x = length of the ball arm in m.

r = radius of rotation when the governor is in mid-position





* For minimum position i.e. when the radius of rotation changes from r to r_1.

then compression of spring or lift of sleeve h_1.

is given by :

$$\sin\theta_1 = \frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \quad \text{--- (i)}$$

* For maximum position i.e when the radius of rotation changes from r to r_2. the compression of the spring or lift of sleeve 'h_2'

$$\sin\theta_2 = \frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \quad \text{--- (ii)}$$

add eqn (i) & (ii)

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad (\because h = h_1 + h_2)$$

lift = $\therefore$ $h = (r_2 - r_1) \cdot \frac{y}{x}$ ------ (iii)

* For minimum position taking moment about pt 'O'

$$M \cdot g + S_1 = \frac{2}{y_1}(F_{c1} \times x_1 - m \cdot g \times a_1) \quad \text{--- (iv)}$$

* For max. position

$$M \cdot g + S_2 / 2 \times y_2 = F_{c2} \times x_2 + m g \times a_2 \quad \text{--- (v)}$$

Substract eqn (iv) from eqn (v)

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m \cdot g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m \cdot g \times a_1)$$

we know that

$$S_2 - S_1 = h \cdot s \quad \text{and} \quad h = (r_2 - r_1)\frac{y}{x}$$

s or δ

Stiffness (K) =
$$K = \frac{S_2 - S_1}{h} = \left[\frac{S_2 - S_1}{r_2 - r_1}\right] \cdot \frac{x}{y}$$

End//

* neglect of $x = x_1 = x_2$ and $y = y_1 = y_2$

moment due to wt. of the balls (i.e mg).

For minimum position:

$$\frac{M \cdot g + S_1}{2} \times y = F_{C1} \times x$$

or $$M \cdot g + S_1 = 2 F_{C1} \times \frac{x}{y} \quad \text{---- (vi)}$$

For maximum position,

$$\frac{M \cdot g + S_2}{2} \times y = F_{C2} \times x$$

or $$M \cdot g + S_2 = 2 F_{C2} \times \frac{x}{y} \quad \text{--- (vii)}$$

substract eqn (vi) from eqn (vii)

$$S_2 - S_1 = 2(F_{C2} - F_{C1}) x/y$$

3

we know that, $S_2 - S_1 = R \cdot S$ and $R = (\sigma_2 - \sigma_1) \cdot \frac{y}{x}$

$\therefore$ S or K $= \frac{S_2 - S_1}{R} = 2\left[\frac{F_{c_2} - F_{c_1}}{\sigma_2 - \sigma_1}\right] \cdot (x+y)^2$

S or KS stiffness

$$\boxed{S = K = \frac{S_2 \cdot S_1}{h}} = 2\left[\frac{F_{c_2} \cdot F_{c_1}}{\sigma_2 \cdot \sigma_1}\right] \cdot (x+y)^2$$

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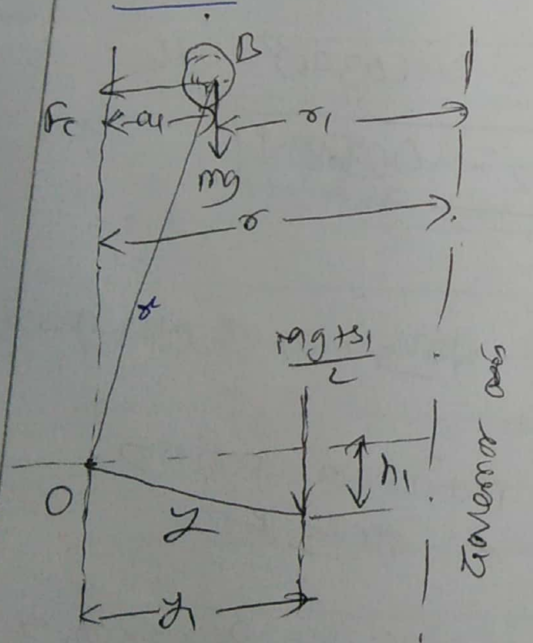
Example (W-2015) :

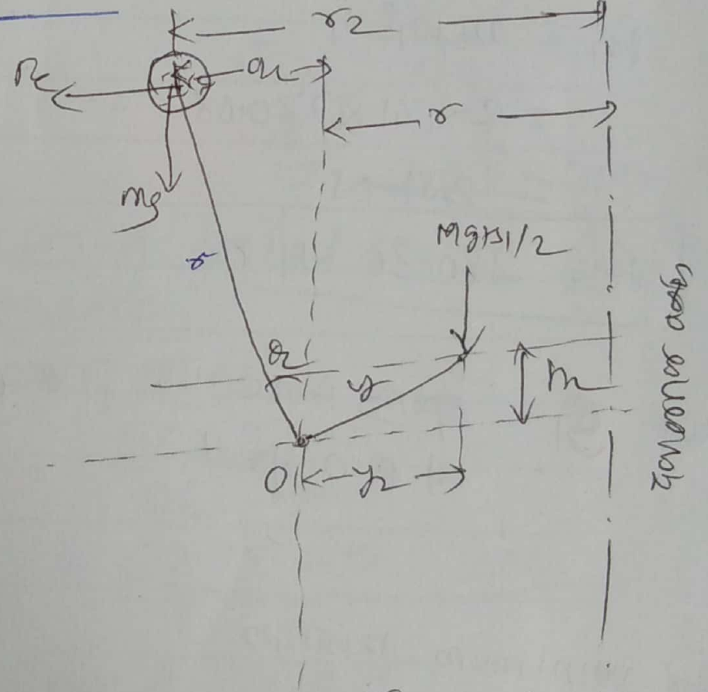
In a spring loaded Hartnell type governor, the extreme radii of rotation of ball are 80 mm & 120 mm. The ball arm and the sleeve arm of the bell crank lever are equal in length. The mass of each ball is 2 kg. If the speeds at the two extreme positions are 400 and 420 rpm.

Find : (i) the initial compression of the central spring and

(ii) The spring constant (10 marks).

Solution :→ Given data :





Given data : $r_1 = 80$ mm $r_2 = 120$ mm

Bell arm and sleeve arm are equal in length i.e $x = y$.

$m = 2$ kg

$N_1 = 400$ rpm $N_2 = 420$ rpm

To find out: Initial compression of central spring
spring constant or stiffness of spring (k).

Calculate the angular speed:

$\omega_1 = \frac{2\pi N_1}{60}$, $\omega_2 = \frac{2\pi N_2}{60}$

$= \frac{2\pi \times 400}{60}$ $= \frac{2\pi \times 420}{60}$

$= 41.86$ rad/sec $= 43.96$ rad/sec

Now calculate the centrifugal force acting at both the conditions. i.e.,

$F_{c1} = m\omega_1^2 r_1$

$= 2 \times (41.86)^2 \times 0.08$

~~= 281 N~~

$F_{c1} = 280.36$ N

$F_c = m\omega_2^2 r_2$

$= 2 \times (43.96)^2 \times 0.12$

$F_{c2} = 463.8$ N

Let S_1 = spring force at min. speed

S_2 = spring force at max. speed.

For minimum position spring force

$M.g + S_1 = \frac{2}{y} \times F_{c1} \times x$

$M = 0$ & $x = y$

$S_1 = 2 \times F_{c1}$

$= 2 \times 280.36$

$S_1 = 560.72$ N

For maximum position spring force

$M.g + S_2 = \frac{2}{y} \times F_{c2} \times x$

here $M = 0$, $x = y$

$S_2 = 2 \times 463.80$

$S_2 = 928$ N

Marks

we know that,

lift of sleeve,

$$h = (r_2 - r_1) \cdot \frac{y}{x} \quad | \text{ since } x = y$$

$$= (120 - 80)$$

$$= 40 \text{ mm}$$

$\therefore$ stiffness of spring $(k) = \frac{S_2 - S_1}{h}$

$$= \frac{928 - 566.72}{40}$$

$$= \frac{361.28}{40}$$

Ans →

$$= 9.03 \text{ N/mm}$$

(1) Initial compression of central spring (δ)

$$= \frac{\text{spring force}}{\text{stiffness of spring}}$$

$$(\delta) = \frac{S_1}{k} = \frac{566.72 \text{ N}}{9.03 \text{ N/mm}}$$

$$= 62.75 \text{ mm}$$ Ans

(2) spring control or stiffness of spring (k)

$$k = 9.03 \text{ N/mm}$$ Ans

Unit 1

(Talodikar Sir)

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unit 1

(Taladik Sir)

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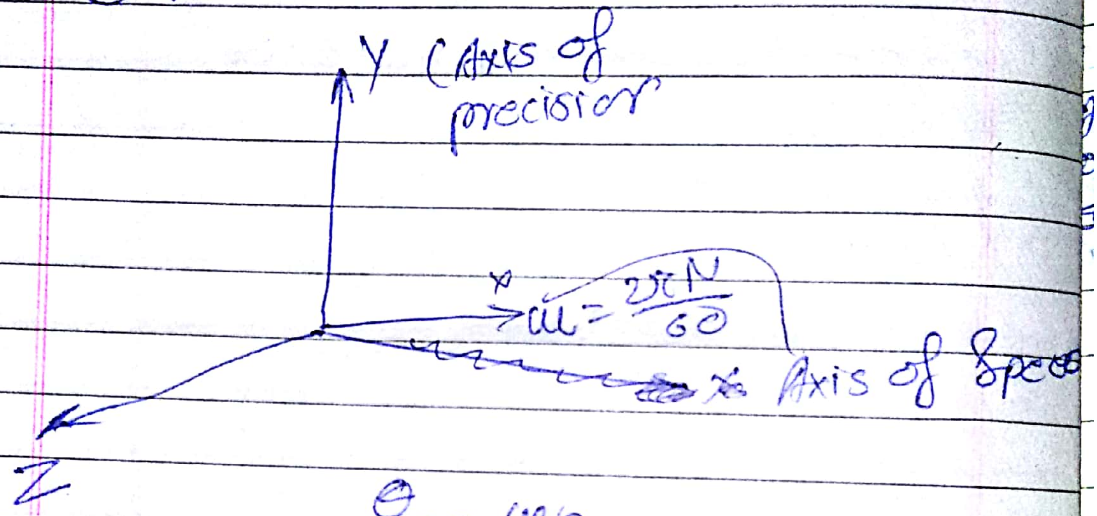
Gyroscopic Couple

(I) Airoplane
(II) Navel Ship
(III) four wheeler
(IV) Two wheeler



$\frac{\theta}{t} = \omega_p$

ω_p → Angular velocity of plane
ω → ——— " ——— disc (Gyrosco

Couple = $C = I \times \omega \times \omega_p$

Defn: The axis on which the rotar is rotating is called axis of spin.

up or down
Raise or dip

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$$I = mk^2 \quad kg\text{-}m^2$$

m = mass

K or R = Radius of Gyration

Angular velocity of rotor → $\omega = \frac{2\pi N}{60}$

Angular velocity of precision $\omega_p = \frac{V}{R}$

R → Radius of Curvature
V → Speed of Airoplane.

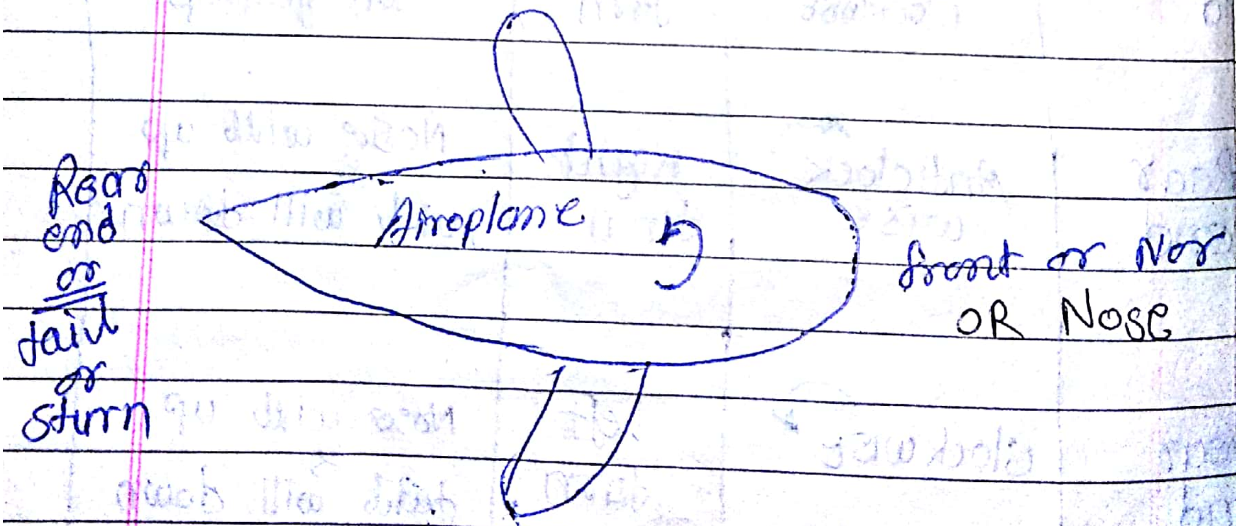
Direction of view	Rotor direction	Arm used	Gyroscopic effect
Rear end	Counter Clockwise	Left turn	Nose goes down & tail goes up
Rear end	Anticlock wise	Right turn	Nose will up & tail will down
Rear end	Clockwise	Left turn	Nose will up & tail will down
Rear end	Clockwise	Right left	Nose down & tail will up

The axis about which the axis spoon itself is called as axis precision.

when the axis of Spoon rotate which the angular velocity (ω_p) It is Subjected to the reactive couple (C) whose magnitude is same but opposite in direction this reactive couple is called as Gyroscopic Couple.

The effect of Gyroscopic Couple rotating body is called as Gyroscopic effect.

Gyroscopic effect on Aeroplane



$$C = I \times \omega \times \omega_p$$

Airoplane

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Ques Airoplane makes a Complete half circle of 50 m radius towards the left when flying at a speed of 200 km/hr. The rotary engine & the propeller of the plane has a mass of 400 kg & Radius of Gyration of 0.3 m. The engine is rotating at a speed of 2400 rpm clockwise. Find the Gyroscopic couple & investigates its effect on the airoplane.

⇒ **Given :**

$m = 400$ kg

Speed $= V = 200$ km/hr $= \frac{200 \times 10^3}{3600} = 55.55$ m/s

Radius of Gyration $= 0.3$ m

$I = mk^2$ $R = 50$ m

$K = 0.3$ m

$\therefore I = 400 \times (0.3)^2$

$I = 36$ kg-m^2

$N = 2400$ rpm

$\therefore \omega = \frac{2\pi N}{60}$

$\omega = 251.32$ rad/s

$\omega_p = \frac{V}{R}$

$$\omega_p = \frac{55.55}{50}$$

$$\boxed{\omega_p = 1.111 \text{ rad/s}}$$

$$\therefore C = I \times \omega \times \omega_p$$

$$= 36 \times 251.32 \times 1.111$$

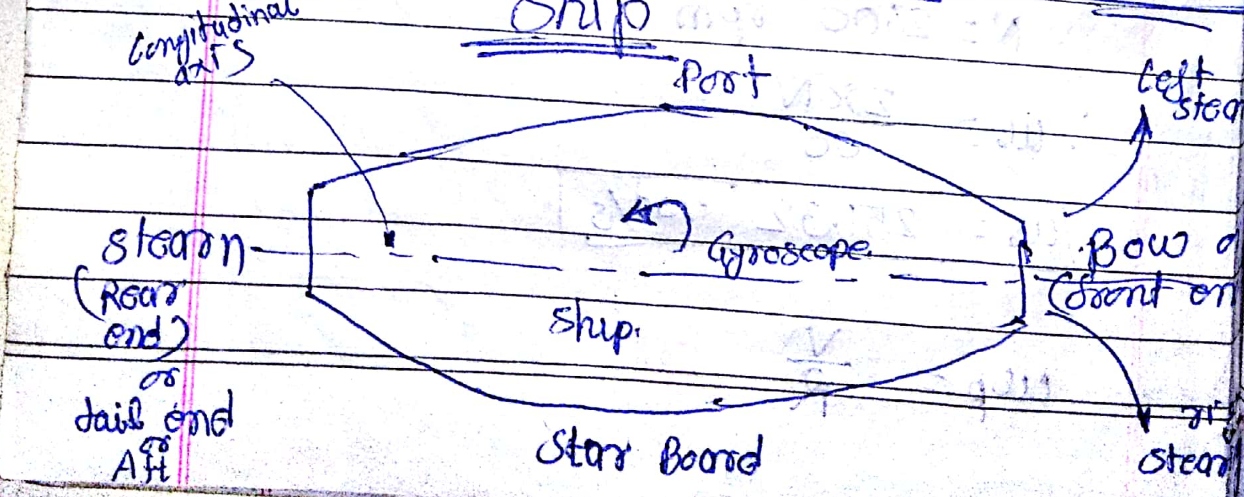
$$\boxed{C = 10051.79 \text{ N-m}}$$

Effect :- at Clockwise,
view from Rear end,
while taking left turn,
Nose goes up &
tail goes down

x — x

Ship

Effect of Gyroscopic Couple on Naval Ship



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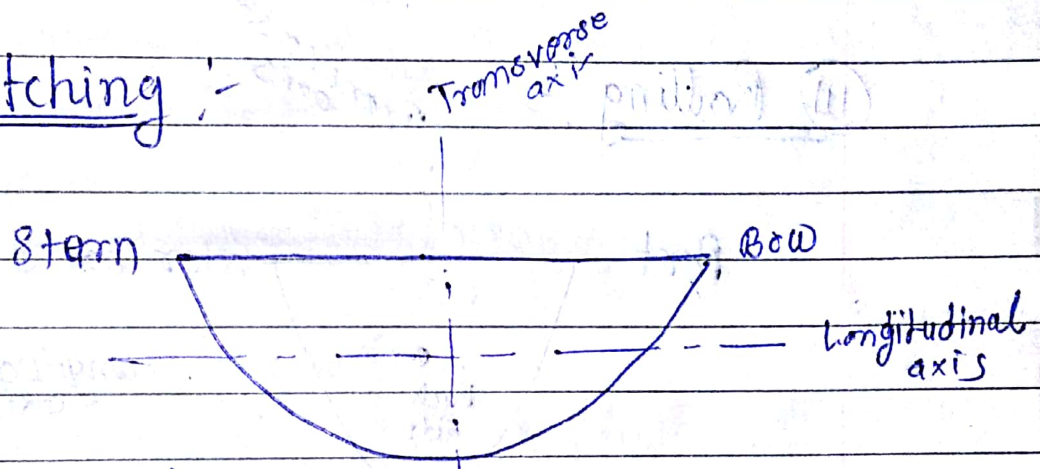
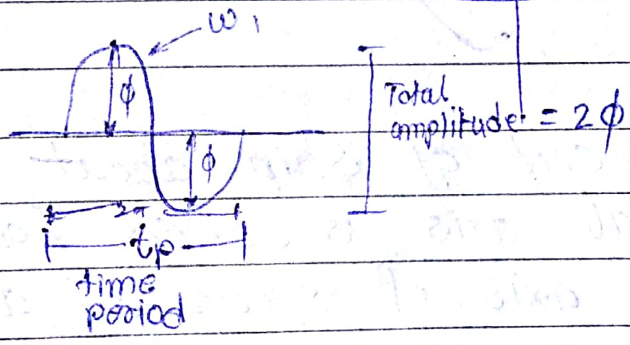
① Steering :-

Left steering
&
Right steering

② The effect of Gyroscopic Couple on steering of Navel ship is same as Airoplane.

(II) Pitching :-





$$C = I \times \omega \times \omega_{p_{max}}$$

where,

Max. Angu. velocity about precession

$$\omega_{p_{max}} = \phi \times \left(\frac{2\pi}{t_p}\right)$$

$$\therefore \omega_{p_{max}} = \phi \times \omega_1$$

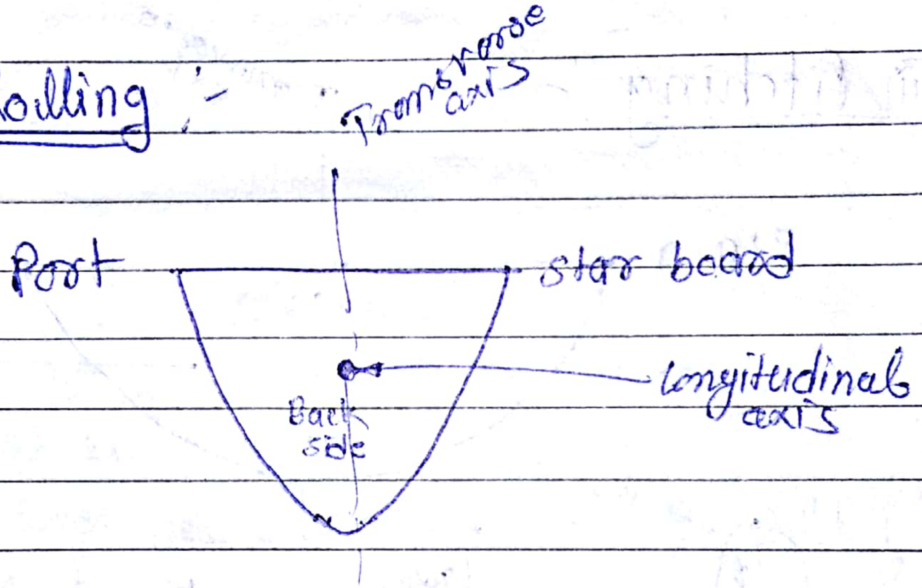
Max. Angular acceleration during pitching = $\phi \omega_1^2$

: effect :- Pitching upward tries to move the ship towards star board side.
& Pitching downward move the ship towards port side.

The moment of the ship about the transverse axis is called Pitching.

Pitching downward is called descending
&
Pitching upward is called ascending

(III) Rolling :-



The moment of Ship about longitudinal axis is called Rolling.
As axis of spin & axis of precision are parallel to each other hence, Gyroscopic Couple does not provide any effect on ship during rolling

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Prob. ① The mass of turbine rotor of ship is 10 tons & has a radius of gyration is 0.8 m. It rotates at 1600 rpm (Clockwise) when looking from stern end. Determine Gyroscopic effect in the following cases :-

(I) Ship is travelling at 120 km/hr. Steering to the left in a Curvature of 80 m. radius.

(II) Ship is pitching and the bow is descending with the max^m velocity. The pitching is S.H.M. The periodic time is 20 sec. & the angular momentum betⁿ extreme position is 10°.

(III) If the ship is rolling & at a certain instant has a angular velocity of 0.04 rad/sec. (clockwise) when looking from stern.

⟹ **Given** :- Diagram

$M = 10 \text{ tons} = 10 \times 10^3 \text{ kg}$

$K = 0.8 \text{ m}.$

$N = 1600 \text{ rpm}$

(i) **steering left**

Speed, $V = 120 \text{ km/hr.} = \frac{120 \times 10^3}{3600} = 33.33 \text{ m/s}$

$\therefore \omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1600}{60}$

$\therefore \boxed{\omega = 167.55 \text{ rad/sec}}$

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Radius = R = 80 m.

$\therefore \omega_p = \frac{V}{R}$

$= \frac{33.33}{80}$

$\boxed{\omega_p = 0.4166 \text{ rad/sec}}$

$\therefore I = mk^2$

$= 10 \times 10^3 \times (0.8)^2$

$\therefore \boxed{I = 6400 \text{ kg-m}^2}$

$\therefore$ Reactive Gyrocouple $= C = I \times \omega \times \omega_p$

$= 6400 \times 167.55 \times 0.4166$

$\therefore C = 446728.512$ N-m.

$\boxed{C = 446.72 \text{ KN-m}}$

effect of Reactive Gyrocouple.

effect ⇒ ~~The ship move~~

Bow - up (Raise)

& stern - down (dip)

(II) Pitching

Reactive Gyrocouple

$t_p = 20$ sec - - - given

$C = I \times \omega \times \omega_{p_{max}}$.

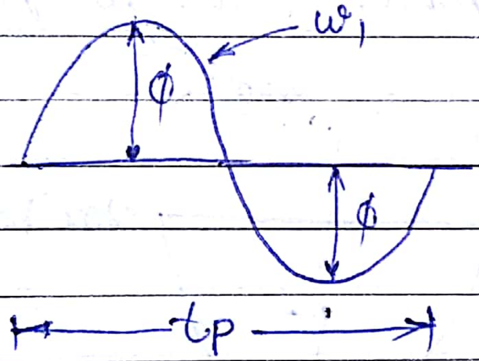
Total angular moment $= 2\phi = 10^\circ$

$\therefore \phi = 5^\circ$

$\therefore$ ~~$\phi = 0.087$ rad~~

$\phi = 5 \times \pi/180$

$\therefore \phi = 0.087$ rad



$\therefore \omega_1 = \dfrac{2\pi}{t_p}$

$= \dfrac{2\pi}{20}$

$\therefore \omega_1 = 0.314$ rad/sec.

$\omega_{p_{max}} = \phi \times \omega_1$

$= 0.087 \times 0.314$

$\therefore \omega_{p_{max}} = 0.027$ rad/sec.

$\therefore C = I \times \omega \times \omega p_{max}$

$= 6400 \times 167.55 \times 0.027$

$C = 28952.64$ N-m

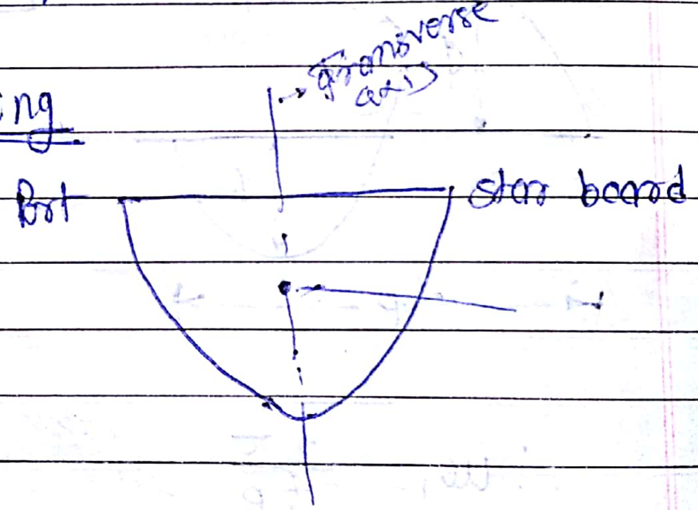
$\therefore$ **C = 28.95 KN-m**

effect of reactive couple

Pitching downward :-

The ship moves towards port side

(III) Rolling



$\omega p = 0.04$ rad/sec ---- given

$C = I \times \omega \times \omega p$

$= 6400 \times 167.55 \times 0.04$

$C = 42892.8$ N-m

C = 42.89 KN-m

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effect :- As axis of spin & axis of precision are parallel to each other Hence Gyroscopic Couple does not provide any effect on ship during rolling.

Prob. (2) Turbine rotor of the Ship has a mass of 3500 kg. It has a radius of Gyration 0.45 m & the speed of 3000 rpm. Clockwise looking from the stern. Find Gyroscopic couple and its effect,

(i) Ship is steering to the left on the Curve of 100 m radius at the speed of 36 km/hr.

(ii) The ship is pitching in SHM. The bow is falling with its max^m velocity. The period of pitching is 40 sec. & the angular movement bet two extreem position of pitching is 12°

(iii) The ship rolls at a Certain instant ~~Given~~, with the angular velocity of ~~m~~ 0.03 rad/sec.

⇒ **Given**:

$m = 3500$ kg.

$K = 0.45$ m

$N = 3000$ rpm

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$\therefore \omega = \frac{2\pi N}{60}$

$\omega = 314.159$ rad/sec.

Radius = $R = 100$ m

$V = 36$ Km/hr. $= \frac{36 \times 10^3}{3600} = 10$ m/s

$\therefore \omega_p = \frac{V}{R}$

$= \frac{10}{314.159}$

$\therefore \omega_p = 0.0318$ rad/s

$I = mK^2$

$= 3500 \times (0.45)^2$

$\therefore I = 708.75$ kg-m^2

$\therefore C = I \times \omega \times \omega_p$

$= 708.75 \times 314.159 \times 0.0318$

~~$\therefore C = 7080.594$ N-m~~

~~$C = 7.08$ KN-m~~

$C = 22.26$ KN-m

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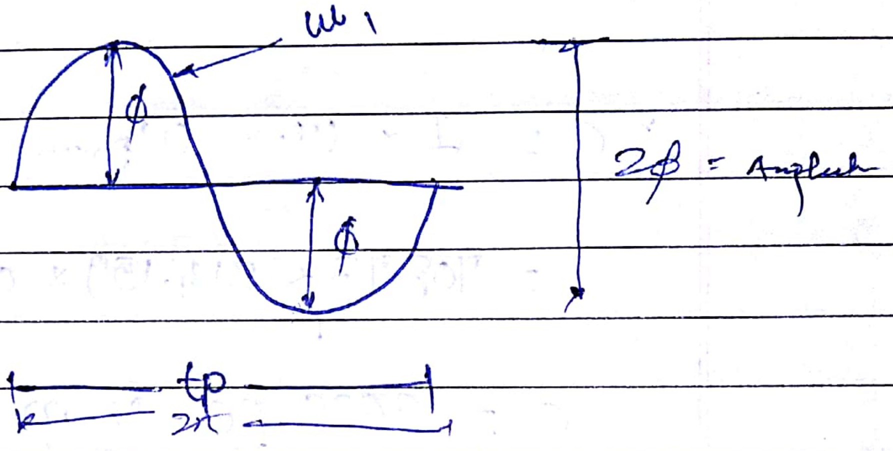
effect Left stron (clockwise).

Bow - up
& stern - down.

(11)



$t_p = 40$ sec. - - - given

Total angular moment $= 2\phi = 12°$

$\therefore \phi = 6°$

$\therefore \phi = 6 \times \pi/180$

$\phi = 0.104$ rad

$C = I \times \omega \times \omega_{p\,max}$

$\omega_1 = \frac{2\pi}{t_p}$

$= \frac{2\pi}{40}$

$\omega_1 = 0.157$ rad/s

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$\omega_{p\,max} = \phi \times \omega_1$

$= 0.104 \times 0.157$

$\boxed{\omega_{p\,max} = 0.0163}$

$\therefore C = I \times \omega \times \omega_{p\,max}$

$= 708.75 \times 314.159 \times 0.0163$

$C = 3629.36 \text{ N-m}$

$\boxed{C = 3.62 \text{ KN-m}}$

Effect: ship moves port side,

(III) Rolling

$\omega_p = 0.03 \text{ rad/s}$ --- given

$C = I \times \omega \times \omega_p$

$= 708.75 \times 314.159 \times 0.03$

$= 6679.80 \text{ N-m}$

$\boxed{C = 6.67 \text{ KN-m}}$

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Ex (11) Aeroplane at 240 km/hr. turns towards the left & Complete cotter circle 60 m. The mass of rotary engine & propeller amount to amount to 450 kg. The radius of Gyration is 320 mm. Engine speed 2000 rpm. (clockwise) when viewed from the rear end. Determine

(i)* Gyroscopic Couple & its effect on aircraft.

(ii)* In what way the effect change when airoplane turns towards the right.

(iii)* Engine rotates counterclockwise when viewed from rear end. & take left turn.

(iv)* Engine take the right turn.

⇒ Given :-

$V = 240 \text{ km/hr} = \frac{240 \times 10^3}{3600} = 66.66 \text{ m/s}$

$R = 60$ m. $m = 450$ kg. $K = 320$ mm

Left turn. $K = 0.32$ m

$N = 2000$ rpm.

Looking from Rear end.

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2 \times \pi \times 2000}{60}$$

$$\omega = 209.42 \text{ rad/s}$$

$I = mK^2$

$= ~~450 \times (320)^2~~ \quad 450 \times (0.32)^2$

$= 46.08 \text{ kg-m}^2$

~~$\omega_p = \frac{V}{R}$~~

$= \frac{66.66}{60}$

$\boxed{\omega_p = 1.111 \text{ rad/s}}$

Gyroscopic Couple (C)

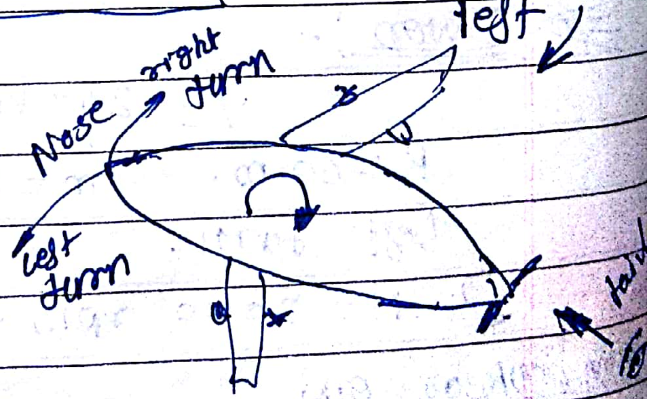
$C = I \times \omega \times \omega_p$

$= 46.08 \times 209.42 \times 1.111$

$= 10721.23 \text{ N-m}$

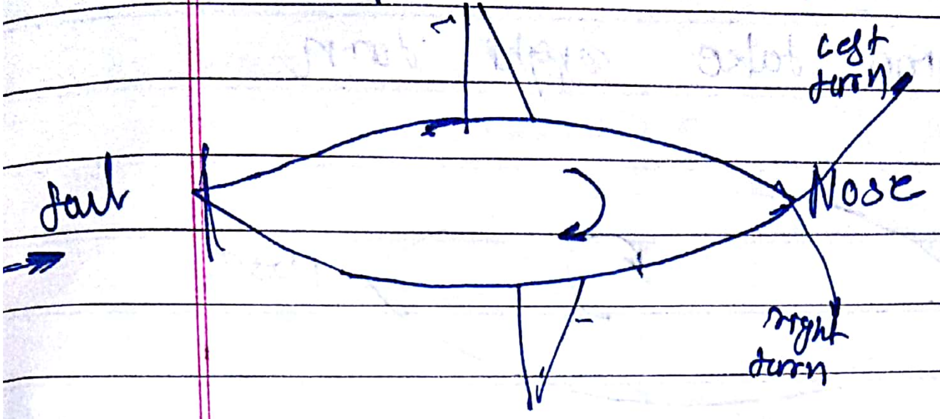
$\boxed{C = 10.72 \text{ KN-m}}$

effect of reactive Gyr~~oscou~~ple	
Nose –	up (Raise)
tail –	dip (down)



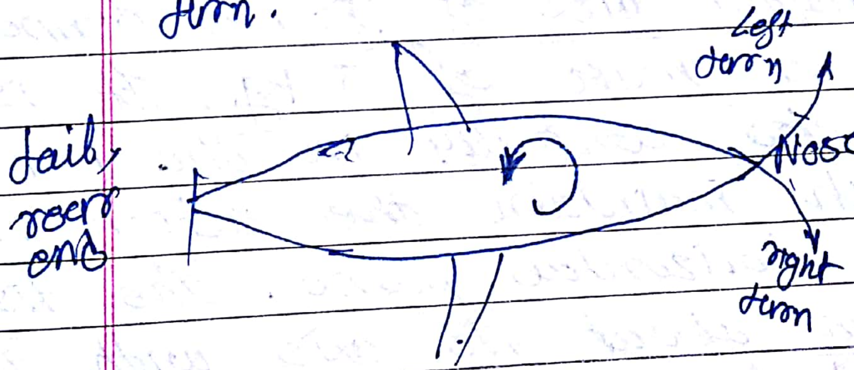
(clockwise)

(II) Aeroplane turns towards the right



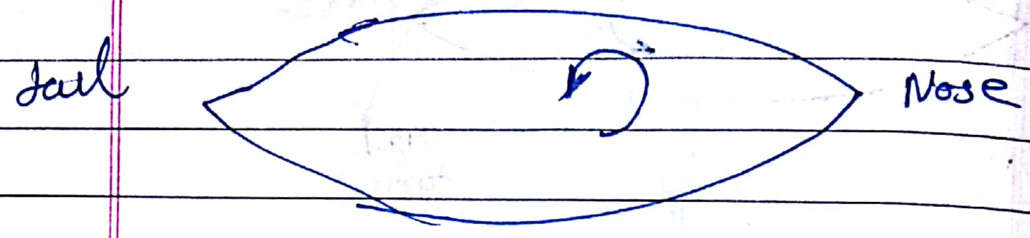
Nose – down
Tail – up

(III) Engine rotates Counterclockwise when viewed from rear end & take left turn.



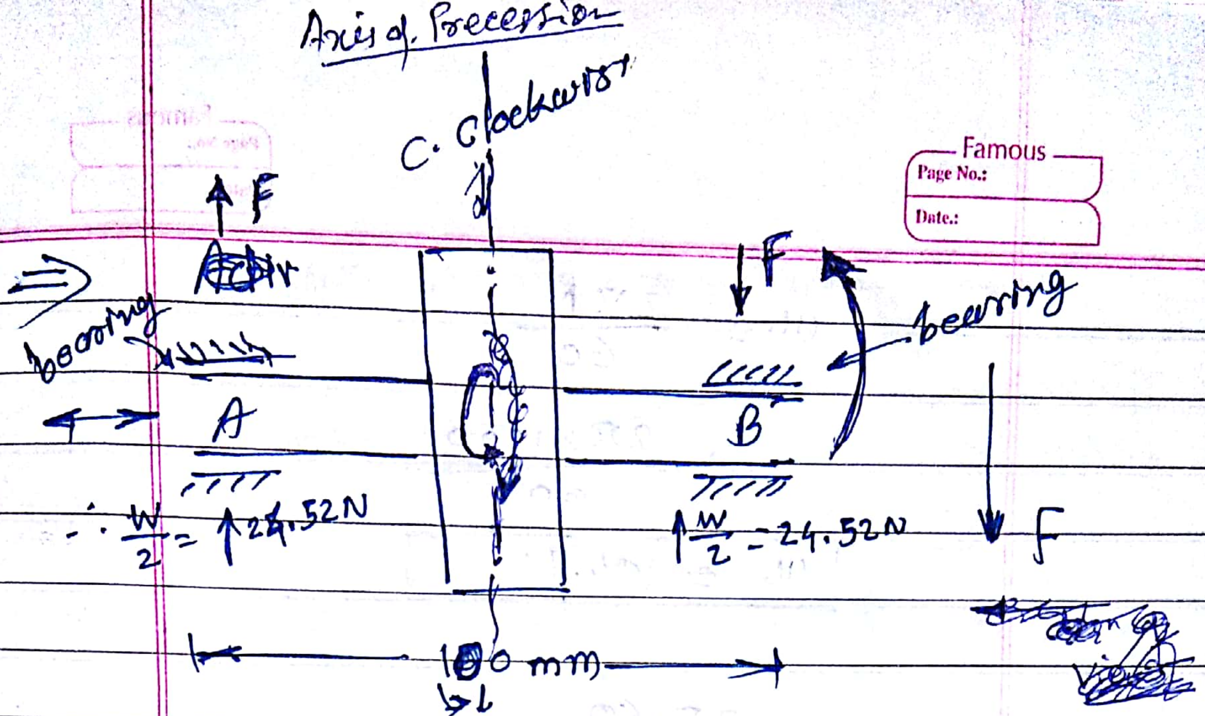
Nose – down
Tail – up

(IV) At Counter clockwise
Engine take right turn.



Nose – up
Tail – down

(III) A uniform disc of 150 mm diameter has a mass of 5 kg. It is mounted centrally in the bearing which maintain the action in the horizontal plane. The disc speen about its axis with the constant velocity of 1000 rpm. The axis of precision is uniformely about vertical axis is 60 rpm. The direction of rotation is shown in figure. The distance betn the bearing is 100 mm. Find the resultant reaction on each bearing due to Gyrocouple.



Active Couple = C = F × l

Reactive Couple = C = I × ω × ωp

d = 150 mm → ∴ R = 75 mm = 0.075 m

m = 5 Kg

weight = W = 5×9.81 N = 1000 rpm.

[W = 49.05 N]

Reactive Gyroscopic Couple

C = I × ω × ωp

$I = mK^2$

$K = \left(\frac{R}{\sqrt{2}}\right)$

$= \left(\frac{0.075}{\sqrt{2}}\right)$

K = ~~2.8125×10⁻³~~ 0.053 m

∴ I = ~~5×(2.8125×10⁻³)²~~ $5\times(0.053)^2$

∴ [I = ~~3.95×10⁻⁵~~ 0.014 kg-m]

$$\omega = \frac{2\pi N}{60}$$

$$= \frac{2\pi \times 1000}{60}$$

$$\boxed{\omega = 104.71}$$

$$\omega_p = \frac{2\pi \times 60}{60}$$

$$\boxed{\omega_p = 6.28 \text{ rad/s}}$$

$\therefore C = I \times \omega \times \omega_p$

$= 0.014 \times 104.71 \times 6.28$

$C = 9.21 \text{ N-m}$

$$\therefore \boxed{C = 9.21 \times 10^{-3} \text{ KN-m}}$$

Now,

Reactive Gyroscopic Couple = Couple applied

$C = f \times d$

$9.21 = F \times 100 \times 10^{-3}$

$$\boxed{F = 92.1 \text{ N}}$$

$F + \frac{W}{2}$

Reaction on bearing

$A = F + 24.5$

$= 92.1 + 24.52$

$= \underline{116.6\ N}$ (↑) upward.

$F - \frac{W}{2}$

Reaction on bearing

$B = F - 24.5$

$= 92.1 - 24.52$

$= \underline{67.6\ N}$ (↓) Downward.

4 Wheeler

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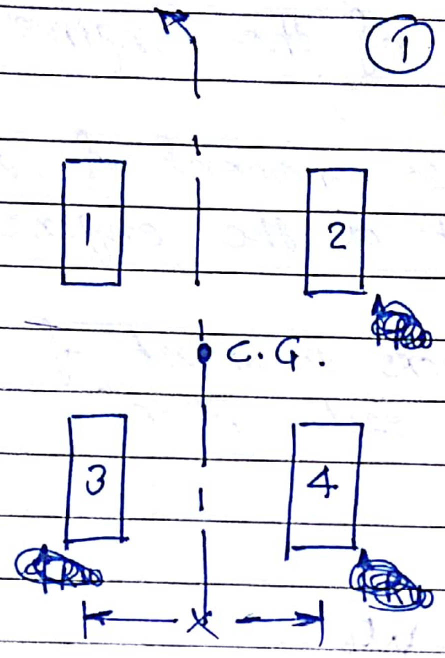
Explanation

Consider the 4 wheeler taking the left turn

① Effect of load (wt) on vehicle



W → total weight of the vehicle
1 & 3 are the inner wheel
2 & 4 are the outer wheel

for left turn

* wheel 1 & 3 become the inner wheel
* wheel 2 & 4 become the outer wheel
* X be the track width.
* C.G. → centre of gravity where whole weight of vehicle is acting.
* h → height of centre of gravity from Road edge.
* r_w → Radius of wheel
* R → Radius of Curvature.
* V → Speed of vehicle.
* ω_p → Angular velocity of precision = $\frac{V}{R}$

* W_w → Angular velocity of wheel.

* W_E → Angular velocity of rotating part of the engine or rotating disc)

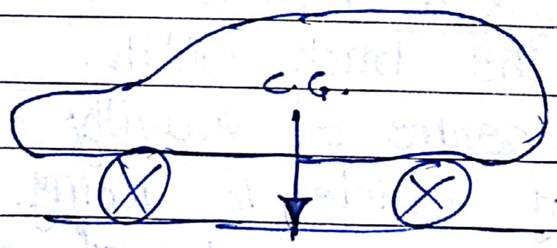
* I_E → Mass moment of rotating part of the engine.

* I_w → Mass moment of inertia of each wheel

* G →

$$G = \frac{W_E}{W_w}$$

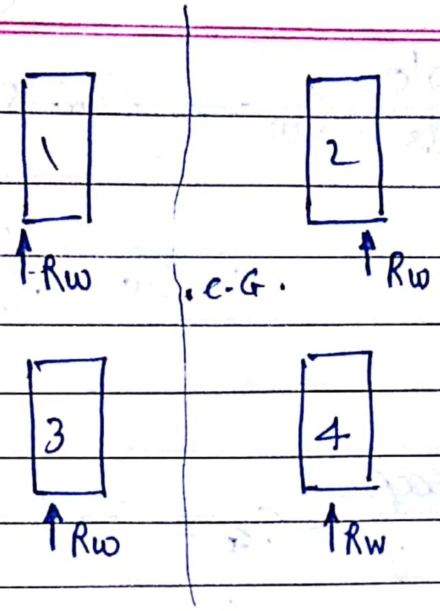
* Effect of weight on vehicle *



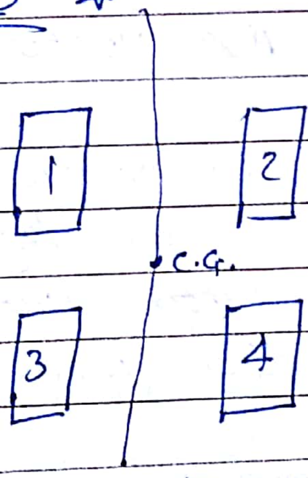
W → Total wt. of the vehicle.

load on each wheel = $\frac{W}{4}$ (↓)

∴ Reaction on each wheel (R_w) = $\frac{W}{4}$ (↑)



* Effect of Gyroscopic Couple on vehicle *



Gyroscopic Couple due to engine $= C_E = I_E \times \omega_E \times \omega_p$

{Rotating Part of Engine}

$$= I_E \times G \times \omega_W \times \frac{V}{R}$$

$G = \frac{\omega_E}{\omega_W}$

$$= I_E \times G \times \frac{V}{r_W} \times \frac{V}{R}$$

Gyroscopic Couple due to 4 wheels $= C_W = 4 \times I_W \times (\omega_W) \times \omega_P$

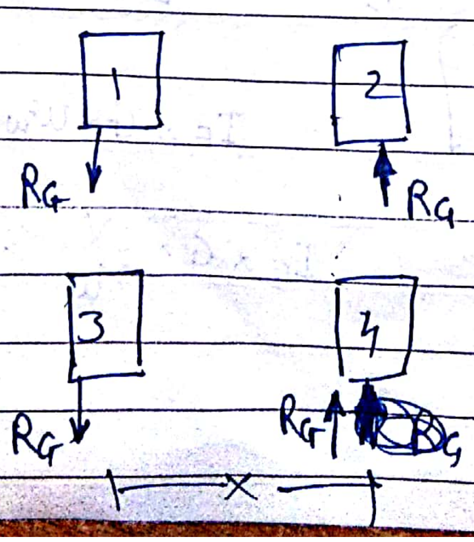
$$= 4 \times I_W \times \frac{v}{r_W} \times \frac{v}{R}$$

∴ Total Gyroscopic Couple $= C_G = C_W \pm C_E$

+ve sign shows that rotating part of engine & vehicle rotates in a same direction & vice versa

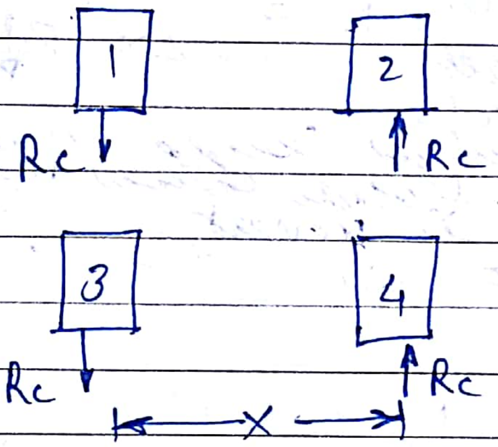
Due to the Gyroscopic Couple (C_G), the inner wheel try to lift while the outer wheel try to press on the ground.

So the reaction on inner wheel is vertically downward.



Reaction on each wheel $= R_G = \dfrac{C_G}{2X}$

* The Effect of Centrifugal Couple or Overturning Couple on vehicle



Since the vehicle moves around the curvature path, the centrifugal force react outward, due to this inner wheel try to lift & the outer wheel try to press on ground.

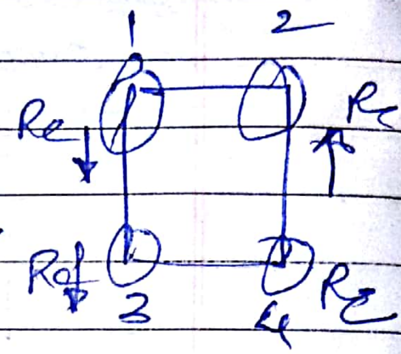
Hence Reaction on inner wheel is vertically downward & Reaction on outer wheel is vertically upward.

$\therefore$ Centrifugal force $= m \times R \times \omega_p^2$

$$= m \times R \times \frac{V^2}{R^2}$$

$$= \frac{mv^2}{R}$$

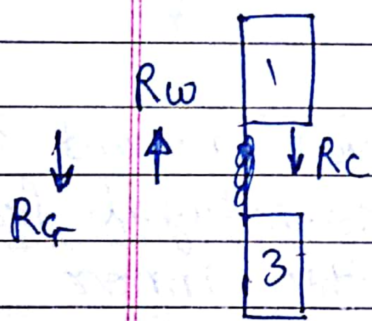
$\therefore$ Centrifugal Couple (Cc) $= \frac{mv^2}{R} \times h$



$\therefore$ Reaction due to Centrifugal force on each wheel $= R_c = \frac{C_c}{2x}$

Due to centrifugal couple inner wheel try to lift & other force press
Hence Reaction is Reverse (inner wheel)

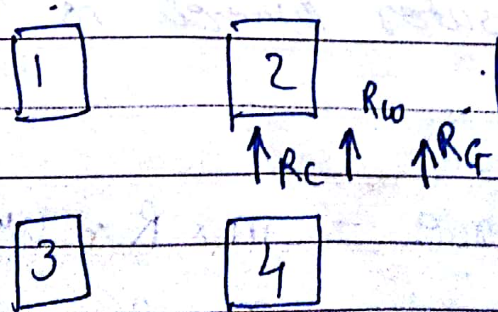
<u>Total Reaction on wheel 1 & wheel 3</u>



$(R_w + R_G - R_c)$

(outer wheel)

Total Reaction on wheel 2 & wheel 4



$R_w + R_c + R_G$

In order to have the Contact of vehicle with the ground on safe side R_w is always greater than or equal to $R_c + R_G$

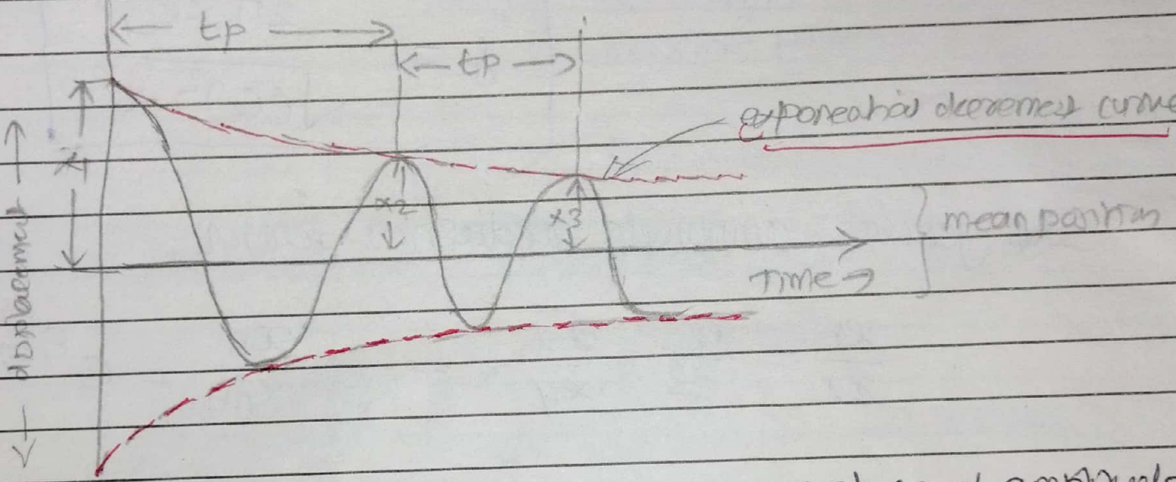
$$\therefore R_w \geq R_c + R_G$$

A new friend, a true friend
A happy-I-met-you friend

* Logarithmic Decrement :

In order to determine the amount of damping present in the system, rate of decay of free oscillation is measured.

Logarithmic decrement is defined as the natural logarithm of the ratio of any two successive amplitude.



If x_1 and x_2 successive values of amplitude on the same side of mean position, as shown in figure.

Logarithmic decrement (δ) is expressed as

$$\delta = \log_e \left(\frac{x_1}{x_2}\right) = \log_e e^{\xi_1 t_p} \quad \text{---(ii)}$$

amplitude reduction factor is expressed as

$$\frac{x_1}{x_2} = e^{\xi_1 t_p} = \text{constant} \quad \text{------(i)}$$

take log both sides

$$\therefore \quad \delta = \log_e \left(\frac{x_1}{x_2}\right) = \xi_1 t_p = \xi_1 \times \frac{2\pi}{\omega_d} \qquad T_d = \frac{2\pi}{\omega_d}$$

$(\because \log_e e = 1)$

$$= \xi_1 \times \frac{2\pi}{\sqrt{(\omega_n)^2 - \xi_1^2}} \qquad \omega_d = \sqrt{\omega_n^2 - \xi_1^2}$$

$$\therefore \delta = \frac{\frac{c}{2m} \times 2\pi}{\sqrt{(\omega_n)^2 - (c/2m)^2}} \quad \Big| \quad \xi = \frac{c}{2m} \text{ put}$$

$$= \frac{\frac{c}{2m} \times 2\pi}{\omega_n \sqrt{1 - \left(\frac{c}{2m\omega_n}\right)^2}} = \frac{c \times 2\pi}{c_c \sqrt{1 - (c/c_c)^2}} \quad \Big| \quad c_c = 2m\omega_n$$

$c_c = 2m\omega_n$

$$\boxed{\delta = \frac{2\pi \times c}{\sqrt{(c_c)^2 - c^2}}}$$

In general, amplitude reduction factor,

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \cdots = \frac{x_n}{x_{n+1}} = e^{\xi t_p} = \text{constant}$$

$$\therefore \text{Logarithmic decrement, } (\delta) = \log_e\left(\frac{x_n}{x_{n+1}}\right) = \frac{2\pi \times c}{\sqrt{(c_c)^2 - c^2}}$$

Ex: A vibrating system consists of a mass of 200 kg, a spring of stiffness 80 N/mm, and a damper with damping coefficient of 800 N/m/s. Determine the frequency of vibration of the system.

Soln => Given data:

$m = 200$ kg

$K = 80$ N/mm $= 80 \times 10^3$ N/m.

$c = 800$ N/m/s.

circular frequency of undamped vibrations,

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{80\times10^3}{200}} = 20 \text{ rad/sec.}$$

circular frequency of damped vibrations

$$\omega_d = \sqrt{(\omega_n)^2 - \varepsilon^2}$$

we know $\varepsilon = \frac{c}{2m} = \frac{800}{2\times200} = 2$

$$\omega_d = \sqrt{(20)^2-(2)^2} = 19.89 \text{ rad/sec}$$

∴ frequency of vibration of the system

$$f_d = \frac{\omega_d}{2\pi} = \frac{19.89}{2\times3.14} = 3.17 \text{ Hz}$$

Ex: An instrument vibrates with a frequency of 1 [Hz when]
there is no damping. When the damping is p[rovided]
the frequency of the damped vibrations was obs[erved]
to be 0.9 Hz.

Find: i) the damping factor

ii) logarithmic decrement (δ).

Soln: Given data:

$f_n = 1\,Hz$,

$f_d = 0.9\,Hz$,

C - damping coeff. N/m/s,
C_c = critical damping coeff. N/m/s,

Damping factor: formulae used:

$\omega_n = 2\pi \times f_n$ $\quad a = C/2m$,

$\omega_d = 2\pi \times f_d$ $\quad D.F = C/C_c$

$\omega_d = \sqrt{(\omega_n)^2 - a^2}$

$\delta = \dfrac{2\pi C}{\sqrt{(C_c)^2 - C^2}}$ $\quad C_c = 2m.\omega_n$.

* (Natural circular frequency of undamped vibrations)

$\omega_n = 2\pi \times f_n = 2\pi \times 1 = 6.28$ rad/sec,

* $\omega_d = 2\pi \times f_d = 2\pi \times 0.9 = 5.65$ rad/sec,

* we also know, circular freq of damped vibration

$\omega_d = \sqrt{(\omega_n)^2 - a^2}$

$5.65 = \sqrt{(6.28)^2 - a^2}$ square both sides

$(5.65)^2 = (6.28)^2 - a^2$

~~$31.92 = 6.28 - a^2$~~

$31.92 = 39.44 - a^2$

$\xi^2 = 7.52 \Rightarrow$ $\boxed{\xi = 2.742}$

* we know $\xi = \frac{c}{2m}$

$2.742 = \frac{c}{2m} \Rightarrow \boxed{c = 5.484m}$ N/m/s

damping coeff.

* critical damping coefficient (C_c) $= 2m\omega_n$

$= 2m \times 6.28$

$= 12.56m$ N/m/s.

* Damping ratio $= \frac{c}{c_c} = \frac{5.484m}{12.56m} = 0.436$ Ans

* Logarithmic decrement (δ):

$$\delta = \frac{2\pi . c}{\sqrt{(C_c)^2 - c^2}}$$

$$= \frac{2 \times 3.14 \times 5.484}{\sqrt{(12.56)^2 - (5.484)^2}}$$

$\boxed{\delta = 3.047}$ Ans

Ex: The following data are given for a vibratory system with viscous damping

mass = 2.5 kg, spring constant = 3 N/mm, amplitude decreases to 0.25 of the initial value after 5 consecutive cycles

Determine the damping coeff. of damper in the system.

Given data : mass (m) = 2.5 kg

spring constant mean stiffness (k) = 3 N/mm
= 3000 N/m

initial amplitude = x_1

final amplitude after 5 consecutive cycle

i.e $x_{1+5} = x_6$.

i.e., $x_6 = 0.25 x_1$

Formulae :

$$\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right) = \xi \cdot t_p = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}}$$

$$\omega_d = \sqrt{(\omega_n)^2 - \xi^2}, \quad \xi = \frac{c}{2m}, \quad \omega_n = \sqrt{\frac{k}{m}}.$$

$$t_p = \frac{2\pi}{\omega_d}.$$

To find :- damping coefficient (c)

* we know,

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{3000}{2.5}} = 34.641 \text{ rad/sec}.$$

* we know

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6}.$$

$$\frac{x_1}{x_6} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6} = \left(\frac{x_1}{x_2}\right)^5$$

or $\frac{x_1}{x_2} = \left(\frac{x_1}{x_6}\right)^{1/5} = \left(\frac{x_1}{0.25 x_1}\right)^{1/5}$

$$= (4)^{1/5} = 1.319$$

we know, $\log_e\left(\frac{x_1}{x_2}\right) = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}} = \xi \cdot t_p = \xi \cdot \frac{2\pi}{\omega_d}$

$$\log_e\left(\frac{x_1}{x_2}\right) = \xi_1 \cdot \frac{2\pi}{\sqrt{(\omega_n)^2 - \xi_1^2}}$$

$$\therefore \log_e(1.319) = \frac{\xi_1 \times 2 \times 3.14}{\sqrt{(34.641)^2 - \xi_1^2}}$$

$$0.2776 = \frac{6.28\xi_1}{\sqrt{1199.95 - \xi_1^2}}$$ square both sides

$$0.077 \times (1199.95 - \xi_1^2) = 39.438\,\xi_1^2$$

$$92.39 - 0.077\,\xi_1^2 = 39.438\,\xi_1^2$$

$$\xi_1^2 = \frac{92.39}{39.515}$$

$$\therefore \boxed{\xi_1 = 1.529}$$

* We know, $\xi_1 = \frac{c}{2m}$

$$1.529 = \frac{c}{2 \times 2.5} \Rightarrow \boxed{c = 7.645 \text{ N/m/sec}}$$

Ex: A shock absorber is to be designed for a vehicle of mass 500 kg such that during the road bump, the damped period of vibrations is 3 sec. and amplitude of vibration reduces to 1/10 in one cycle.

Find: i) spring stiffness
ii) Damping coefficient of shock absorber.

Soln ⇒ Given data: $m = 500$ kg.
$T = 3$ sec

x_1 = initial amplitude, x_2 = final amplitude because the

the cycle is one, amplitude reduces to 1/10.

i.e $x_2 = \frac{1}{10} x_1$ ∴ ~~$\frac{x_1}{x_2} = 10$~~ or $\frac{x_1}{x_2} = 10$

Formulae: $\omega = \frac{2\pi}{T}$, $\omega_n \Rightarrow$ natural circ freq
ω_d → damped circu-freq

$T_p = \frac{1}{f_n}$

$\varepsilon = \frac{c}{2m}$, $\omega_d = \sqrt{(\omega_n)^2 - \varepsilon^2}$

δ ~~ε~~ $= \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}}$

$c_c = 2m \cdot \omega_n$.

damped freq (circular):

* $\omega_d = \frac{2\pi}{T} = \frac{2 \times 3.14}{3} = 2.093$ rad/sec.

* damped ~~Natural~~ circular freq. : $\omega_d = \sqrt{(\omega_n)^2 - \varepsilon^2}$.

* we also know,

$\delta = \log_e\left(\frac{x_1}{x_2}\right)$

$\delta = \log_e(10)$

$\boxed{\delta = 2.302}$

* $\delta = \varepsilon t_p$ put the values

$2.302 = \varepsilon \times 3$ ∴ $\boxed{\varepsilon = 0.767}$

* $\omega_d = \sqrt{(\omega_n)^2 - \varepsilon^2} \Rightarrow 2.093 = \sqrt{(\omega_n)^2 - (0.767)^2}$ squaring

$4.381 = \omega_n^2 - 0.588$

$\therefore \omega_n^2 = 4.969 \Rightarrow \boxed{\omega_n = 2.229 \text{ rad/sec}}$

* $\omega_n = \sqrt{\frac{k}{m}}$

$2.229 = \sqrt{\frac{k}{500}}$ squaring

$4.968 = \frac{k}{500} \Rightarrow \boxed{k = 2484.22 \text{ N/m}}$ Ans

* Damping factor $= \frac{c}{c_c}$

$\xi = \frac{c}{2m} \Rightarrow 0.767 = \frac{c}{2 \times 500} \Rightarrow \boxed{c = 767 \text{ N/m/sec}}$ Ans

$c_c = 2m\omega_n = 2 \times 500 \times 2.229 = 2229$ N/m/sec

$D.F = c/c_c = \frac{767}{2229} = 0.344$

Q: A vibrating system consists of a mass of 50 kg, a spring of stiffness 30 kN/m, and a damper. The damping provided is 20% of the critical value.

Determine: i) the damping factor
ii) the critical damping coefficient.
iii) the natural frequency of a damped vibration.
iv) the ratio of two consecutive amplitudes.

soln Given data: mass (m) = 50 kg
stiffness (k) = 30,000 N/m,

damping (C) = 20% C_c = 0.2 C_c

so damping ratio = $\frac{C}{C_c} = \frac{0.2 C_c}{C_c}$ = 0.2

* circular freq of undamped vibration,

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$= \sqrt{\frac{30000}{50}} = 24.494 \text{ rad/sec.}$$

* critical damping coefficient (C_c) = $2m \cdot \omega_n$

$$C_c = 2 \times 50 \times 24.494 = ~~2049.4~~$$
$$= 2449.4 \text{ N/m/s.}$$

* $C = 0.2 C_c = 0.2 \times$ ~~2049.4~~ 2449.4 = ~~409.88~~
$= 489.90$ N/m/se

Ans

* $\omega_d = \sqrt{(\omega_n)^2 - a^2}$

here, $a = \frac{c}{2m}$

$$= \frac{489.90}{2 \times 50}$$

$$= 4.899$$

$$\therefore \omega_d = \sqrt{(24.494)^2 - (4.899)^2}$$

$$\omega_d = 23.999 \text{ rad/sec}$$ Ans.

* we know, $\delta = \log_e\left(\frac{x_n}{x_{n1}}\right) = \frac{2\pi C}{\sqrt{(C_c)^2 - C^2}}$

$$= \frac{2 \times 3.14 \times 489.90}{\sqrt{(2449.4)^2 - (489.90)^2}}$$

$$= 1.281$$

$\delta = 1.281$

but, $\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right)$

$1.281 = \log_e\left(\frac{x_n}{x_{n+1}}\right)$

$\therefore \boxed{\frac{x_n}{x_{n+1}} = 3.6036}$ Ans

S198 (CS-15):

Same Ex: A mass of 1 kg is to be supported on a spring having constant of K = 9800 N/m. The damping coefficient is 4.9 N/m/sec. Determine the damped natural frequency of the system.
Find also the logarithmic decrement and amplitude of vibration after 8th cycle if, the initial displacement is 0.4 cm.

Soln =>
Given data: ->

mass (m) = 1 kg.
stiffness of spring (K) = 9800 N/m.

damping coefficient (c) = 4.9 N/m/sec

amplitude of 1st cycle = x_1 = 0.004 m.
amplitude of 8th cycle after = x_{8+1} = x_9.

formulae:

$\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right) = a \cdot t_p = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}}$

$\omega_d = \sqrt{(\omega_n)^2 - a^2}$, $a = \frac{c}{2m}$, $\omega_n = \sqrt{\frac{k}{m}}$

Damping factor = c/c_c. $c_c = 2m\omega_n$

To find : δ, amplitude x_9 after 8th cycle

* $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{9800}{1}} = 98.994$ rad/sec.

* critical damping coefficient $(C_c) = 2m.\omega_n$

$= 2 \times 1 \times 98.994$

$= 197.99$ N/m/sec.

* damping factor $= \frac{C}{C_c} = \frac{4.9}{197.99}$

$= 0.0247$

* we also know, $\varepsilon = \frac{C}{2m}$

$= \frac{4.9}{2 \times 1} = 2.45$

* log. decrement $(\delta) = \frac{2\pi C}{\sqrt{(C_c)^2 - (C)^2}}$

$= \frac{2 \times 3.14 \times 4.9}{\sqrt{(197.99)^2 - (4.9)^2}}$

$\boxed{\delta = 0.155}$ Ans

* we know, $\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5} = \frac{x_5}{x_6} = \frac{x_6}{x_7} = \frac{x_7}{x_8} = \frac{x_8}{x_9}$

$\therefore \frac{x_1}{x_9} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5} \times \frac{x_5}{x_6} \times \frac{x_6}{x_7} \times \frac{x_7}{x_8} \times \frac{x_8}{x_9} = \left(\frac{x_1}{x_2}\right)^8$

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or $\frac{x_1}{x_2} = \left(\frac{x_1}{x_9}\right)^{1/8}$ here $x_1 = 0.004$ m

$= \left(\frac{0.004}{x_9}\right)^{1/8}$ x_9 is unknown.

$= \left(\frac{0.004}{x_9}\right)^{0.125}$

* we also know. $\delta = \log_e \frac{x_1}{x_2}$

$0.155 = \log_e \frac{0.004}{x_2}$ | take antiloge.

$1.167 = \frac{0.004}{x_2}$

$\frac{0.004}{0.00342} = \left(\frac{x_1}{x_9}\right)^{1/8}$

$\therefore x_2 = 0.00342$

$\therefore 1.169 = \left(\frac{0.004}{x_9}\right)^{0.125}$ take loge both sides

$0.1566 = 0.125 \log_e \left(\frac{0.004}{x_9}\right)$

$\log_e \left(\frac{0.004}{x_9}\right) = 1.2532$

$\therefore \frac{0.004}{x_9} = 3.5016$

$\therefore x_9 = 0.001142$ m

amplitude after 8th cycle is $\Rightarrow$ or $x_9 = 0.1142$ cm

Ans

Ex: A body weighing 25 kg is supported by a spring

W/03: of 140 N/cm, the assembly incorporates a dashpot which causes the amplitude to reduce to one-eighth of its original value after three complete oscillations,

Determine the following: -

① The Nat. freq of und. vibrations

② The Nat. freq of damped vibrations

③ The coefficient of damping

④ The damping ratio

⑤ The time req. for the amplitude to decay to less than 1000 of its original value.

Soln ⇒ Given data:

mass of body (m) = 25 kg

Stiffness (k) = 140 N/cm,

amplitude of first cycle = x_1

$x_4 = \frac{1}{8} x_1$ → amplitude of fourth cycle = x_4

$\therefore x_1 = 8x_4 \Rightarrow \frac{x_1}{x_4} = 8$ ✓

To find: wn, wd, Coeff. of damp, damping factor, time.

Formulae: $w_n = \sqrt{k/m}$, $w_d = \sqrt{(w_n)^2 - a^2}$, $a = c/2m$,

$T_p = 2\pi/w_d$, $\delta = \log_e\left(\frac{x_n}{x_{n+1}}\right)$

$C_c = 2mw_n$ $= \frac{2\pi c}{\sqrt{(C_c)^2 - (c)^2}}$

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Nat. freq of unda. vibrations (ω_n):

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{14000}{25}}$$

$$\boxed{\omega_n = 23.664 \text{ rad/sec.}}$$ Ans

$$\omega_n = 2\pi f_n \Rightarrow 23.664 = 2\pi \times f_n$$

$$\therefore \boxed{f_n = 3.768 \text{ Hz}}$$ Ans

* we know $C_c = 2m.\omega_n$

$$= 2 \times 25 \times 23.664$$

$$= 118.32 \text{ N/m/s.}$$

$$\boxed{C_c = 118.32 \text{ N/m/s.}}$$

* Amplitude is given as (3 oscillations complete)

$$\frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4}$$

$$\therefore \frac{x_1}{x_4} = \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} = \left(\frac{x_1}{x_2}\right)^3$$

$$\therefore \left(\frac{x_1}{x_2}\right) = \left(\frac{x_1}{x_4}\right)^{1/3}$$

$$= (8)^{1/3}$$

$$= 2.$$

* we also know,

log. decrement, $\delta = \log_e \frac{x_1}{x_2} = \log_e 2 = 0.693.$

$$\boxed{\delta = 0.693}$$

$$\delta = \frac{2\pi c}{\sqrt{(c_c)^2 - c^2}} \Rightarrow 0.693 = \frac{2 \times 3.14 \times c}{\sqrt{(118.32)^2 - c^2}}$$

squaring

$$(0.693)^2 \times ((118.32)^2 - c^2) = 39.438 \times c^2$$

~~$6723.30 - 0.480c^2 = 39.438c^2$~~

~~$6723.30 = 39.918c^2$~~

$$0.480 \times 118.32 - 0.480c^2 = 39.438c^2$$ ~~$\therefore c = 12.978$~~

$$\therefore 39.918c^2 = 56.793$$

$$\therefore c =$$

~~* $\xi = \frac{c}{2m} = \frac{12.978}{2 \times 25} =$~~

$$0.693\sqrt{(118.32 - c^2)} = 6.28c$$

$$\sqrt{118.32 - c^2} = 9.062c \quad \text{squaring both sides}$$

$$(118.32)^2 - c^2 = 82.120c^2$$

~~* $\xi = \frac{c}{2m} = \frac{1.192}{2 \times 20}$~~ ~~$\therefore c = 1.192$~~ ~~Please check~~

~~$\xi = 0.0238$~~

* damping ratio $= \frac{c}{c_c} = \frac{1.192}{118.32} = 0.010$

~~* Time period $(T_p) = \frac{2\pi}{\omega_d}$~~

$$14000 = 83.120c^2$$ ~~$\omega_d = \sqrt{\omega_n^2 - \xi^2}$~~

~~$= \sqrt{(23.664)^2 -}$~~

~~$\therefore c = 12.978$~~

$$c = 129.78$$

* damping ratio $= \frac{c}{c_c} = \frac{12.978}{118.32}$

damping ratio = 0.109 Ans | $\xi_1 = c/2m$

* $\omega_d = \sqrt{\omega_n^2 - \xi_1^2} = \sqrt{(23.664)^2 - \left(\frac{12.978}{2\times 25}\right)^2}$

$\omega_d = 23.59$ rad/sec Ans

* we also know $\omega_d = 2\pi f_d$

$\therefore f_d = \frac{23.59}{6.28}$

$f_d = 3.756$ Hz Ans

* Time req. to decay of amp to 1/1000th of ori. value

i.e., $\frac{x_1}{x_4} = 1000$

we know $\log_e\left(\frac{x_1}{x_4}\right) = \xi_1 . t_p$

$\log_e(1000) = \cancel{0.2595} \times t_p$

$6.907 = \cancel{0.2595} \times t_p$

$\xi_1 = \frac{c}{2m}$ (129.78)
$= \frac{12.978}{2\times 25}$
$= \cancel{0.2595}$.

$\xi_1 = \frac{c}{2m}$
$= \frac{129.78}{50}$
$\xi_1 = 2.59$

$\therefore$ ~~$t_p = 26.62$ sec~~ ~~Ans~~

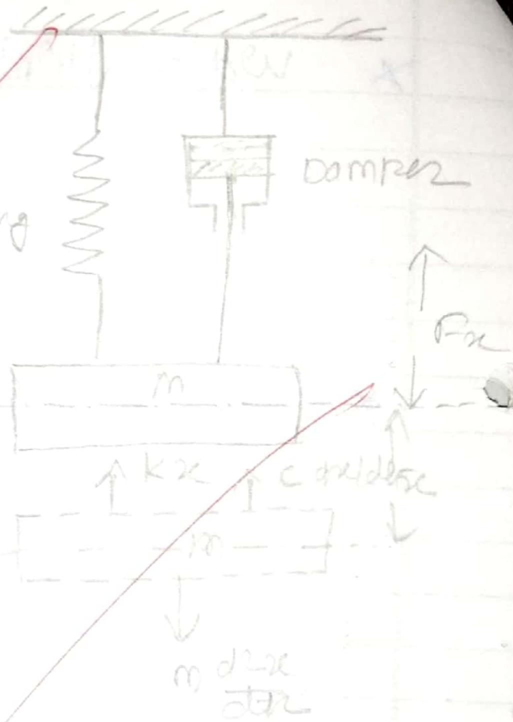
$t_p = 2.68$ sec.

$t_p = 2.661$ sec Ans

* Forced vibration of a single degree of free[dom]

Def^n: When the body vibrates under the influence [of] external force, then the body is said to be un[der] forced vibration.

Consider a body consisting of spring, mass, and damper. Let the system is acted upon by an external periodic force.

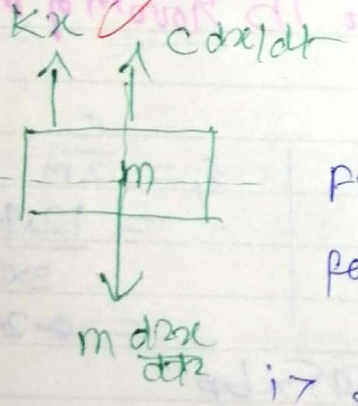


(Also known as simple Harmonic disturbing force.)

$F_x = F \cos \omega t$

here F = static force.

ω = Angular velocity of the periodic disturbing force.



From the free body diagram, forces acting on the body are –

i) Spring force 'kx' in upward direction.

ii) Damping force '$c\frac{dx}{dt}$' in upward dir^n.

iii) Inertia force '$m\frac{d^2x}{dt^2}$' in downward dir^n.

iv) Harmonic force '$F \cos \omega t$'.

After some time 't' the mass is displaced downward ex (Torsional vibration): A shaft of 150 mm diameter and 2 m long has one of its end fixed and the other end carries a disc of mass 500 kgs at a radius of gyration of 450 mm. The modulus of rigidity of shaft material is 80 GN/m². Determine the frequency of torsional vibration?

Sol'n ⇒ Given data:

dia of shaft (d) = 0.15 m
length (l) = 2 m
mass (m) = 500 kgs
radius of gyration (k) = 0.45 m
(C) = 80×10^9 N/m²

To find: frequency (f_n):

formulae: $\frac{T}{\theta} = \frac{CJ}{l}$, $\omega_n = \sqrt{\frac{K_t}{I}}$, $\frac{T}{\theta} = K_t$.

$t_p = \frac{2\pi}{\omega}$, $f_n = \frac{1}{t_p}$

* mass M.I (I) = mk^2
$= 500 \times (0.45)^2 = 101.25$ kg-m²

* $J = \pi/32\, d^4 = \pi/32 (0.15)^4 = 4.967 \times 10^{-5}$ m⁴

* we know, $K_t = \frac{CJ}{l} = \frac{80 \times 10^9 \times 4.967 \times 10^{-6}}{2\text{ m}}$

$= 3.9736 \times 10^6$ N/m.
$= 1.988 \times 10^6$ N/m

* $\omega = \sqrt{\frac{K_t}{I}} = \sqrt{\frac{3.9736 \times 10^6}{101.25}} = 198.10$ rad/sec $= 140.14$ rad/sec