

Unit:-1

- * Introduction to C.S. (7M-Theory) Unit-1 & 2
- * Mathematical modelling

Unit:-2

- * Block diagram reduction Techniques
- * Signal flow graph

Unit:-3

- * Transient Response (7M-T)
- * Steady state Response & error

Unit:-4

- * Mathematical stability (Routh's stability) (7M-Theory)
- OR
- * Root Locus (Graphical) (13M)

Unit:-5

- * Bode plot (Graphical) (13M)
- * Inverse Bode plot "
- * Polar plot (Graphical) 7 & 8 & 13 M

Unit:-6

- * State space Analysis (7M-T)

Introduction TO Control System:-

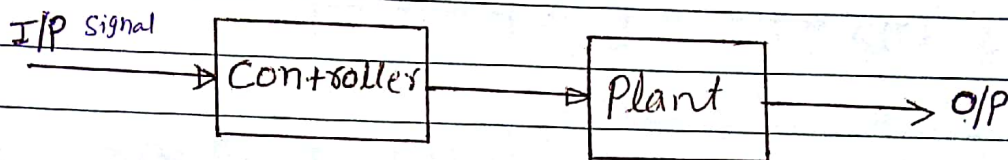
Control System = Control + System

* Control System:-

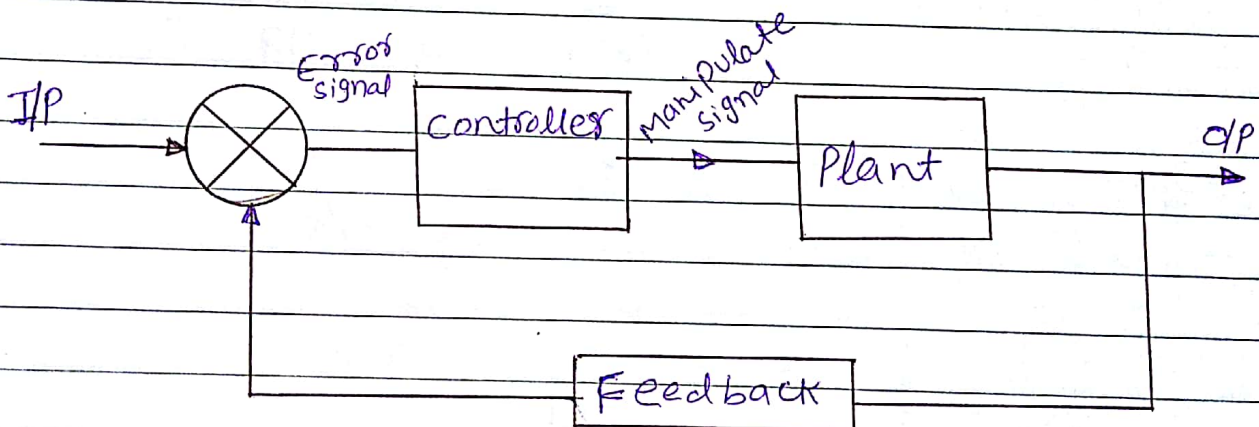
It is an arrangement of different physical element connected in such a manner so as to regulate direct or command it self to achieve a certain objective.

* There are two types of C.S.:-

- (i) Open loop C.S.
- (ii) Closed loop C.S.



Open loop

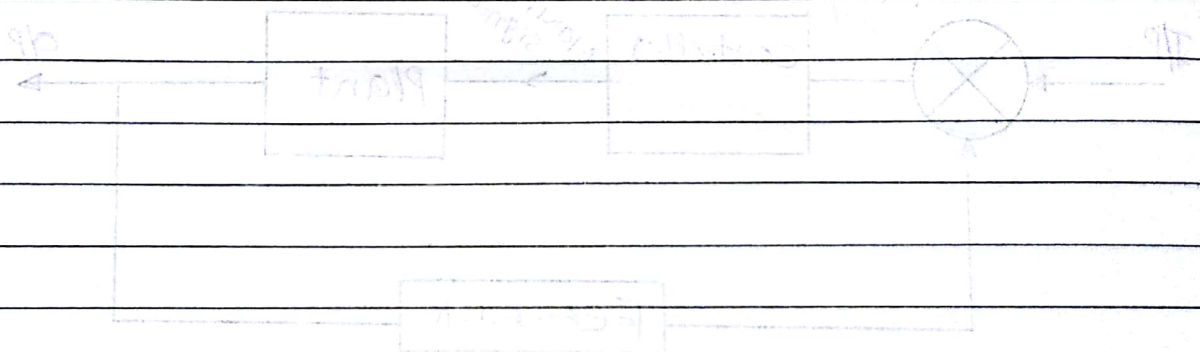
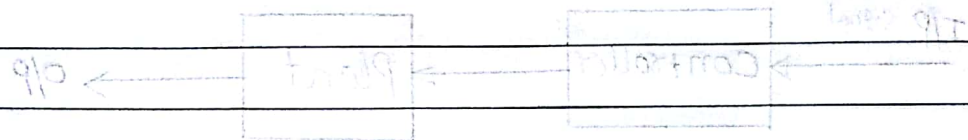


Q- Explain open loop & closed loop c.s.

Q- Compare open loop & closed loop c.s.

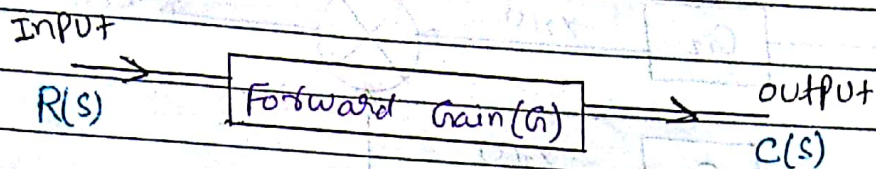
Q- State merits and Demerits of closed loop and open loop c.s.

Q- Explain Briefly any two examples of open loop and closed loop c.s.



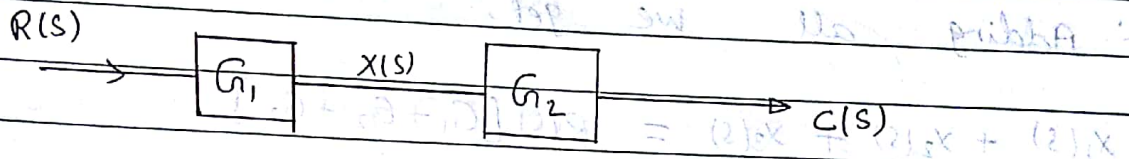
Block diagram Reduction Techniques * Signal Flow graph

A) Block diagram Reduction Techniques:-



$\therefore$ Transfer Function (T.F.) = $\frac{C(s)}{R(s)} = G$

Rule(I):- Series operation



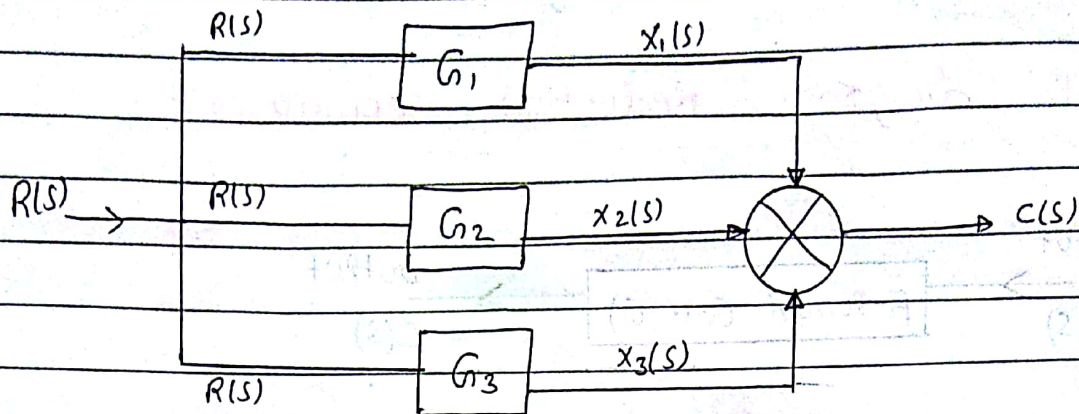
$R(s) G_1 = X(s)$
 $X(s) G_2 = C(s)$

$\therefore R(s) G_1 \cdot G_2 = C(s)$

$\frac{C(s)}{R(s)} = G_1 \cdot G_2$

$\therefore$ $T.F. = \frac{C(s)}{R(s)} = G_1 \cdot G_2$

Rule-(2):- parallel operation



$$X_1(s) = R(s) G_1$$

$$X_2(s) = R(s) G_2$$

$$X_3(s) = R(s) G_3$$

∴ Adding all we get.

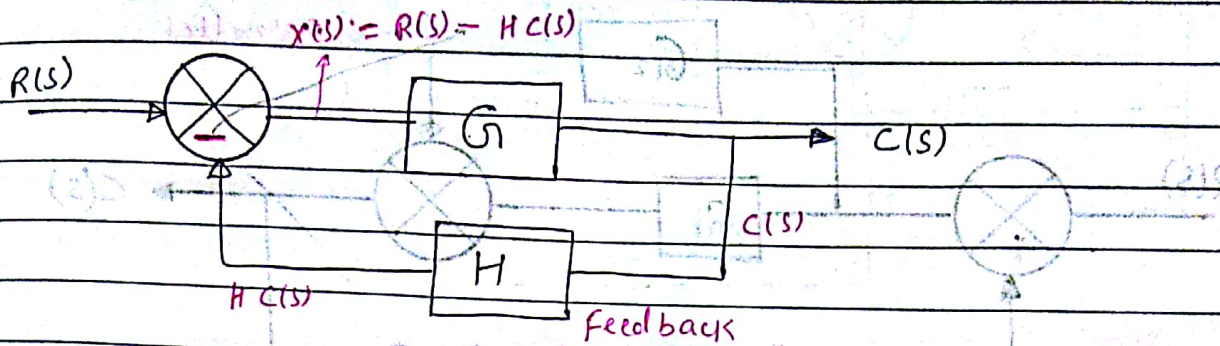
$$X_1(s) + X_2(s) + X_3(s) = R(s) [G_1 + G_2 + G_3]$$

$$∴ C(s) = R(s) [G_1 + G_2 + G_3]$$

$$\frac{C(s)}{R(s)} = G_1 + G_2 + G_3$$

re, $T.F = \frac{C(s)}{R(s)} = G_1 + G_2 + G_3$

✓ Rule-(3) (A) (-ve) Feedback



$$X(s) = R(s) - H C(s)$$

$$X(s) G = C(s)$$

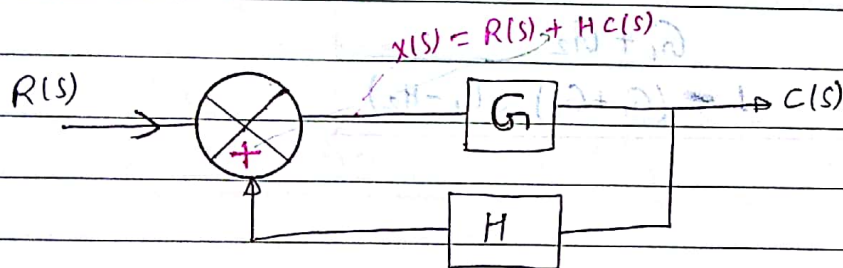
$$[R(s) - H C(s)] G = C(s)$$

$$\therefore G[R(s)] = C(s) + G H C(s)$$

$$G[R(s)] = [1 + G H] C(s)$$

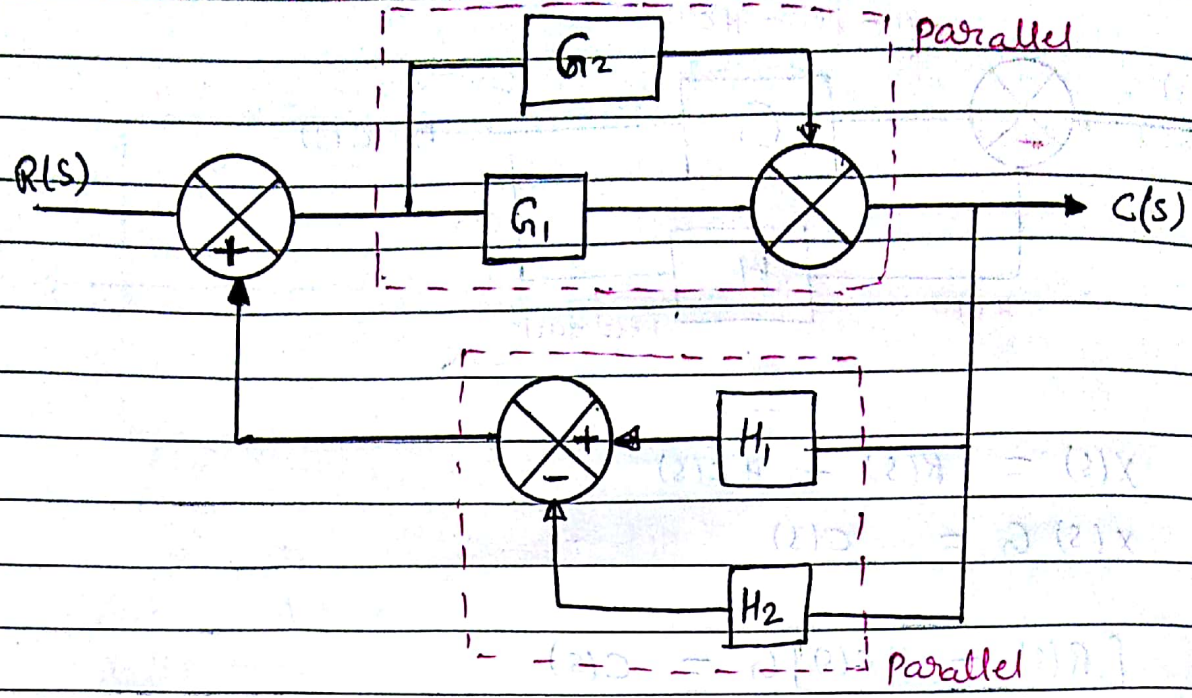
$$T.F. = \frac{C(s)}{R(s)} = \frac{G}{1 + G H}$$

(B) (+ve) Feedback

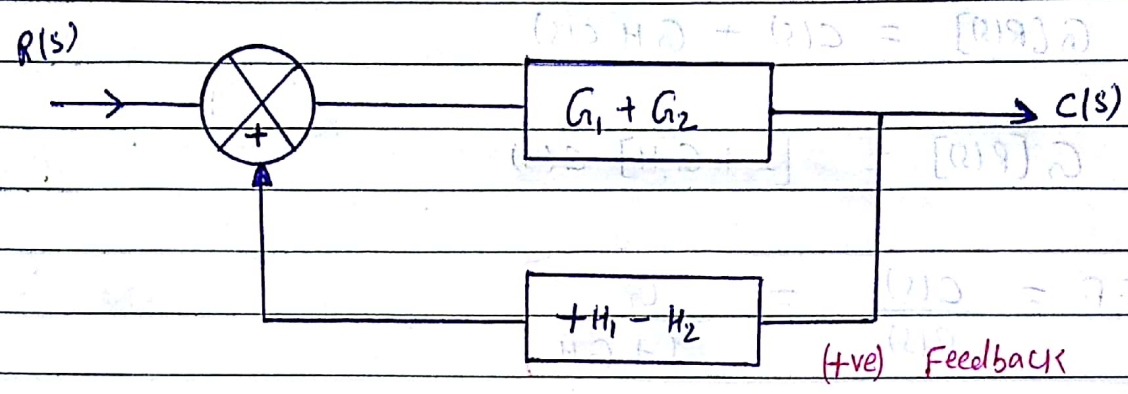


$$T.F. = \frac{G}{1 - G H}$$

Q-1) Find The Transfer function.

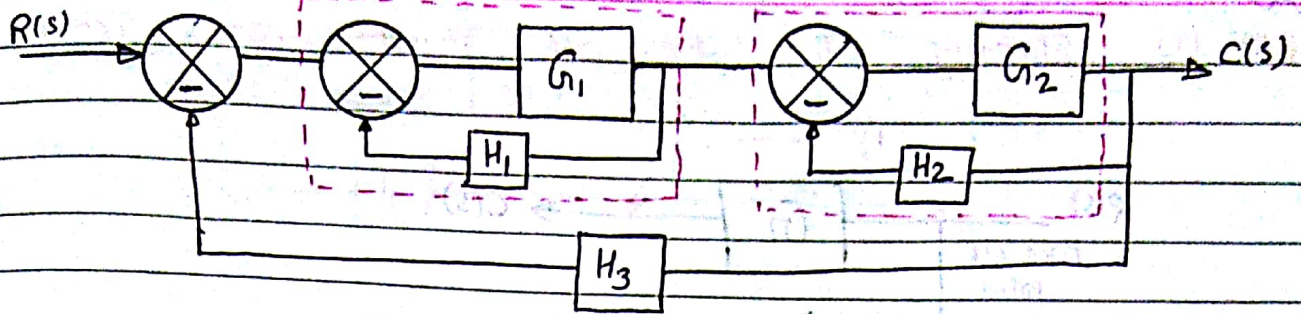


Sol:-

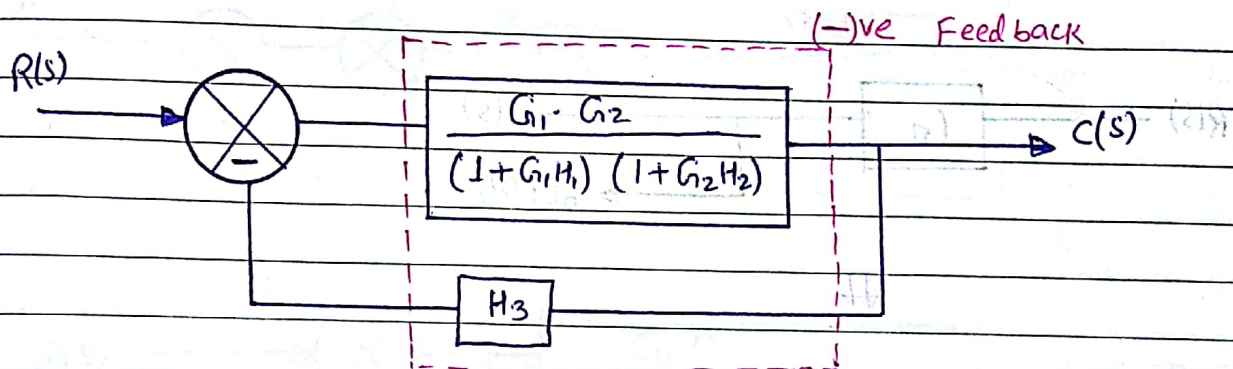
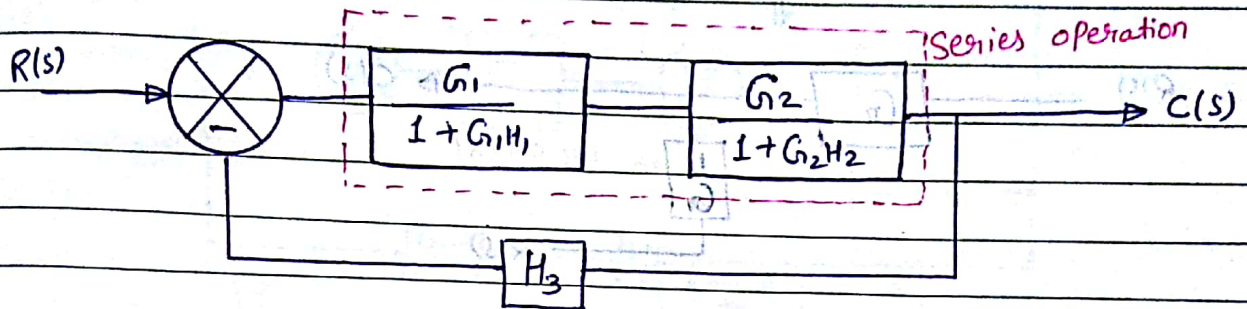


$$T.F. = \frac{C(s)}{R(s)} = \frac{G_1 + G_2}{1 - (G_1 + G_2)(H_1 - H_2)}$$

Q-2

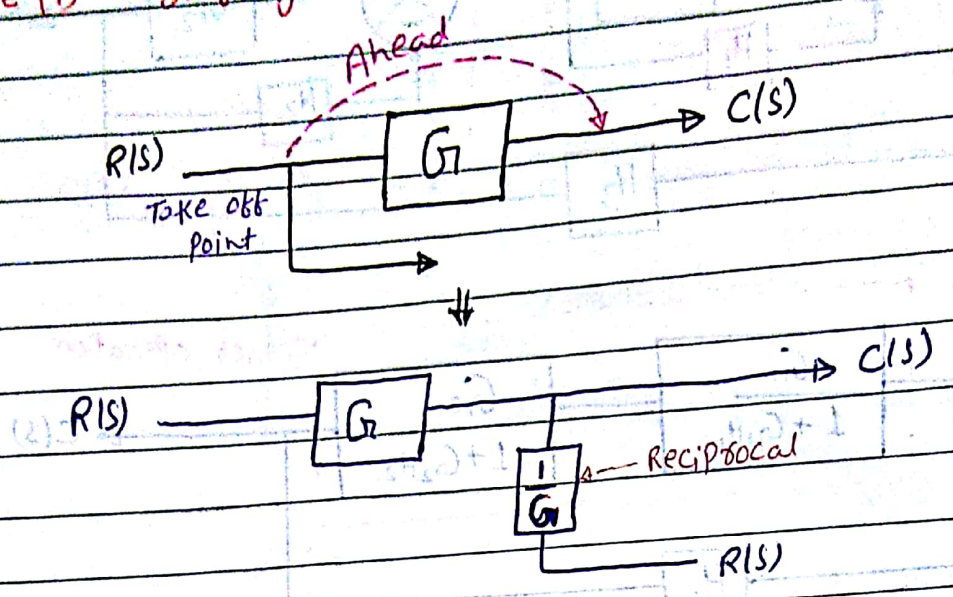


Solⁿ:-

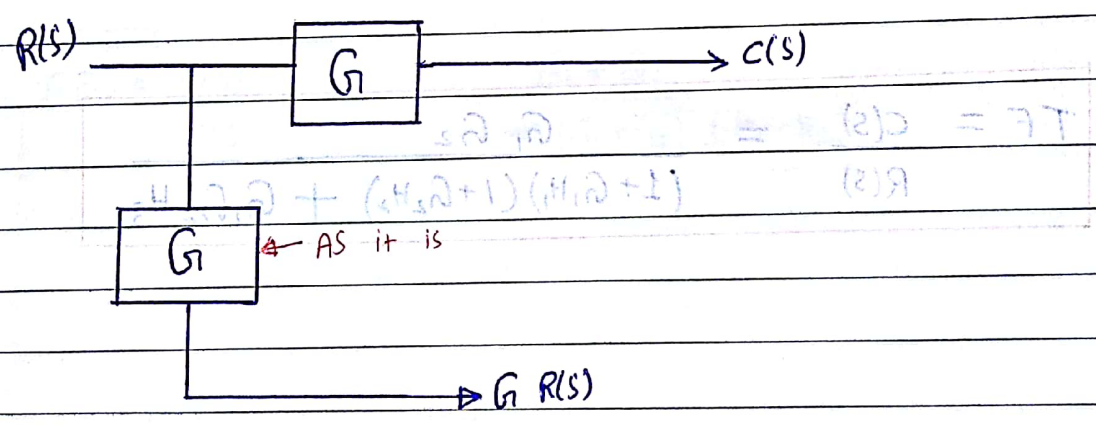
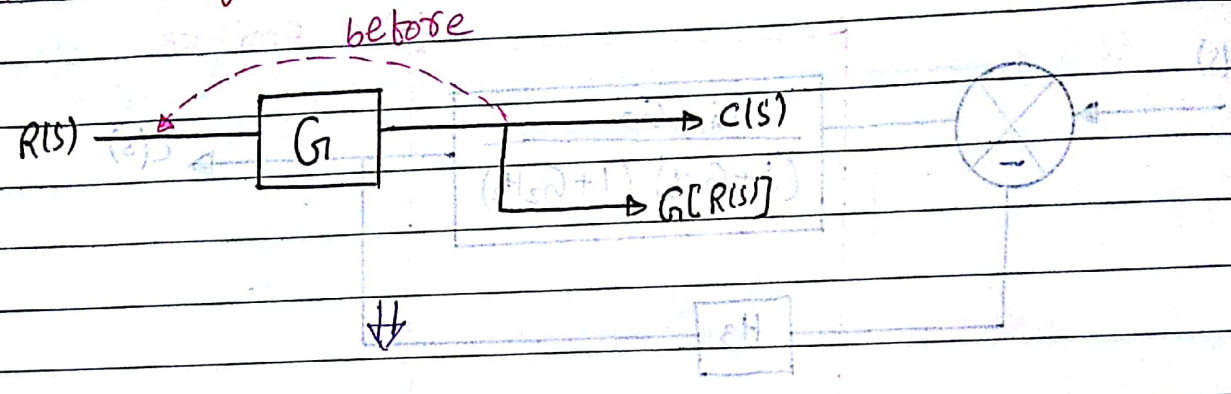


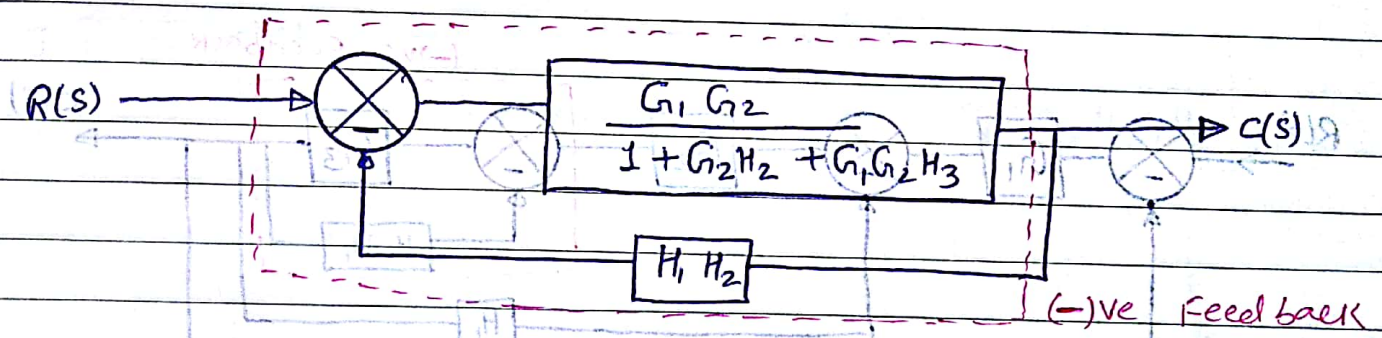
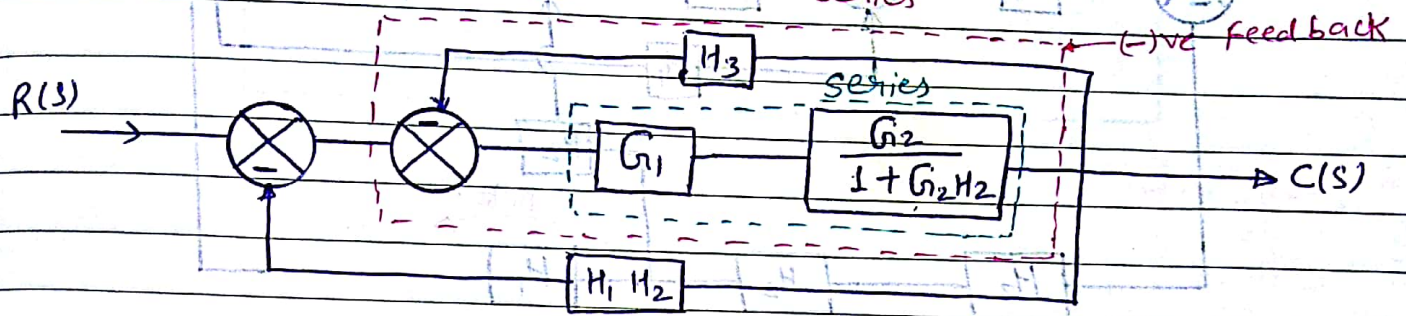
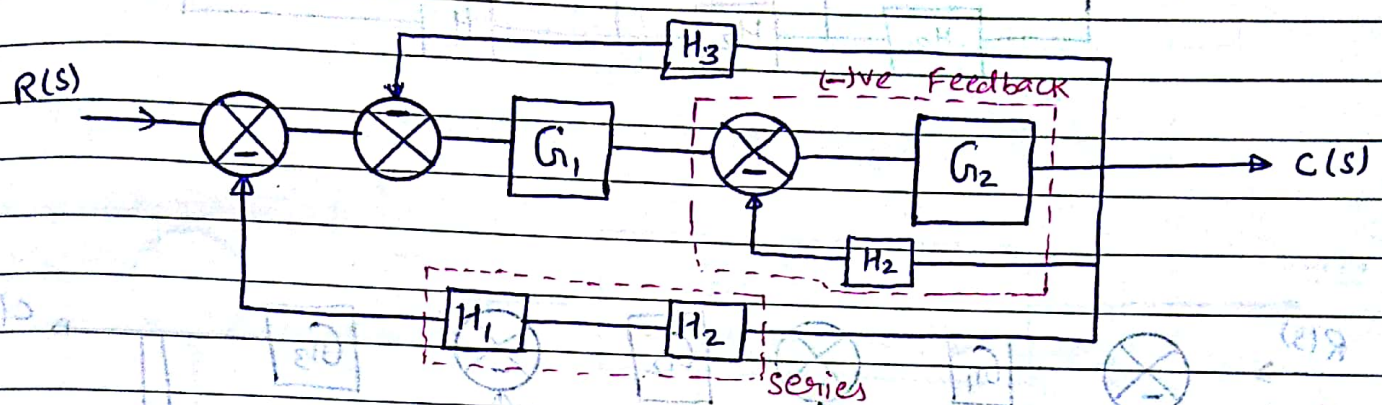
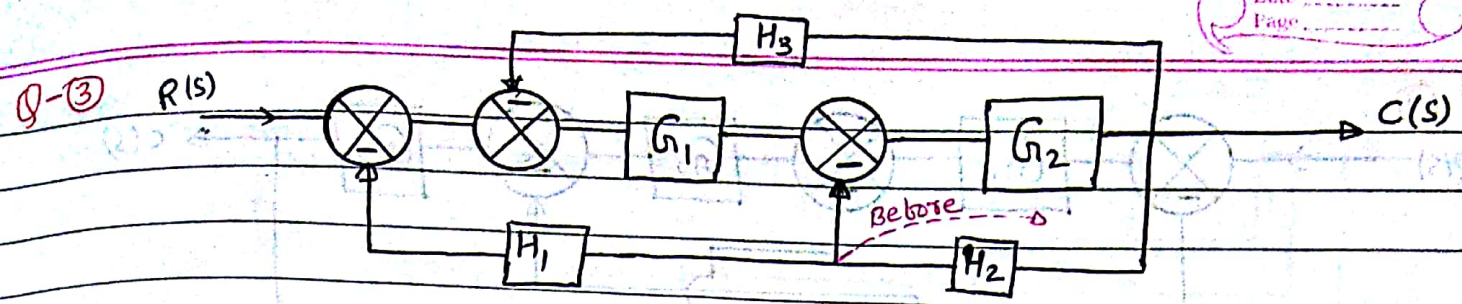
$$\therefore T.F = \frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2}{(1 + G_1 H_1)(1 + G_2 H_2) + G_1 G_2 H_3}$$

✓✓ Rule-(4) Shifting at Take off point ahead element



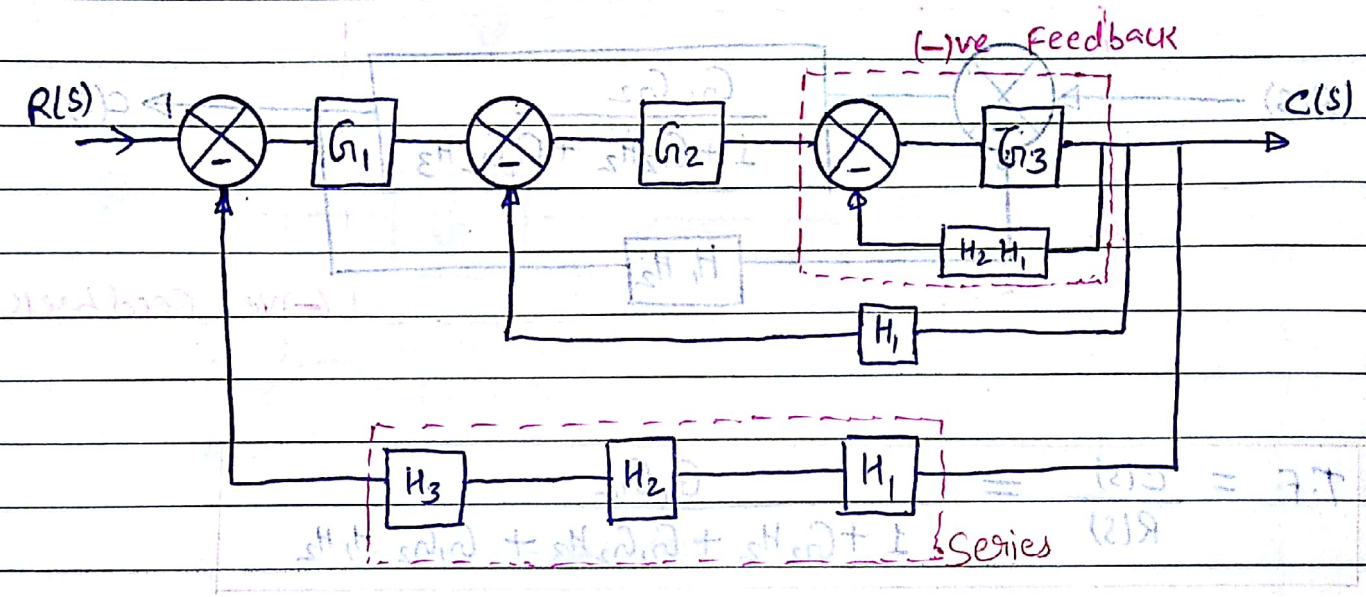
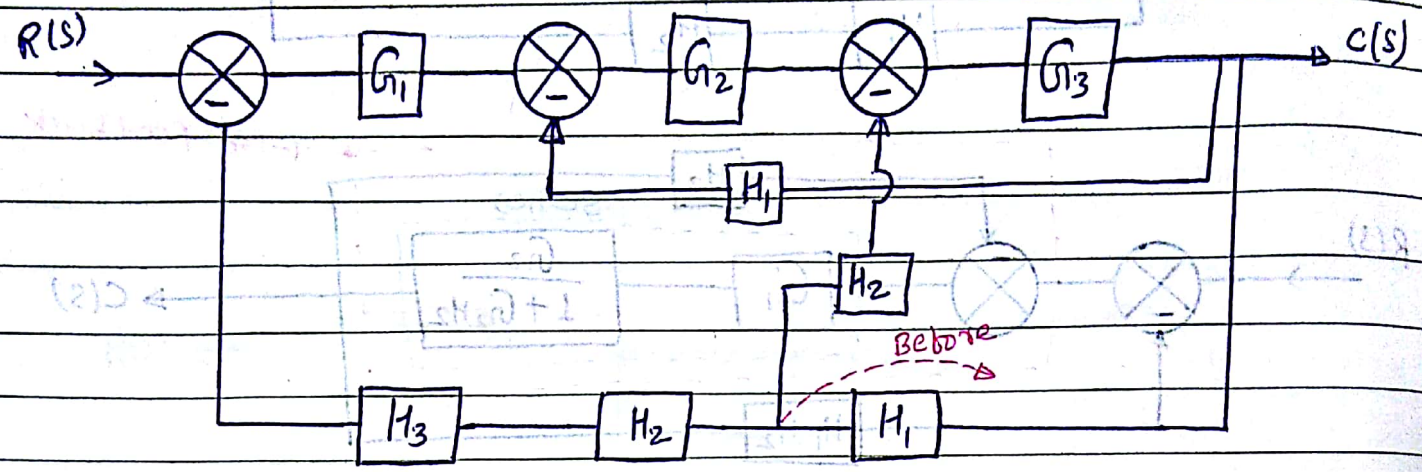
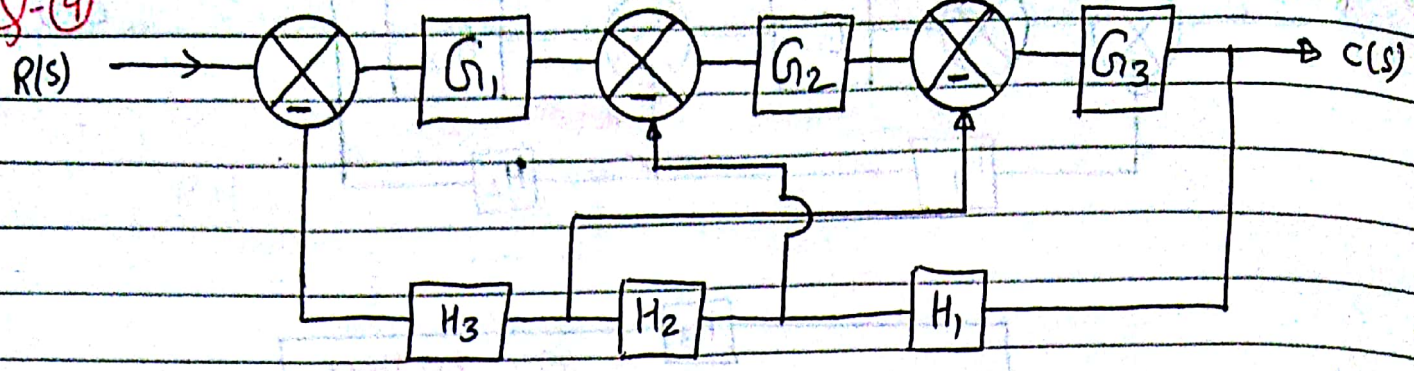
Rule-(5) Shifting at Take off point before element.



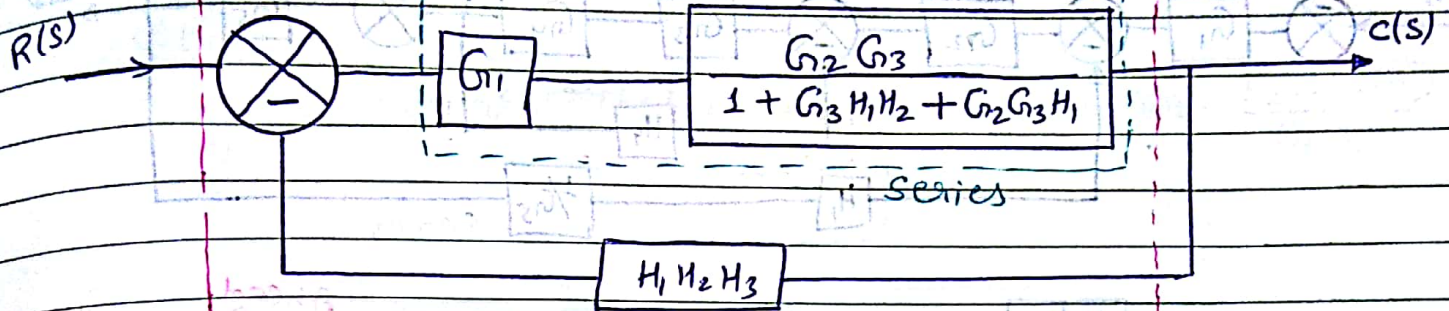
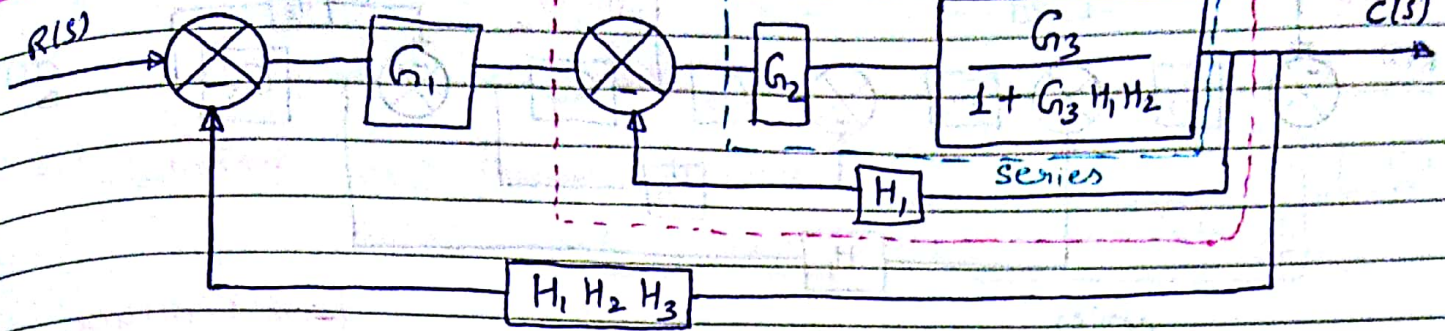


$$T.F = \frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 H_2 + G_1 G_2 H_3 + G_1 G_2 H_1 H_2}$$

Q-4

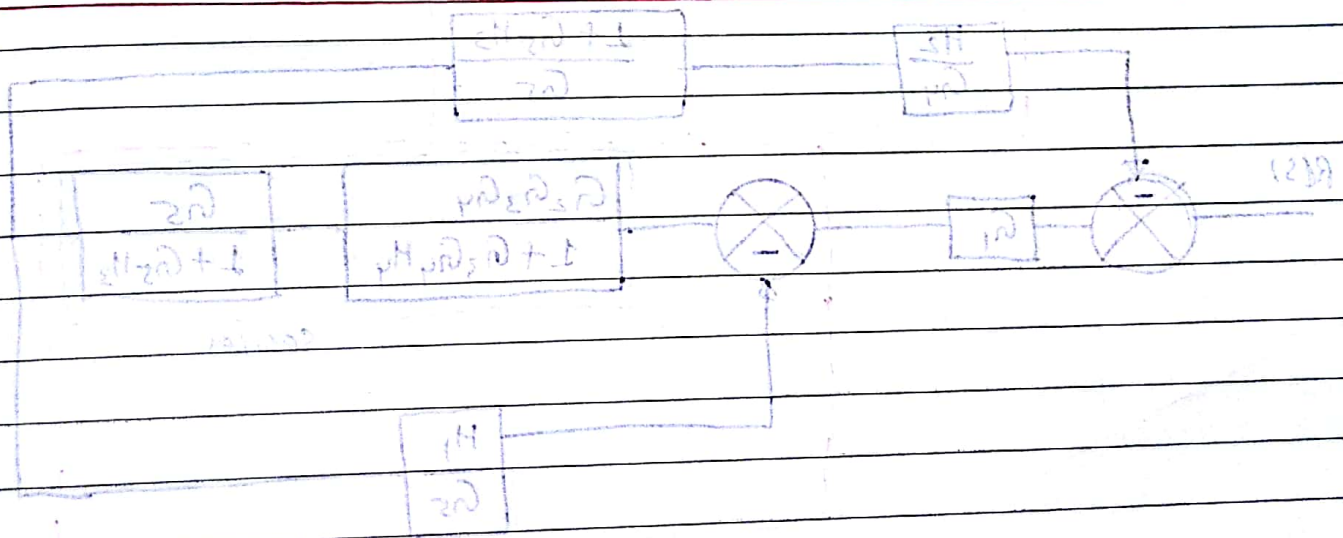


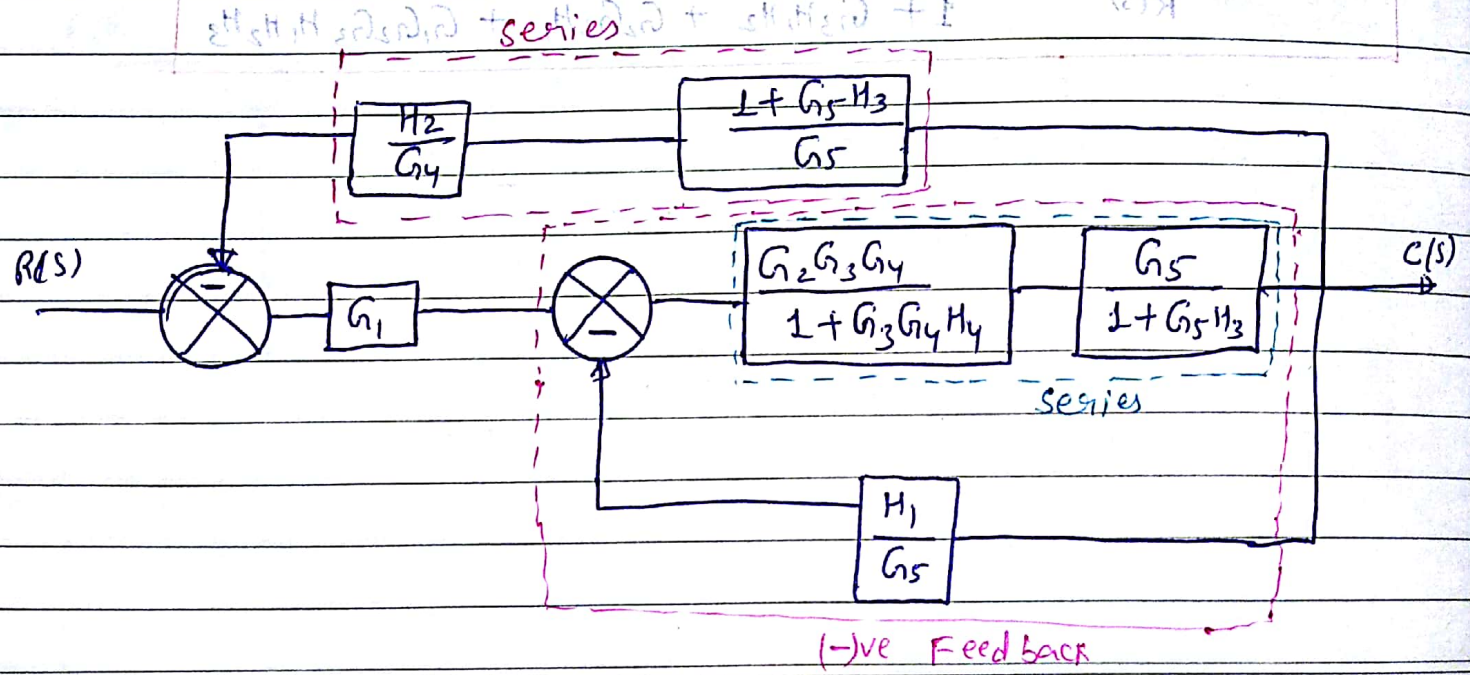
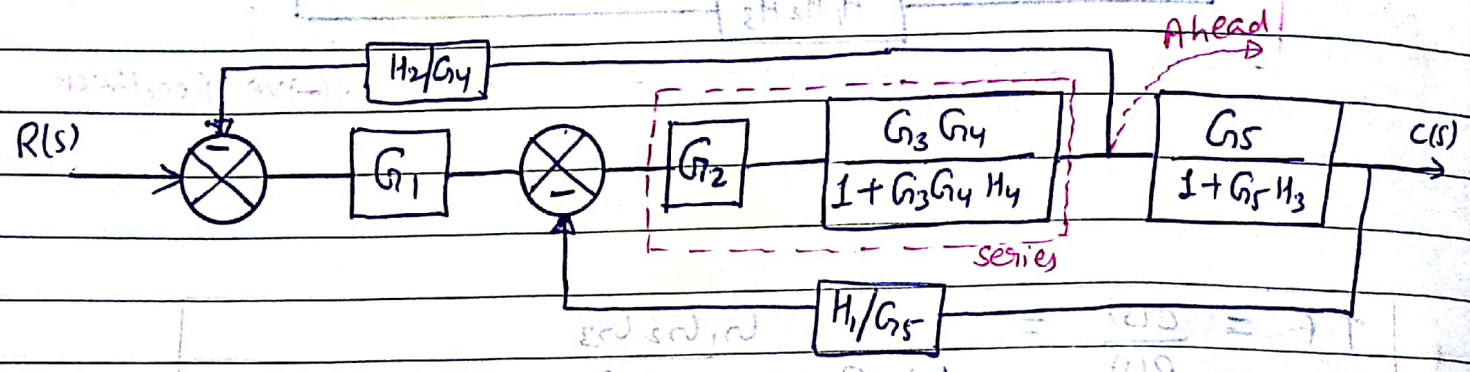
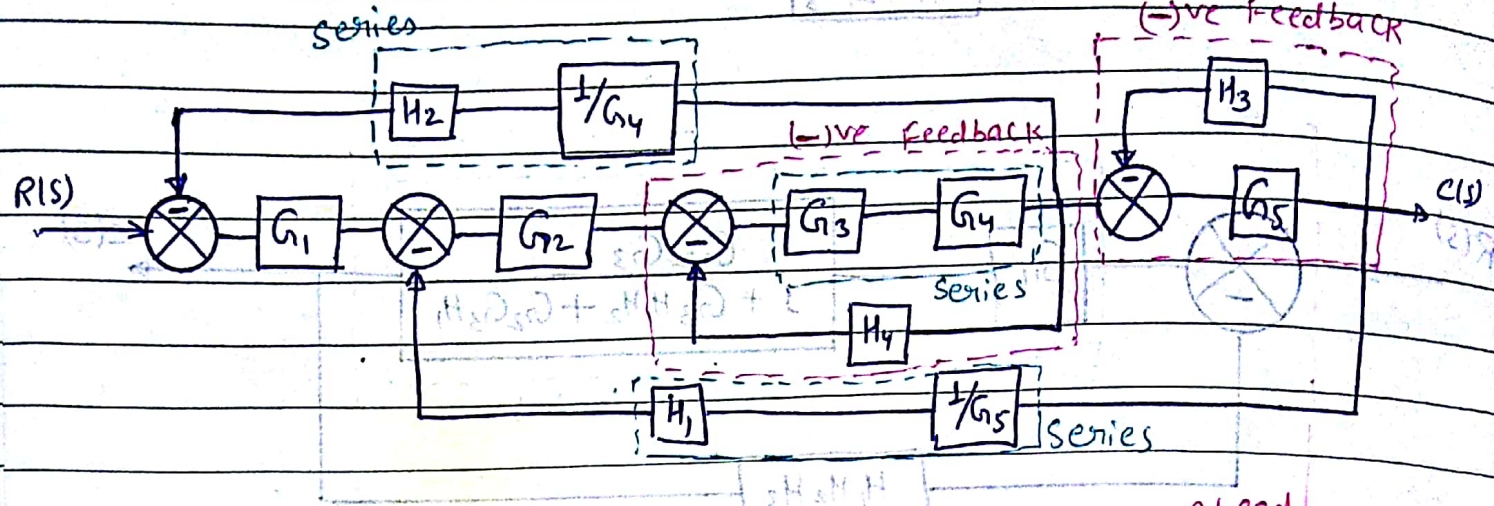
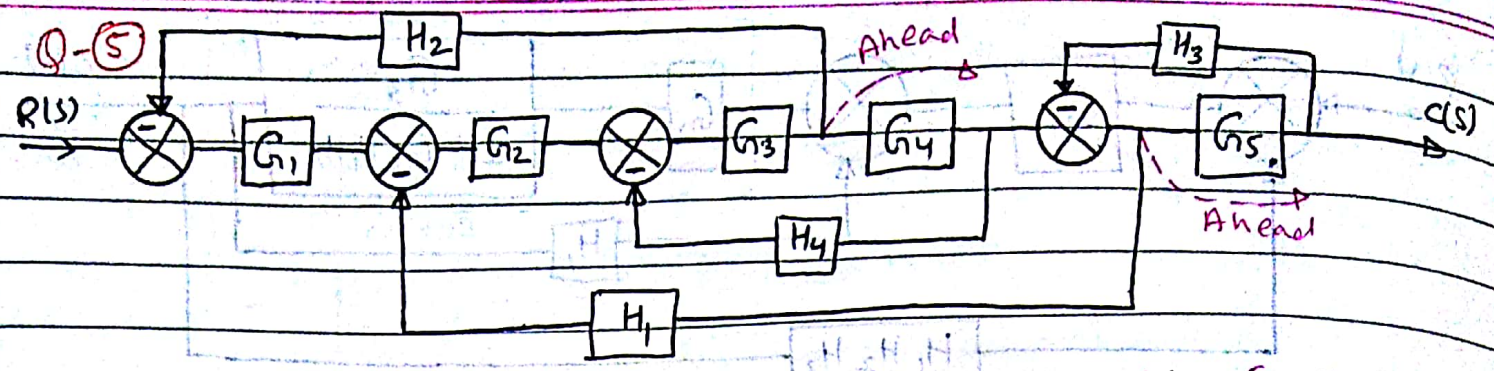
(-)ve feedback



(-)ve feedback

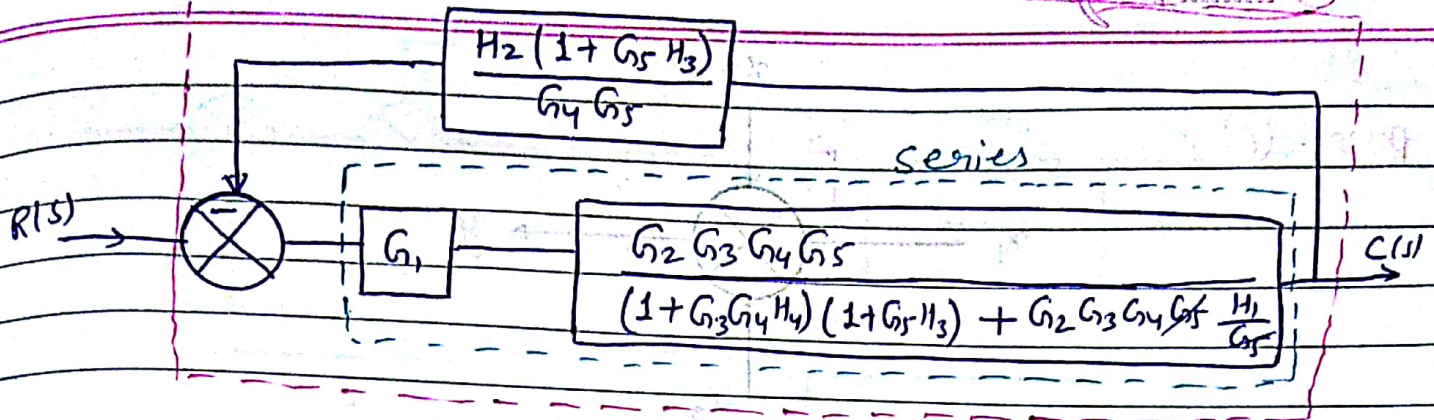
$$T.F. = \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_3 H_1 H_2 + G_2 G_3 H_1 + G_1 G_2 G_3 H_1 H_2 H_3}$$





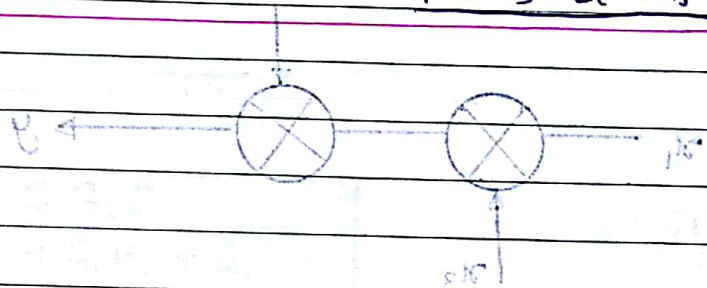
(-)ve Feedback

Date _____
Page _____

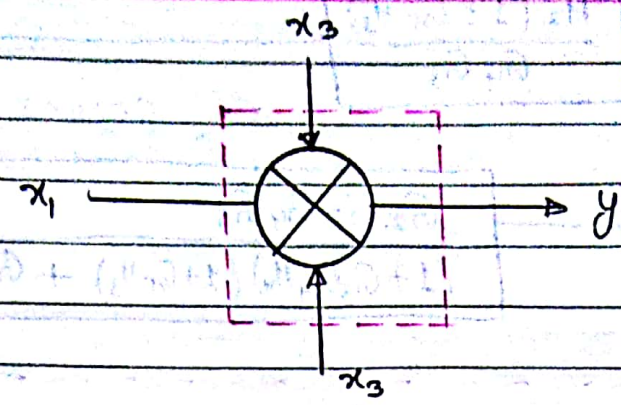


$$T.F. = \frac{G_1 G_2 G_3 G_4 G_5}{(1+G_3G_4H_4)(1+G_5H_3) + G_2G_3G_4H_1 + G_1G_2G_3G_4G_5 \times \frac{H_2(1+G_5H_3)}{G_4G_5}}$$

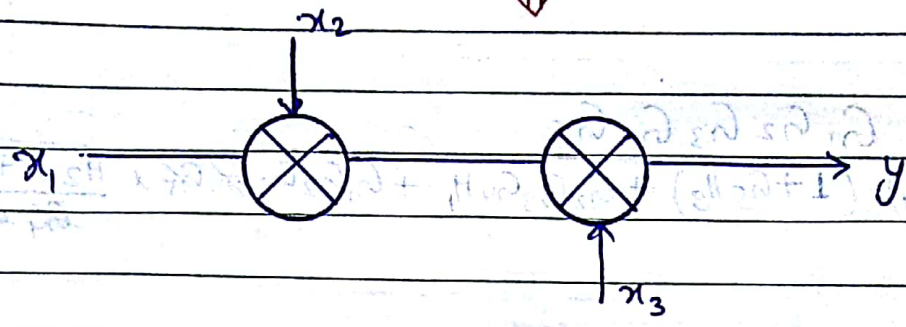
$$T.F. = \frac{G_1 G_2 G_3 G_4 G_5}{(1+G_3G_4H_4)(1+G_5H_3 + G_2G_3G_4H_1 + G_1G_2G_3H_2(1+G_5H_3))}$$



Rule: - (6)

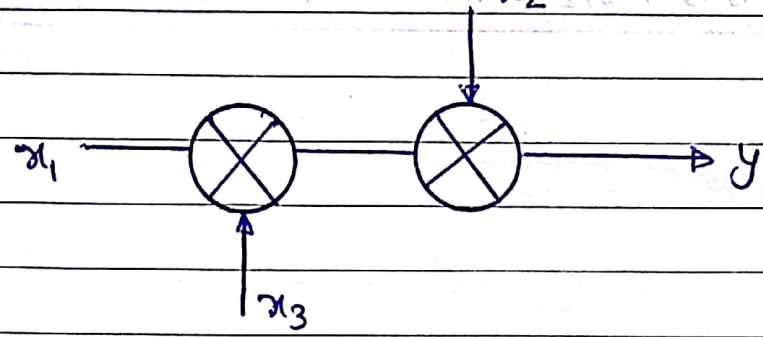


⇓

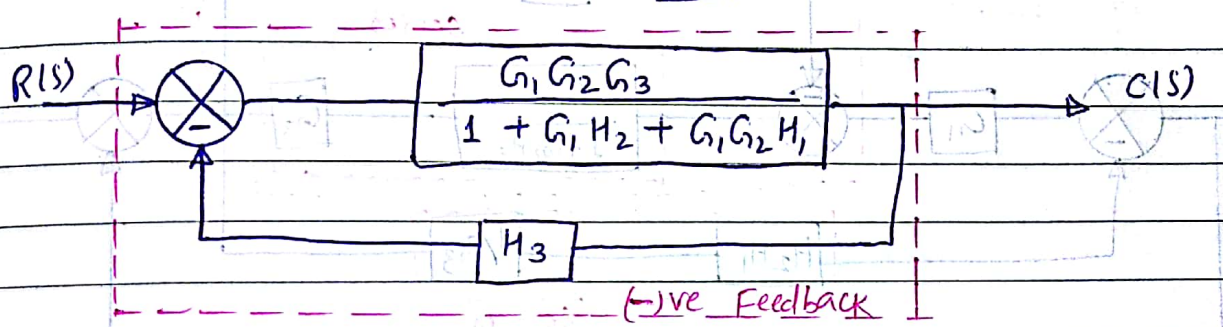
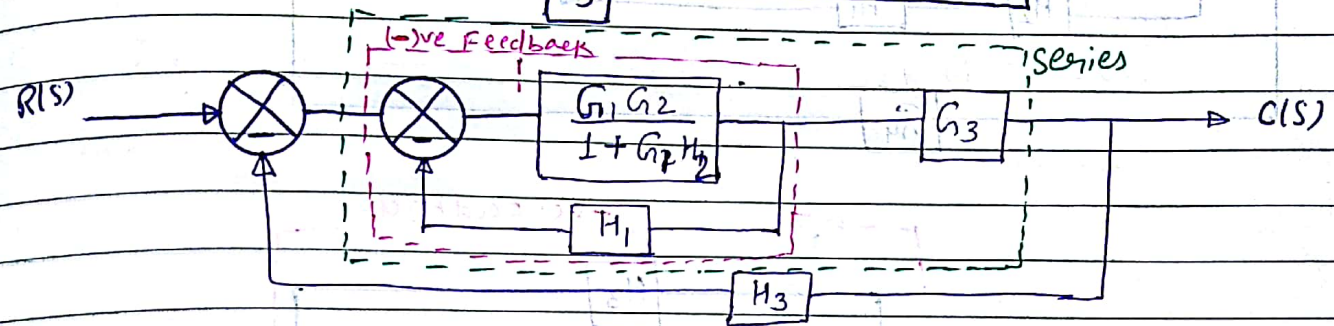
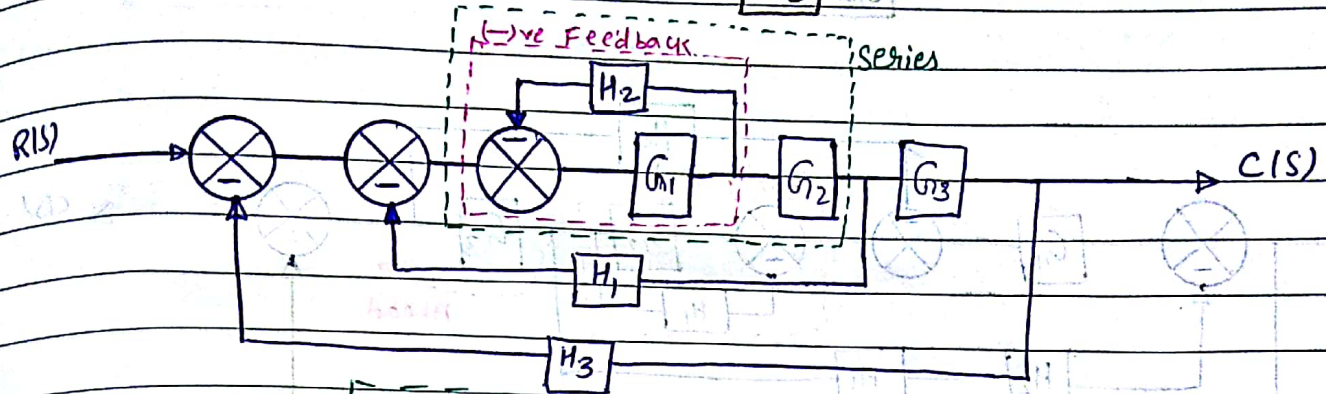
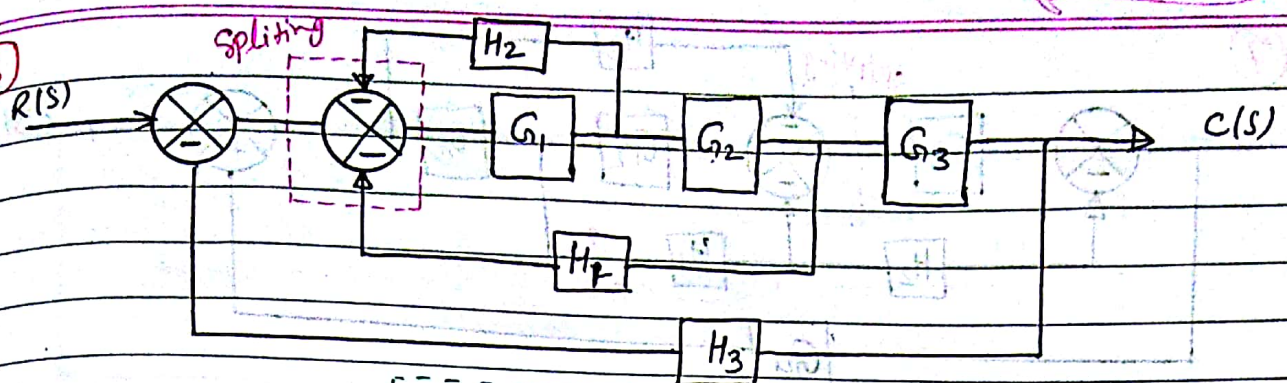


Interchanging the Signal

$$(e^{j\omega T} + 1) e^{j\omega T} + e^{j\omega T} (e^{j\omega T} + 1) = 2e^{j\omega T}$$

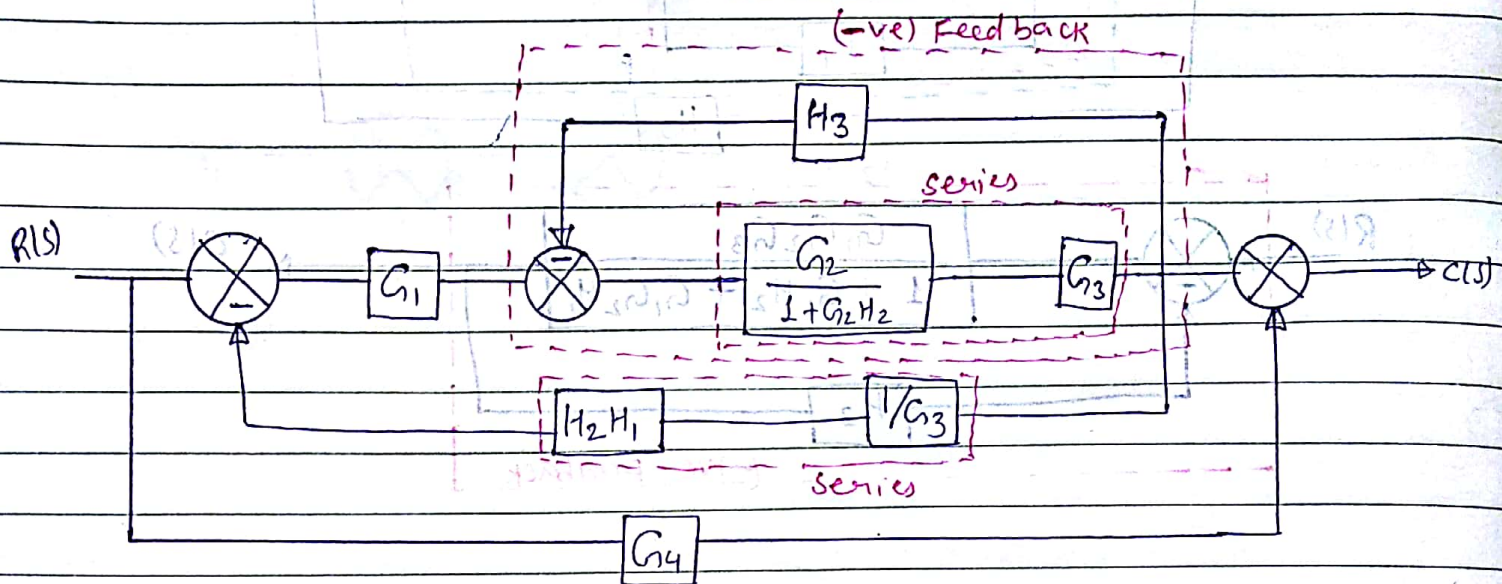
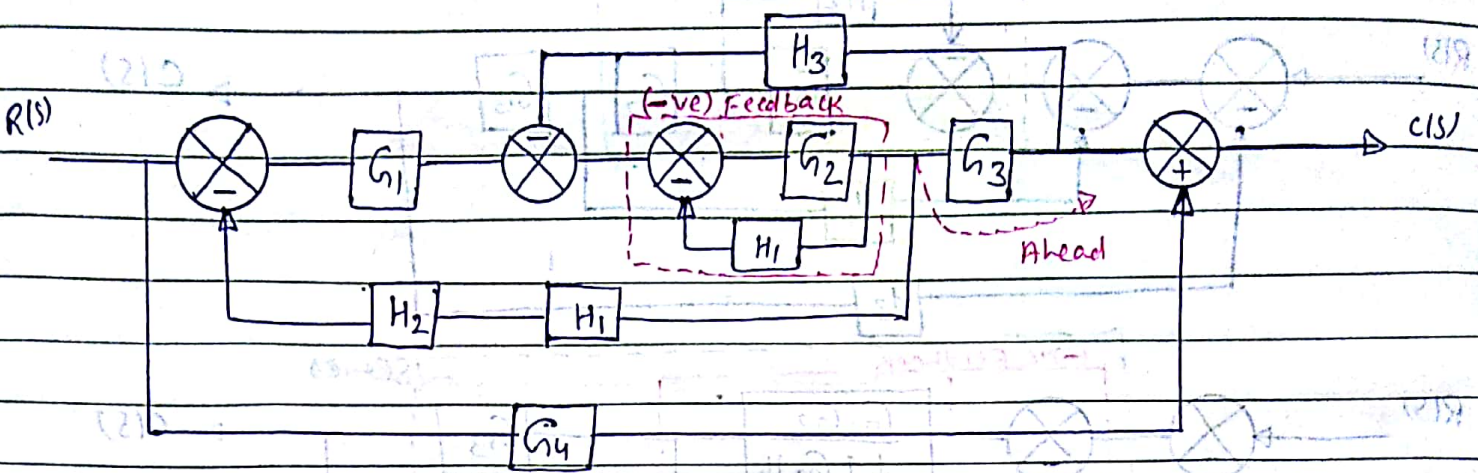
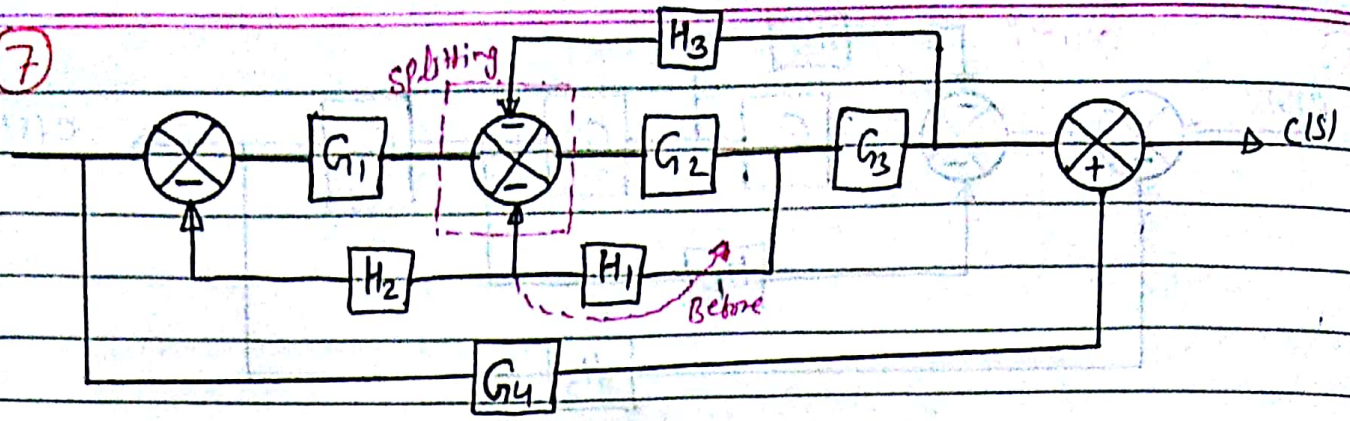


Q-6

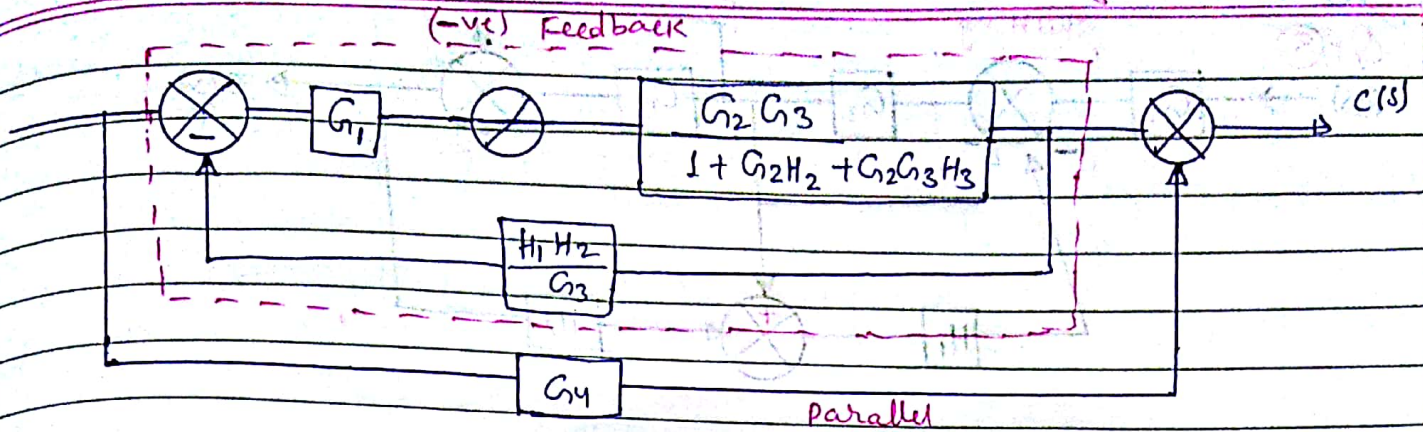


$$T.F = \frac{G_1 G_2 G_3}{1 + G_1 H_2 + G_1 G_2 H_1 + G_1 G_2 G_3 H_3}$$

Q-7

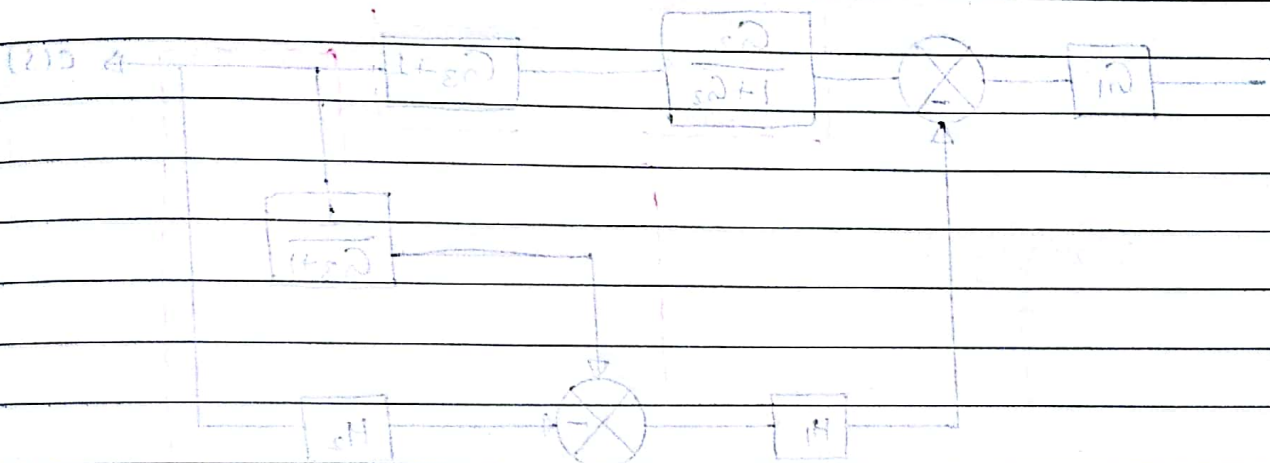


$$s^2 H_2 G_2 G_3 + H_2 G_2 + s H_1 + 1$$

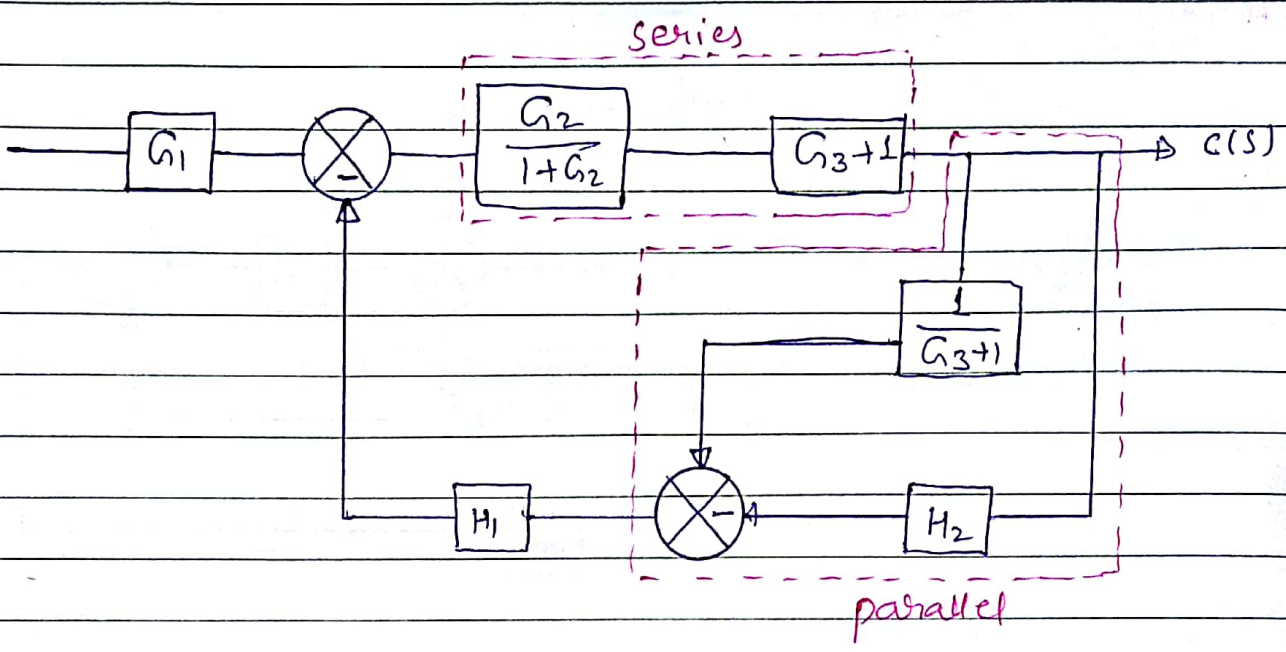
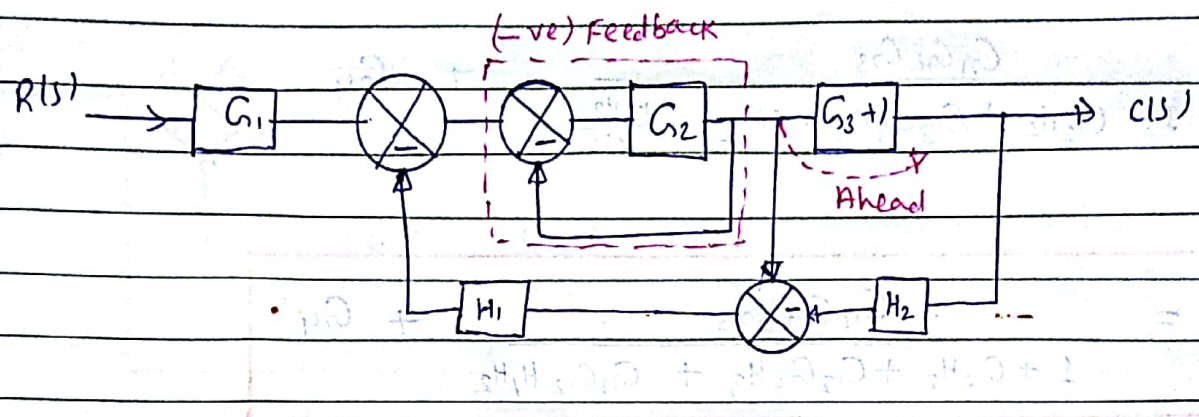
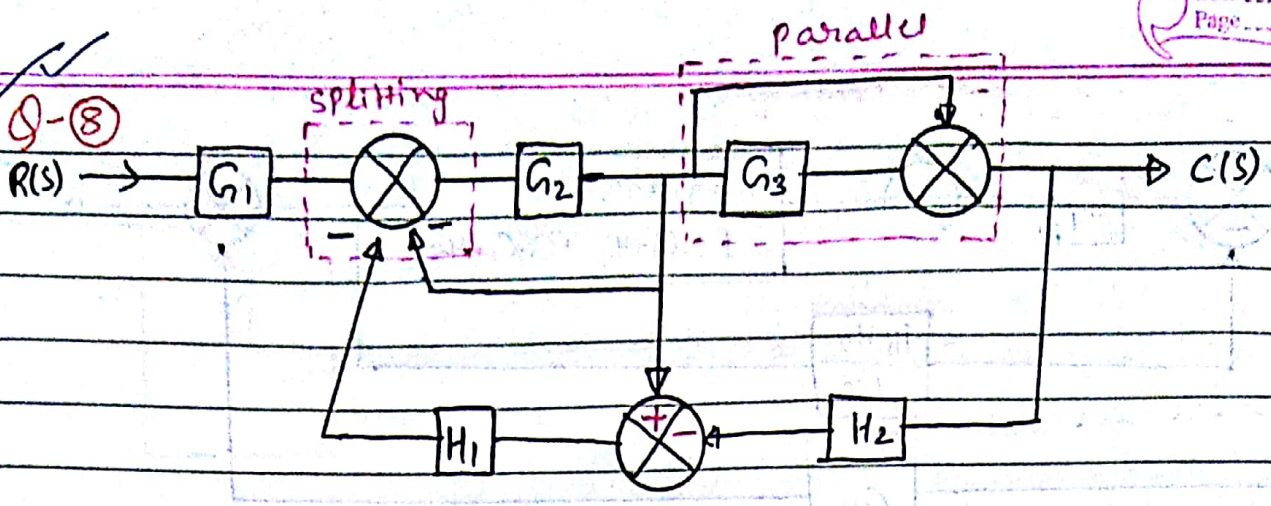


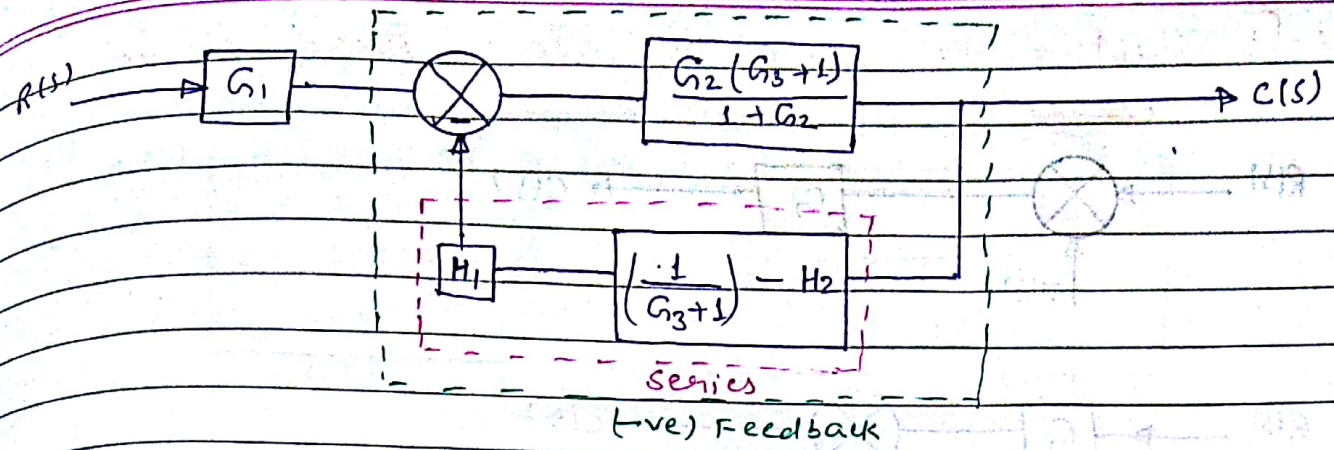
$$T.F. = \frac{G_1 G_2 G_3}{1 + G_2 H_2 + G_2 G_3 H_3} + G_4$$

$$T.F. = \frac{G_1 G_2 G_3}{1 + G_2 H_2 + G_2 G_3 H_3 + G_1 G_2 H_1 H_2} + G_4$$



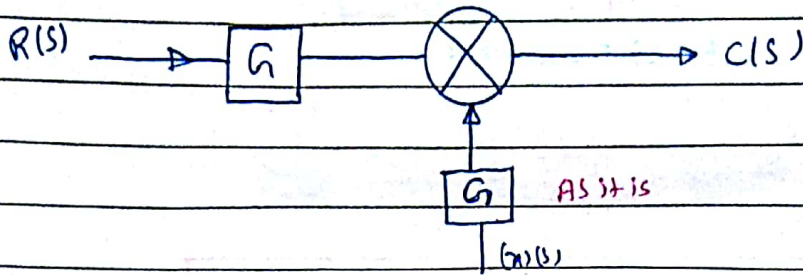
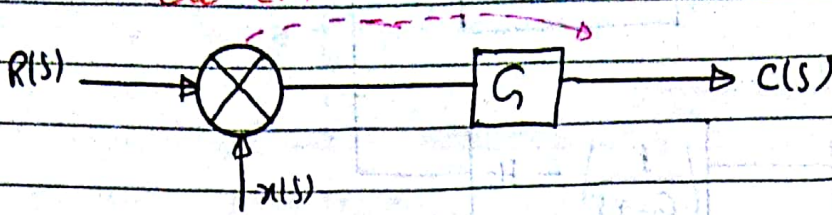
Q-8



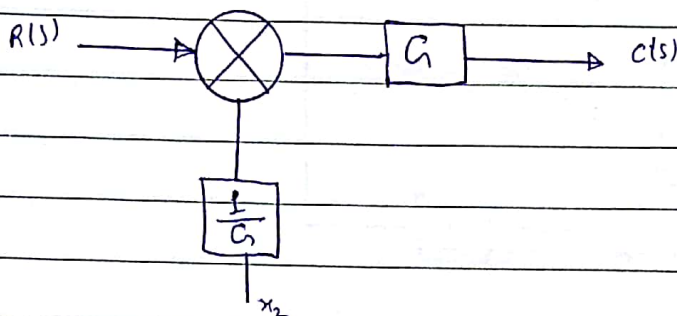
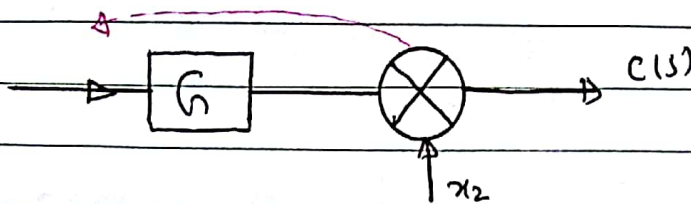


$$T.F = \frac{C(s)}{R(s)} = \frac{G_1 \cdot G_2 (G_3 + 1)}{1 + G_2 + G_2 (G_3 + 1) H_1 \left[\frac{1}{G_3 + 1} - H_2 \right]}$$

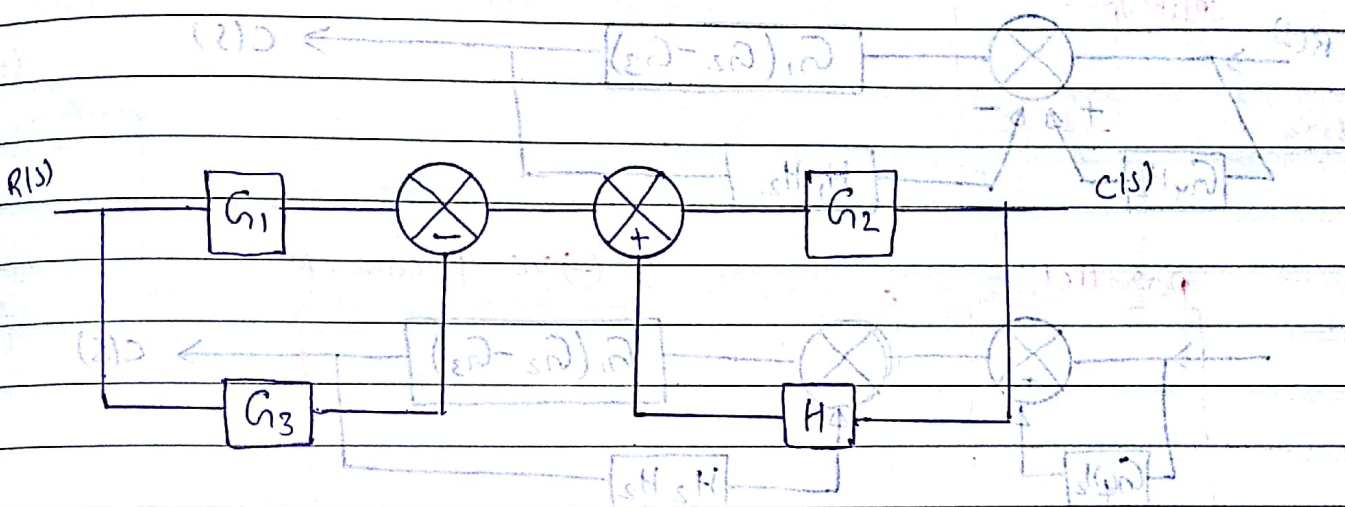
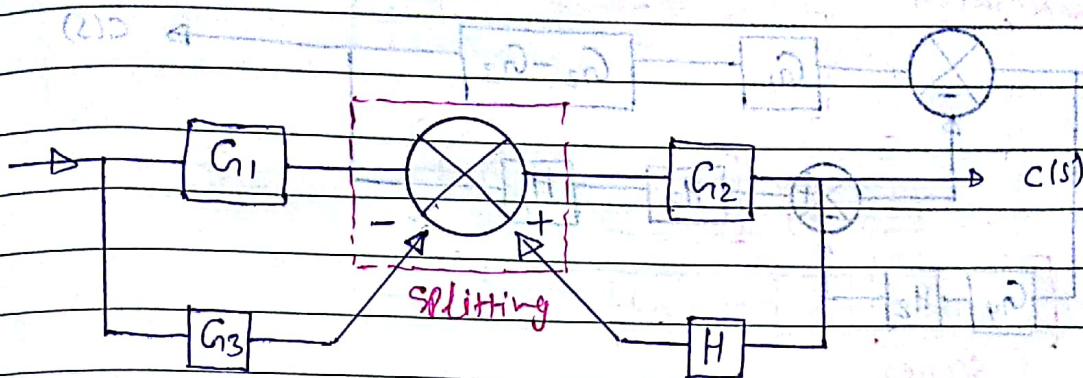
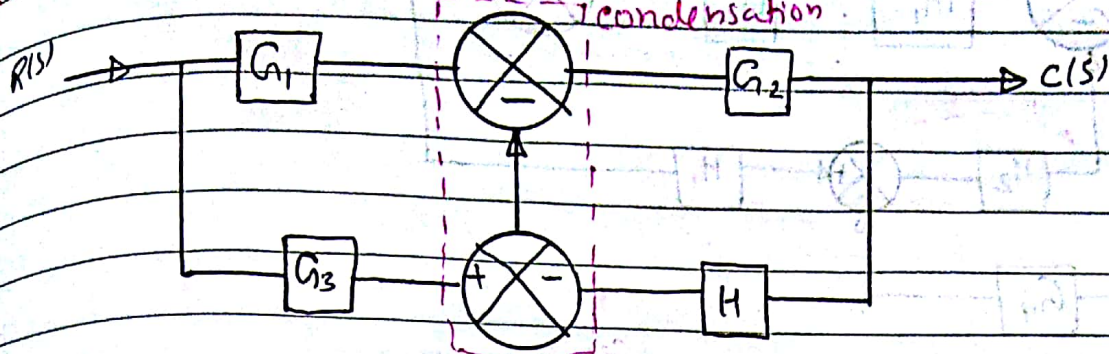
Rule-⑦ Shifting the Summation block Ahead of all element.



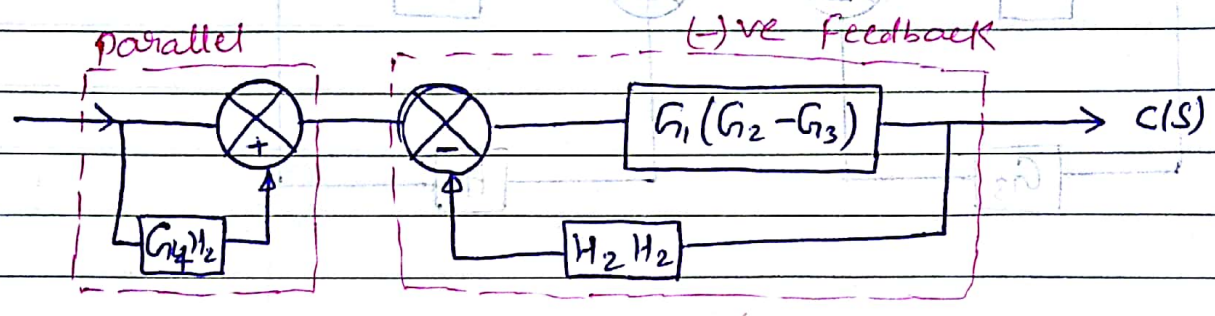
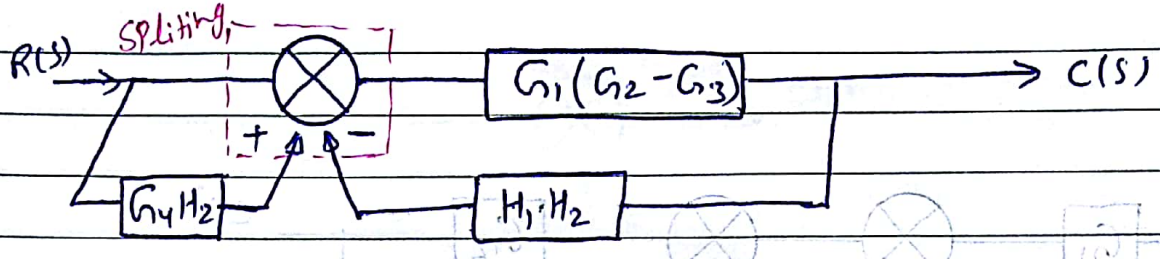
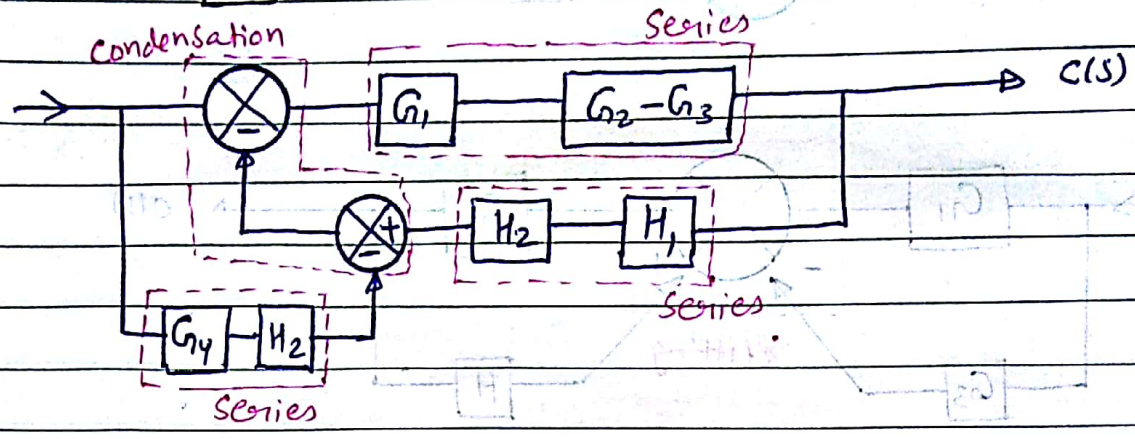
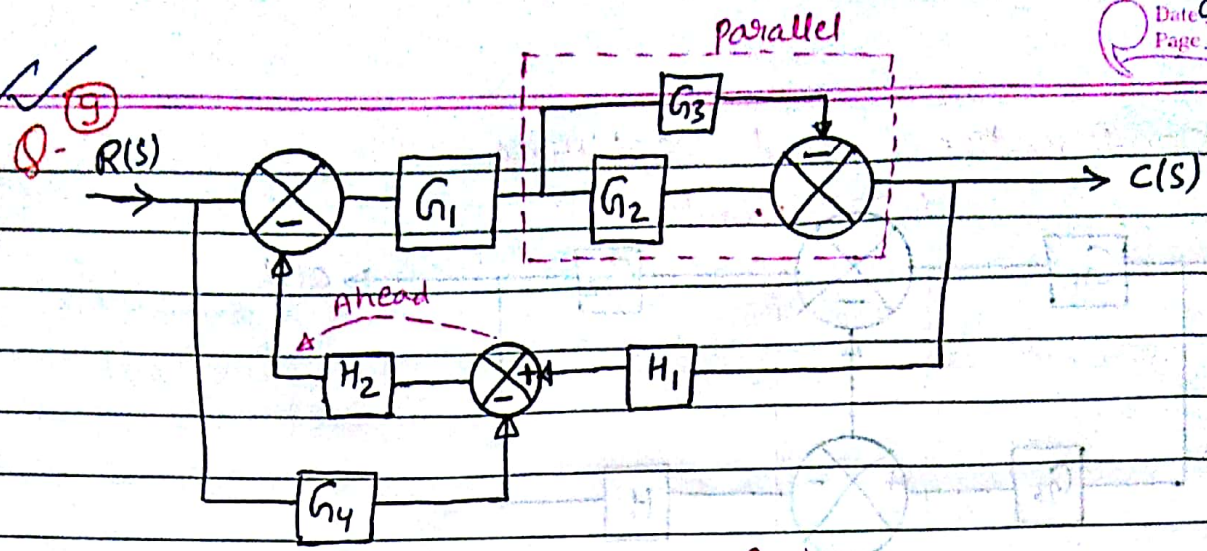
Rule ⑧ Shifting the Summation block before element



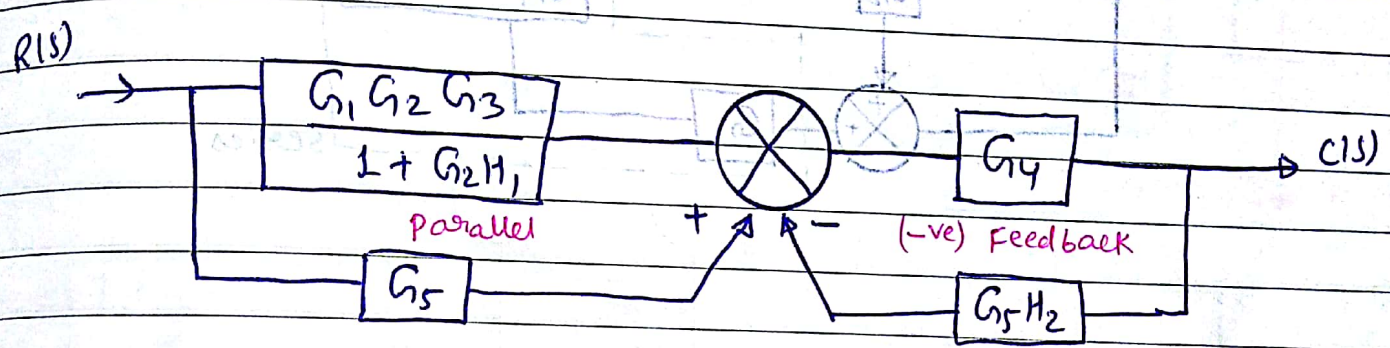
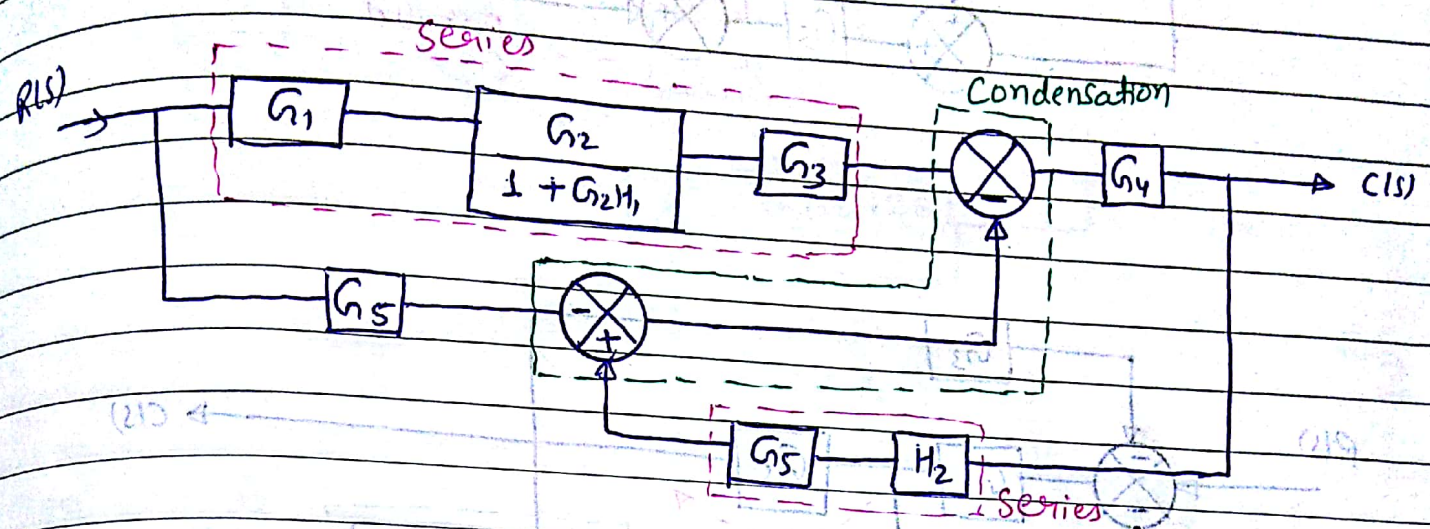
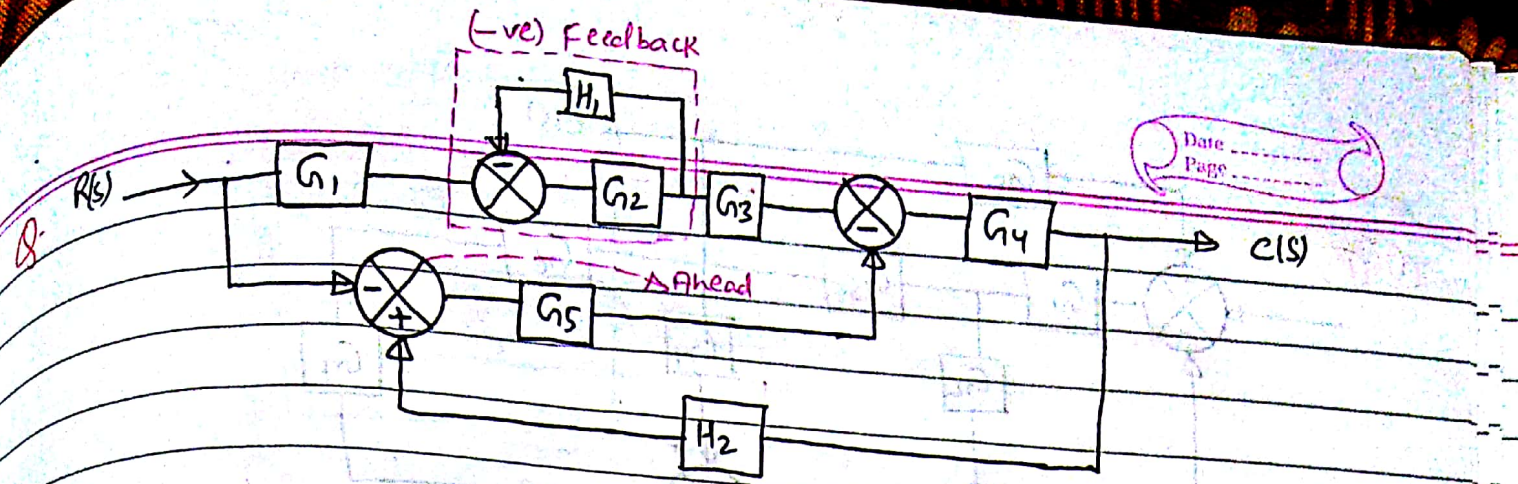
Rule ③ Condensation And Splitting



$$\left[\frac{(s^2 - s^2) \cdot 2}{sH(s^2 - s^2) \cdot 2 + 1} \right] [sH \cdot 2 + 1] = 2T$$

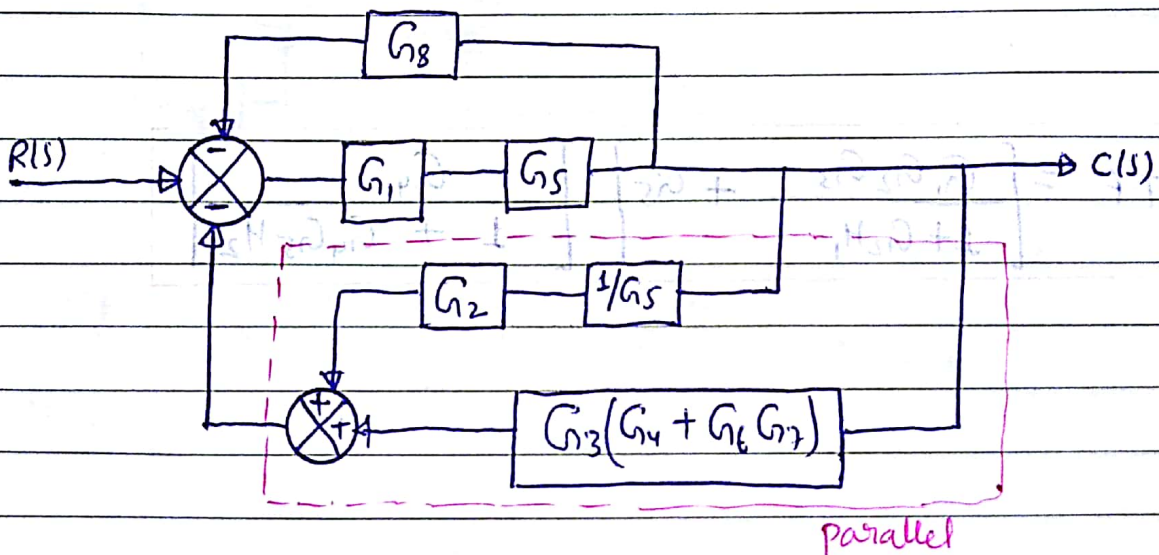
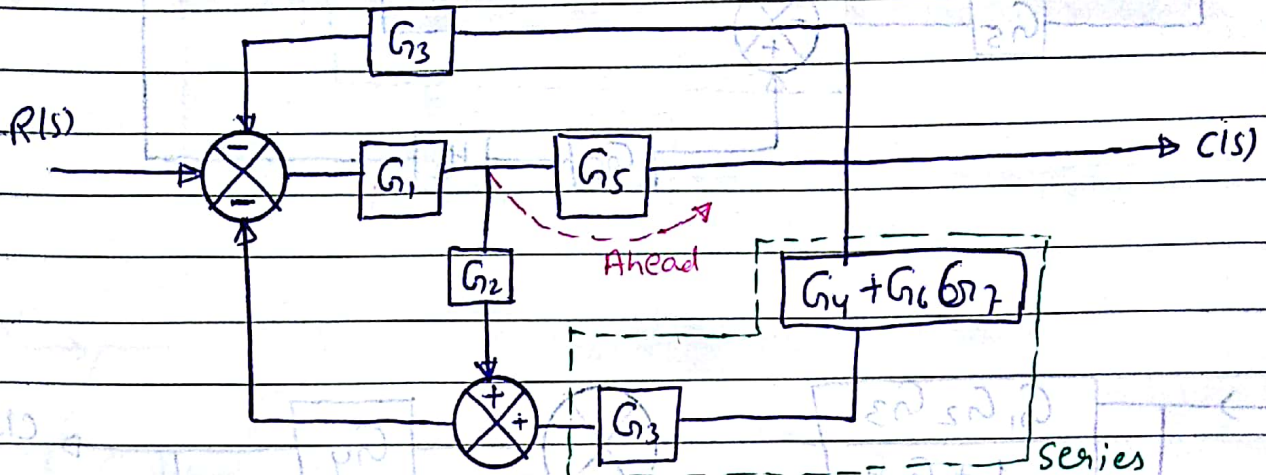
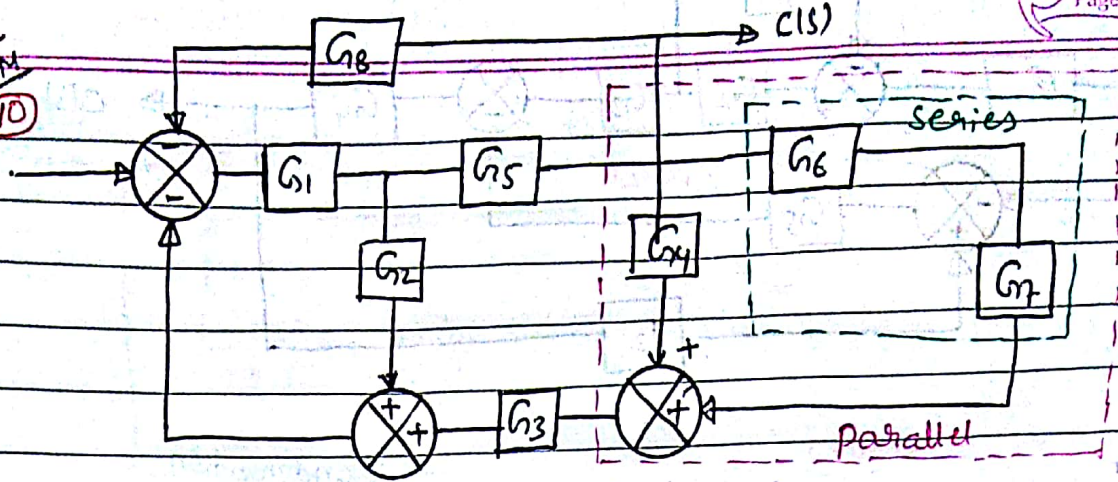


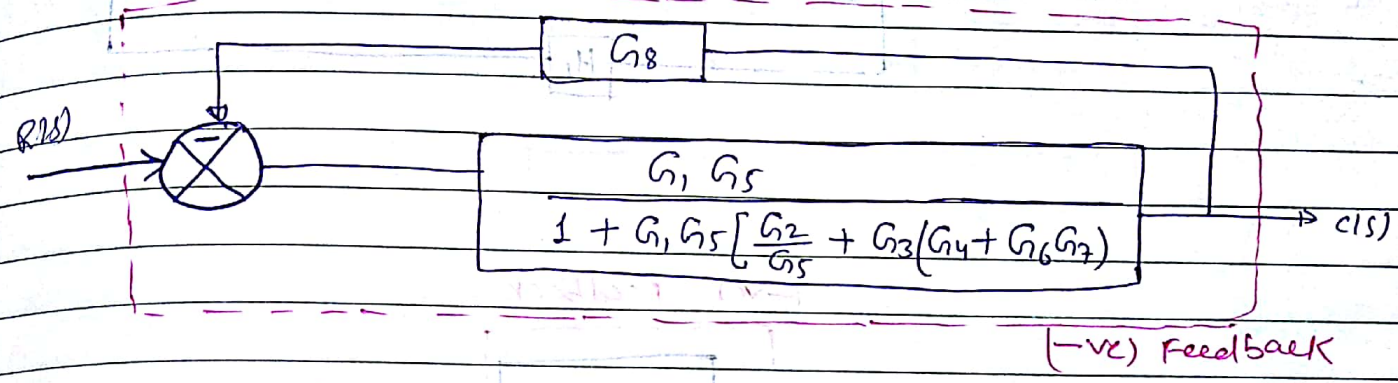
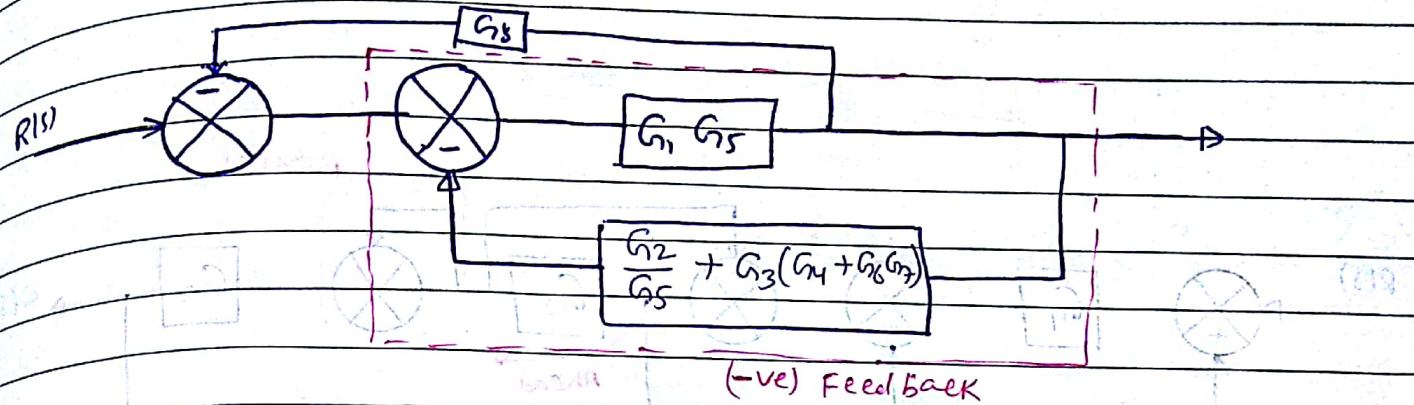
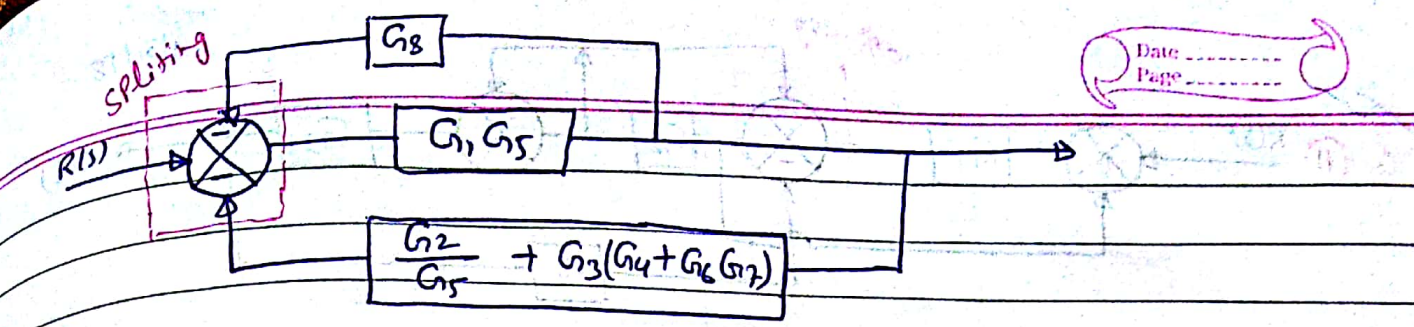
$$T.F = [1 + G_4 H_2] \left[\frac{G_1 (G_2 - G_3)}{1 + G_1 (G_2 - G_3) H_1 H_2} \right]$$



$$T.F = \left[\frac{G_1 G_2 G_3}{1 + G_2 H_1} + G_5 \right] \left[\frac{G_4}{1 + G_4 G_5 H_2} \right]$$

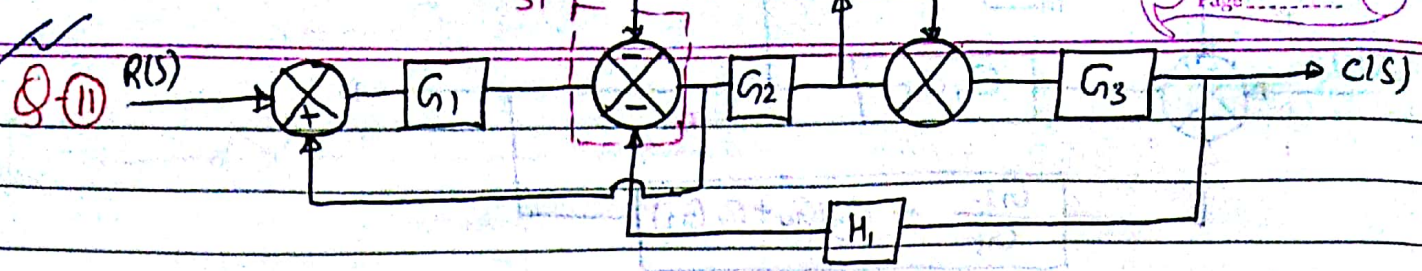
S-16
Q-10



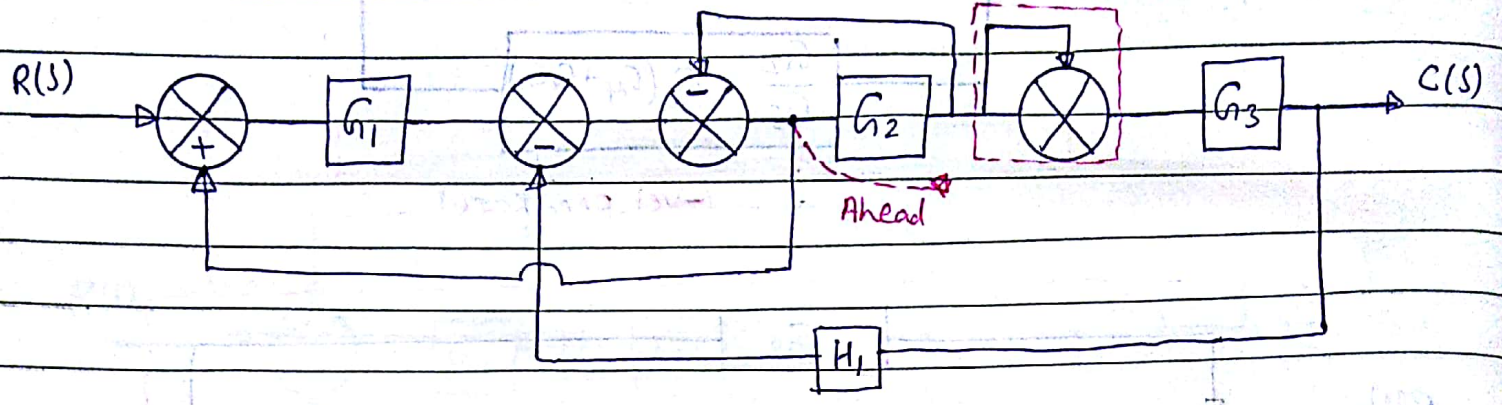


T.F =
$$\frac{G_1 G_5}{1 + G_1 G_5 \left[\frac{G_2}{G_5} + G_3(G_4 + G_6 G_7) \right] + G_1 G_5 G_8}$$

splitting

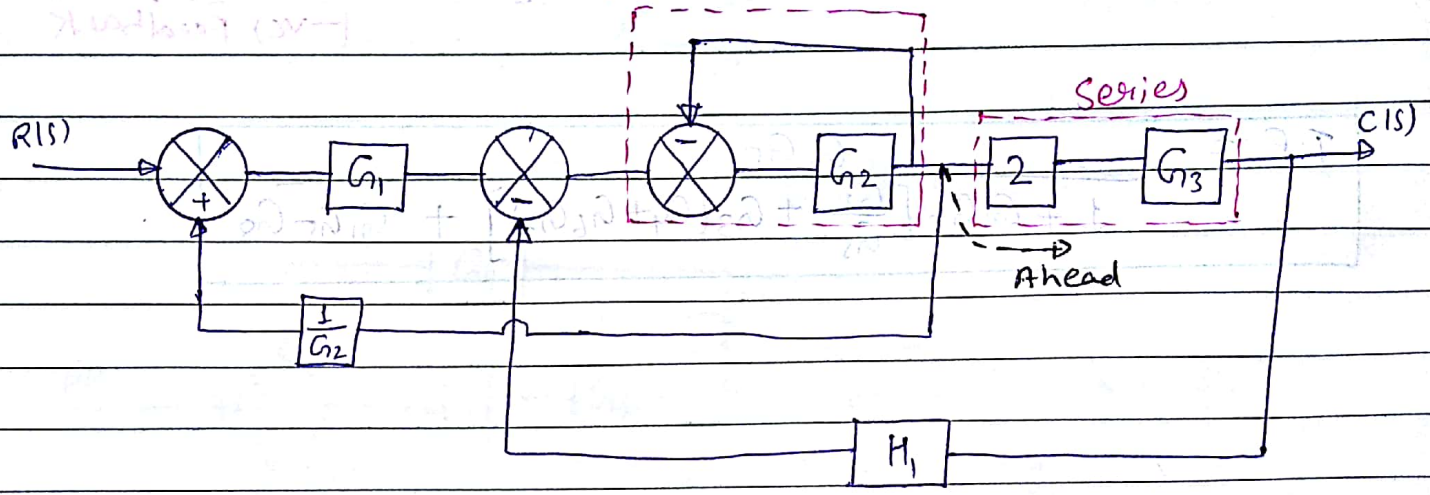


parallel



Ahead

(-ve) Feedback

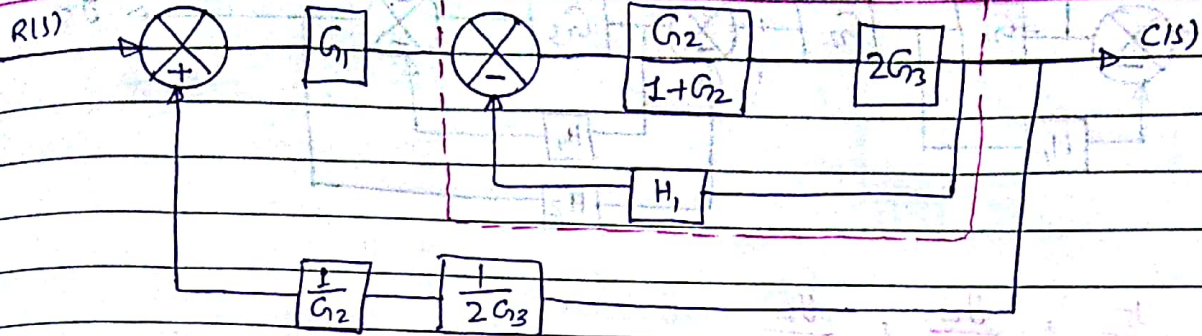


Series

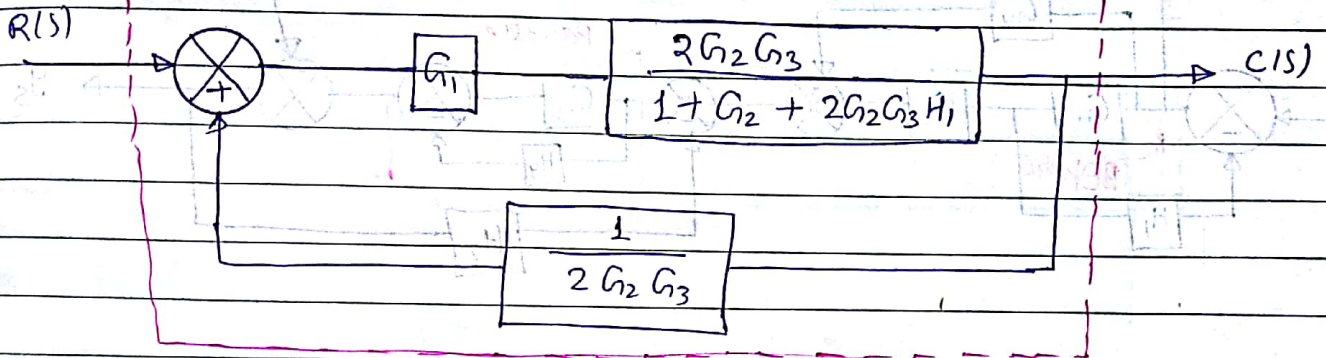
Ahead

Series & (-ve) Feedback

Date _____
Page _____

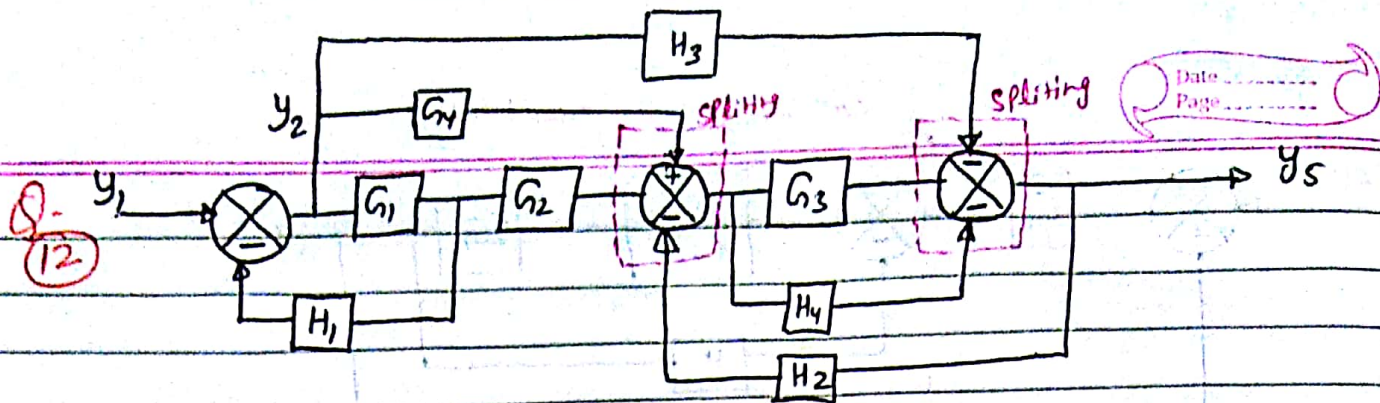


series & (-ve) Feedback

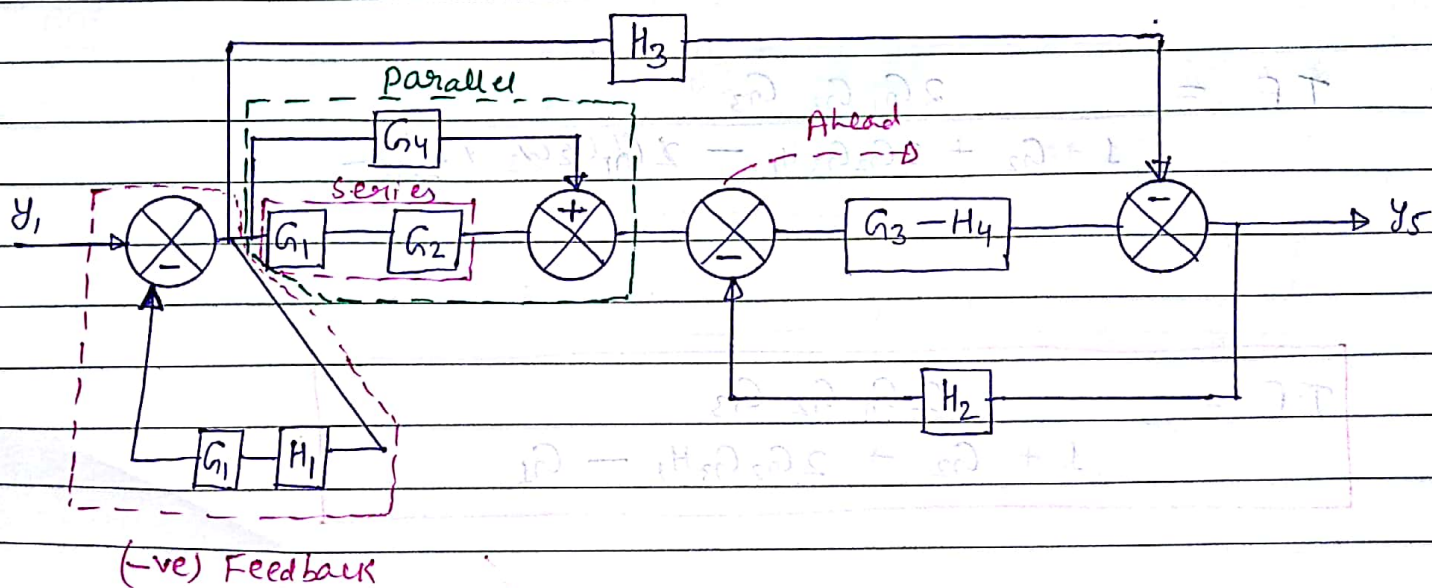
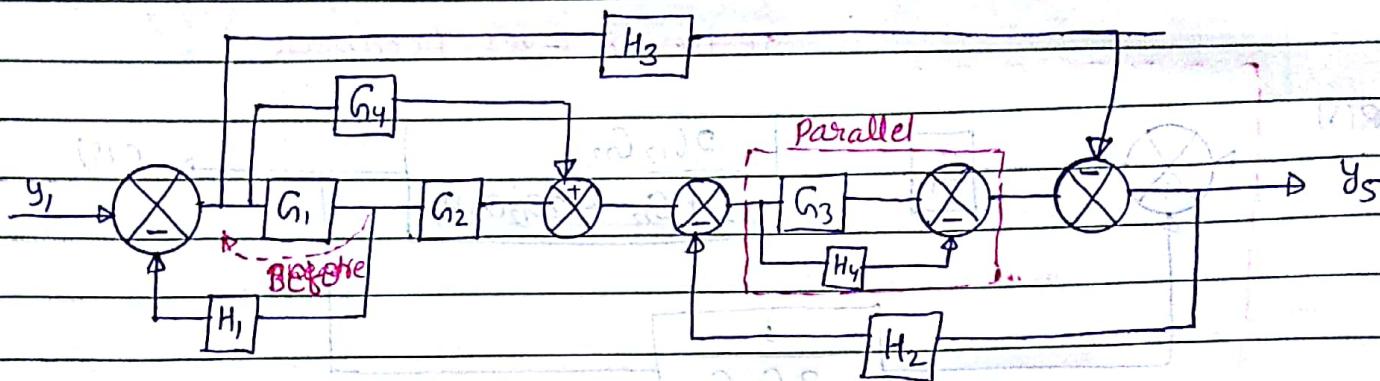


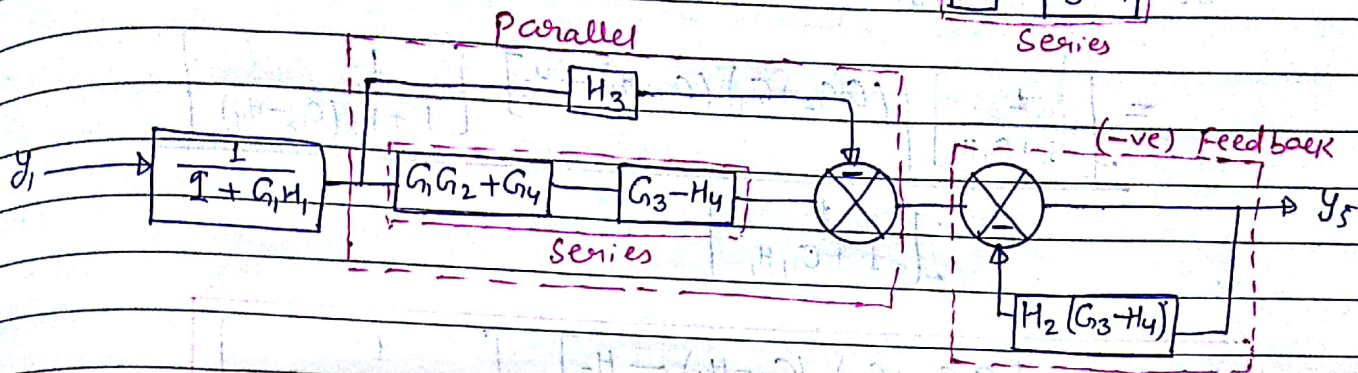
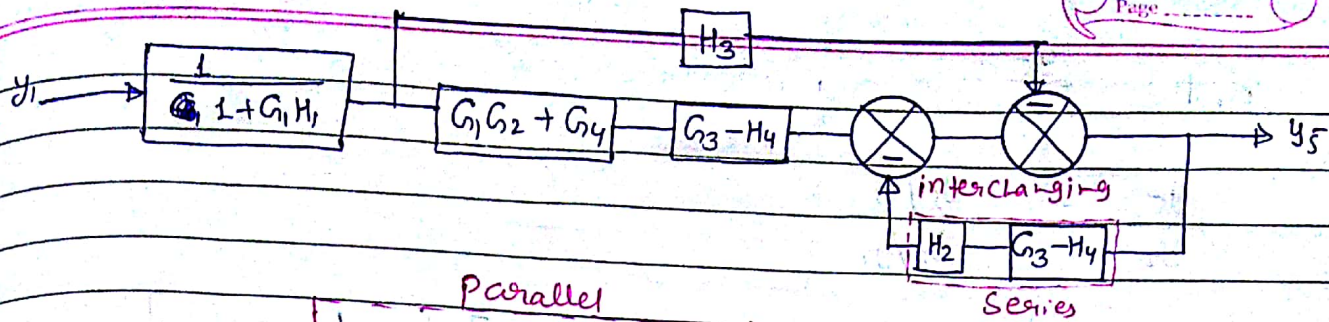
$$T.F = \frac{2G_1 G_2 G_3}{1 + G_2 + 2G_2 G_3 H_1 - 2G_1 G_2 G_3 \times \frac{1}{2G_2 G_3}}$$

$$T.F = \frac{2G_1 G_2 G_3}{1 + G_2 + 2G_2 G_3 H_1 - G_1}$$



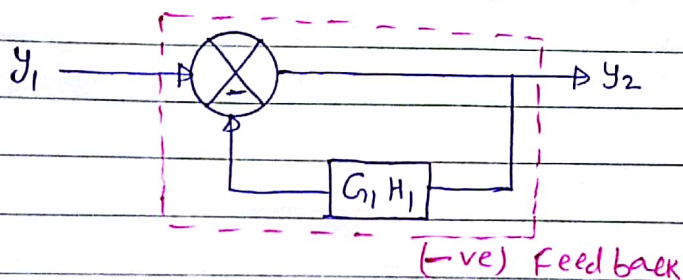
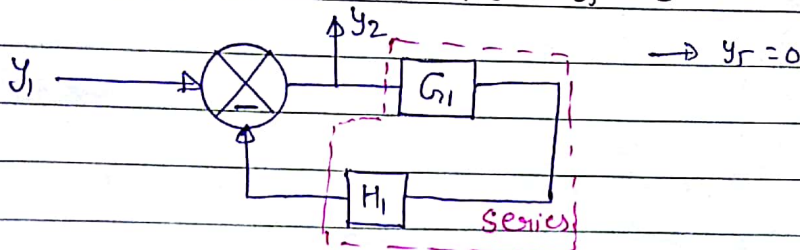
Find:- $\frac{y_5}{y_1}$, $\frac{y_2}{y_1}$, $\frac{y_5}{y_2}$





$$\frac{y_5}{y_1} = \left(\frac{1}{1+G_1H_1} \right) \left[(G_1G_2+G_4)(G_3-H_4) - H_3 \right] \left[\frac{1}{1+H_2(G_3-H_4)} \right]$$

* Find:- $\frac{y_2}{y_1}$, Take $y_r = 0$



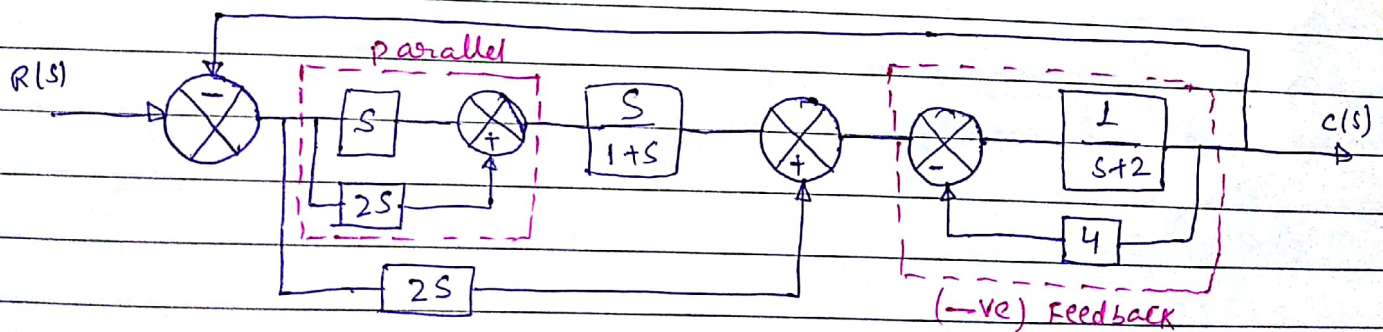
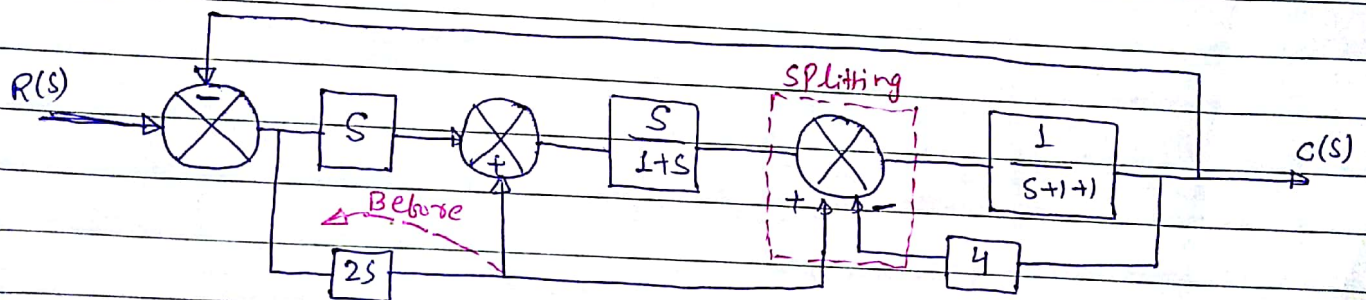
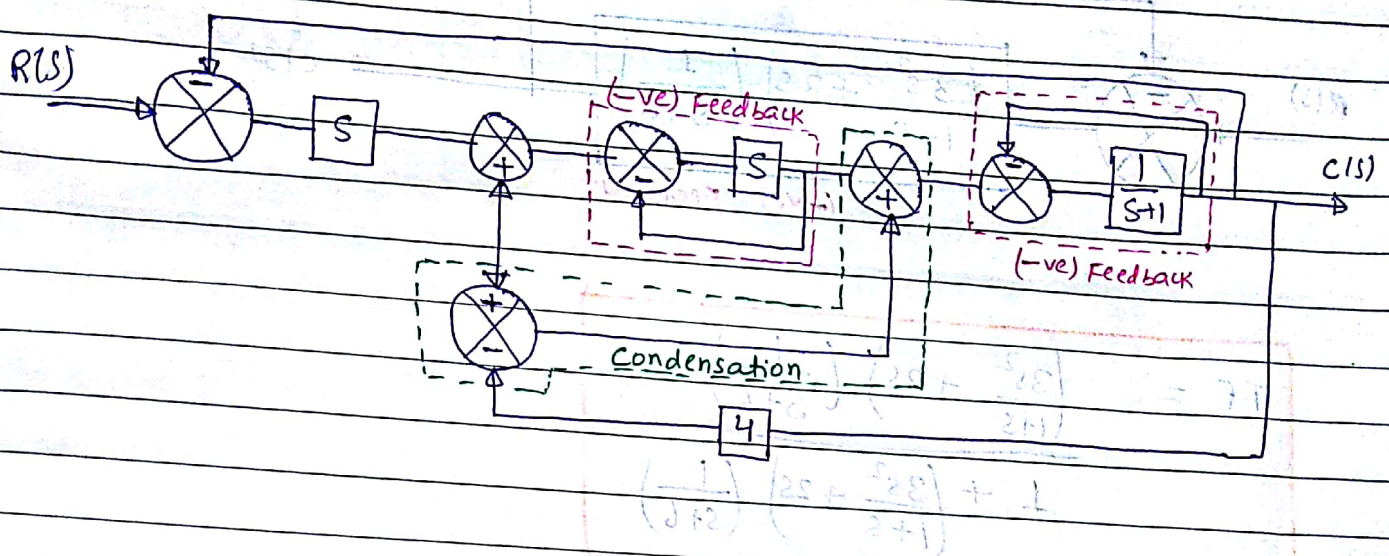
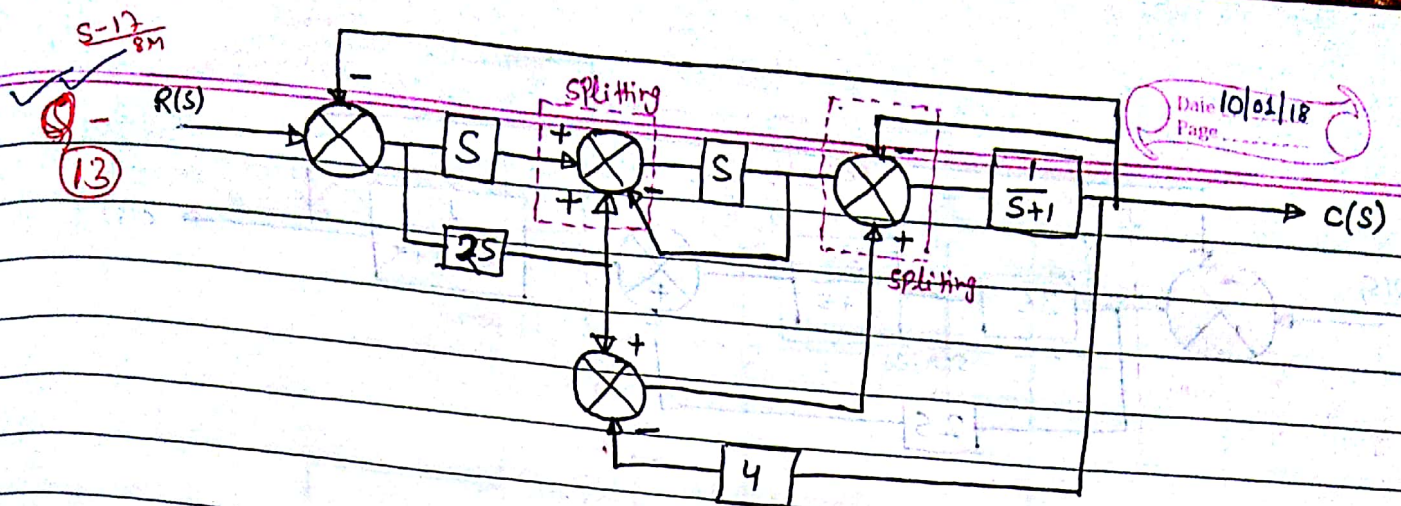
$$\frac{y_2}{y_1} = \frac{1}{1+G_1H_1}$$

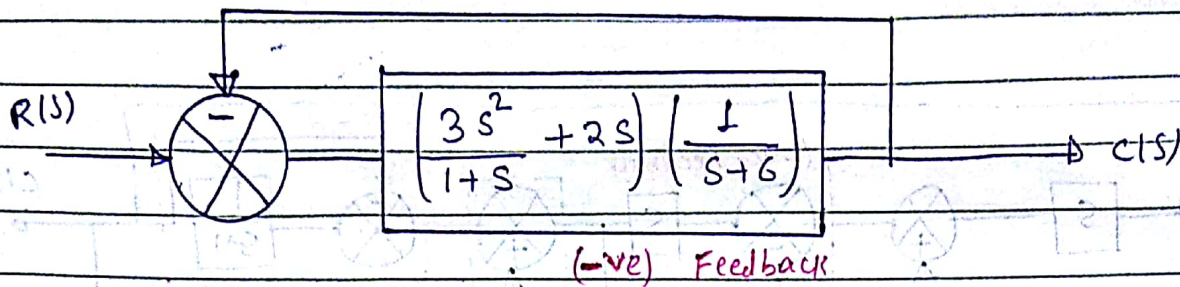
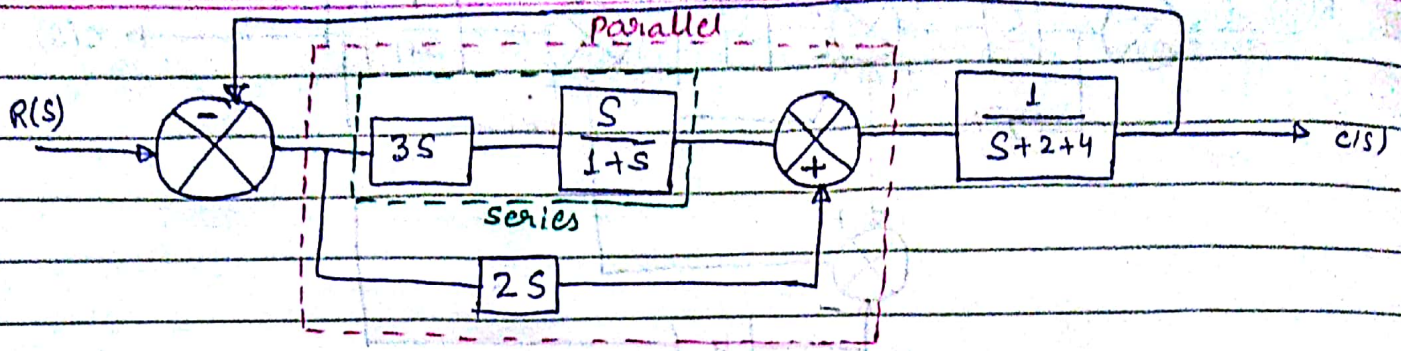
Find:- $\frac{y_5}{y_2}$

$$\frac{y_5}{y_2} = \frac{y_5/y_1}{y_2/y_1}$$

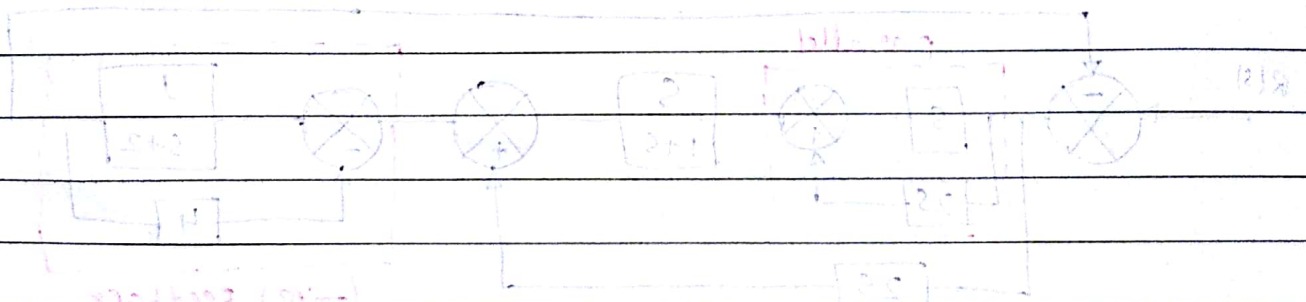
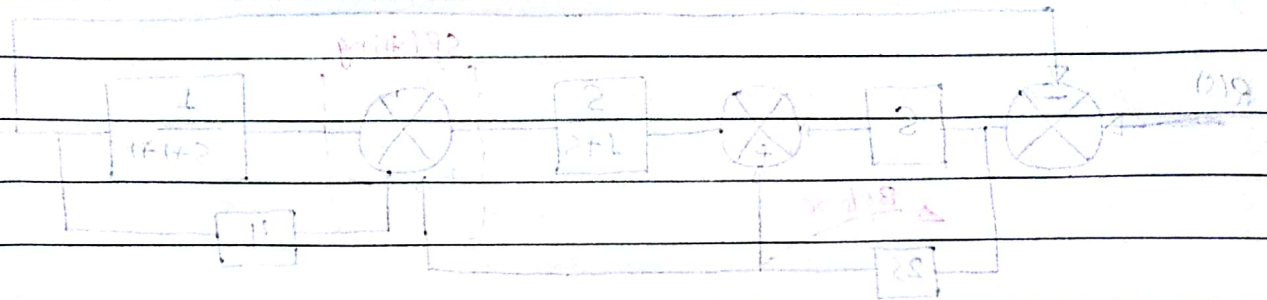
$$= \frac{\left[\frac{1}{1+G_1H_1} \right] \left[(G_1G_2+G_4)(G_3-H_4) - H_3 \right] \left[\frac{1}{1+H_2(G_3-H_4)} \right]}{\left[\frac{1}{1+G_1H_1} \right]}$$

$$\frac{y_5}{y_2} = \frac{[G_1G_2+G_4](G_3-H_4) - H_3}{1+H_2(G_3-H_4)}$$

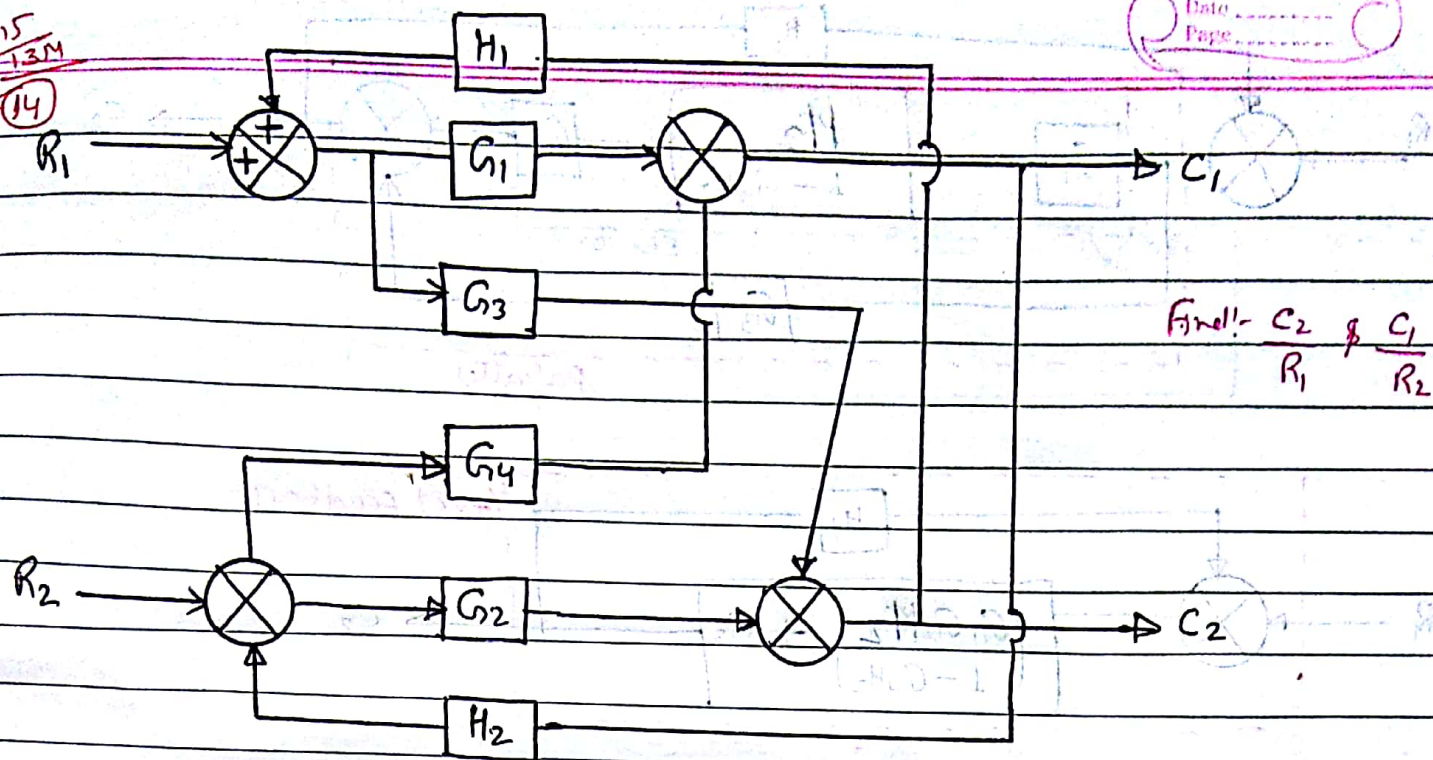




$$T.F = \frac{\left(\frac{3s^2 + 2s}{1+s} \right) \left(\frac{1}{s+6} \right)}{1 + \left(\frac{3s^2 + 2s}{1+s} \right) \left(\frac{1}{s+6} \right)}$$

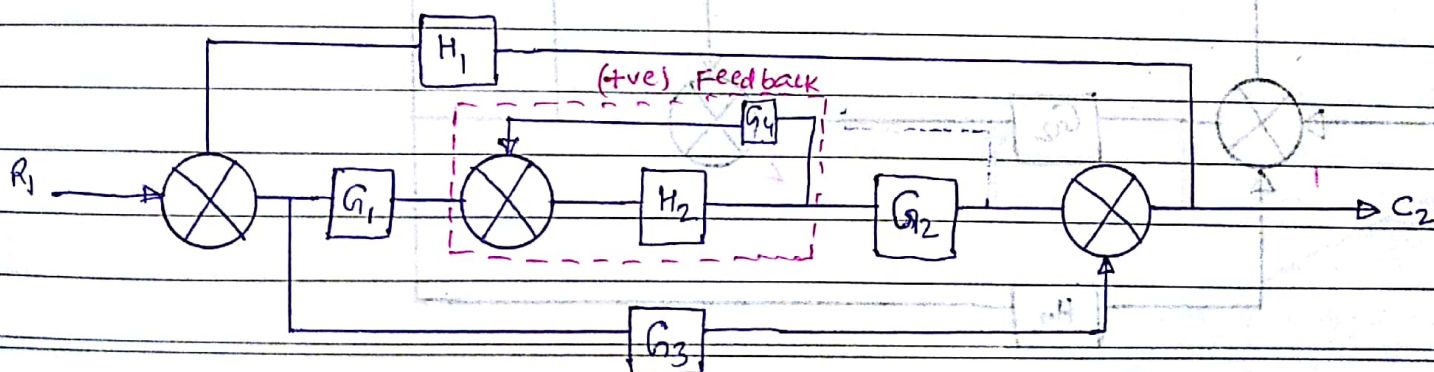
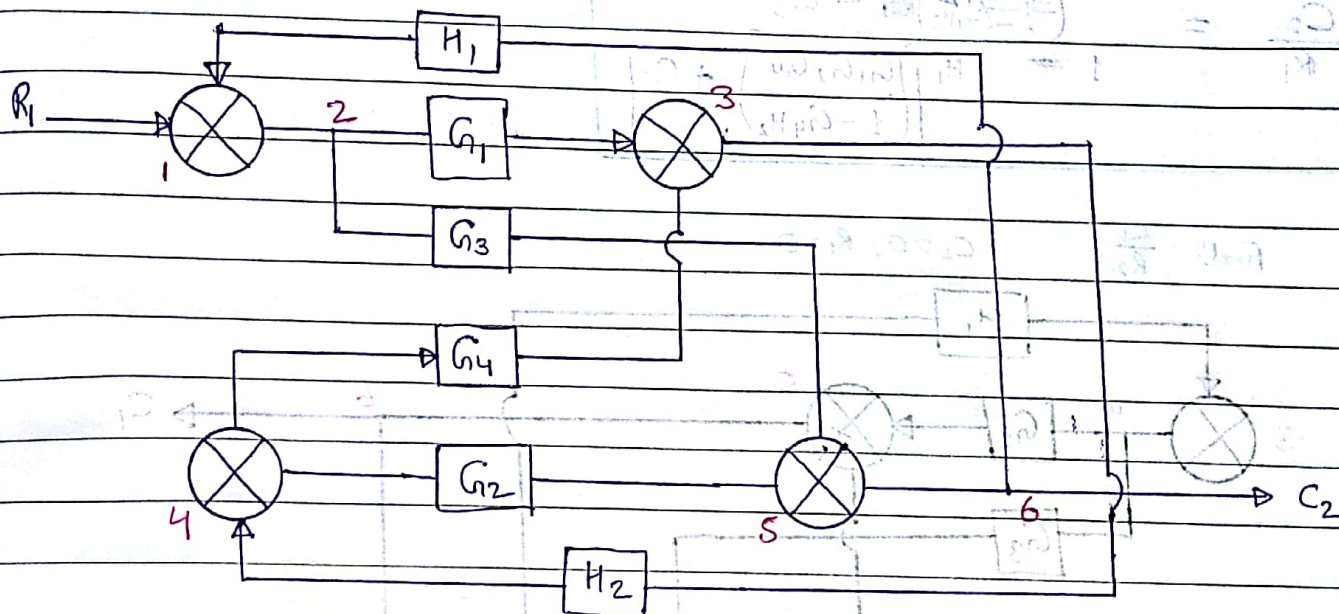


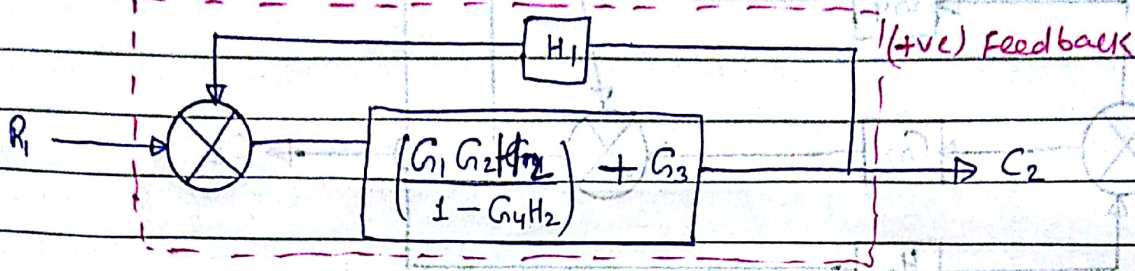
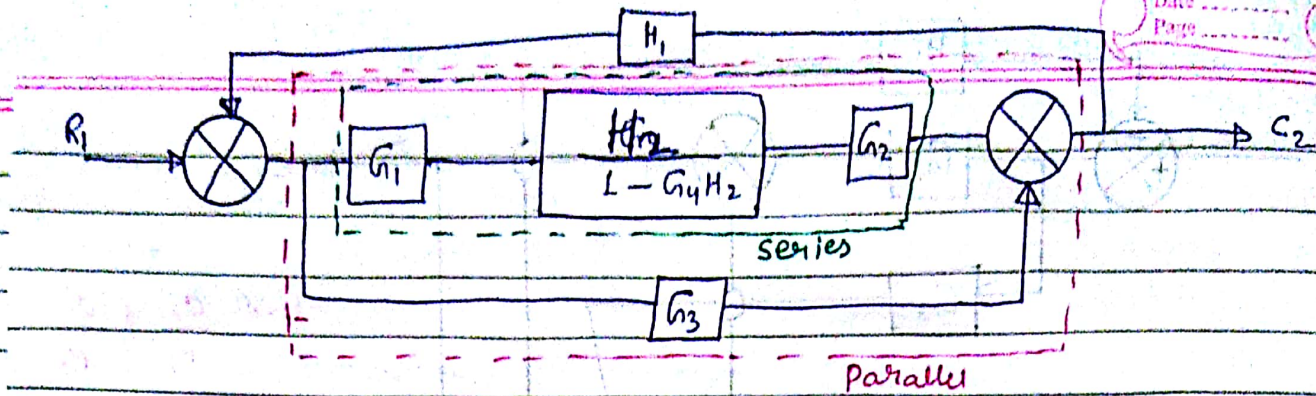
S-15
13M
Q-14



Find:- $\frac{C_2}{R_1}$ & $\frac{C_1}{R_2}$

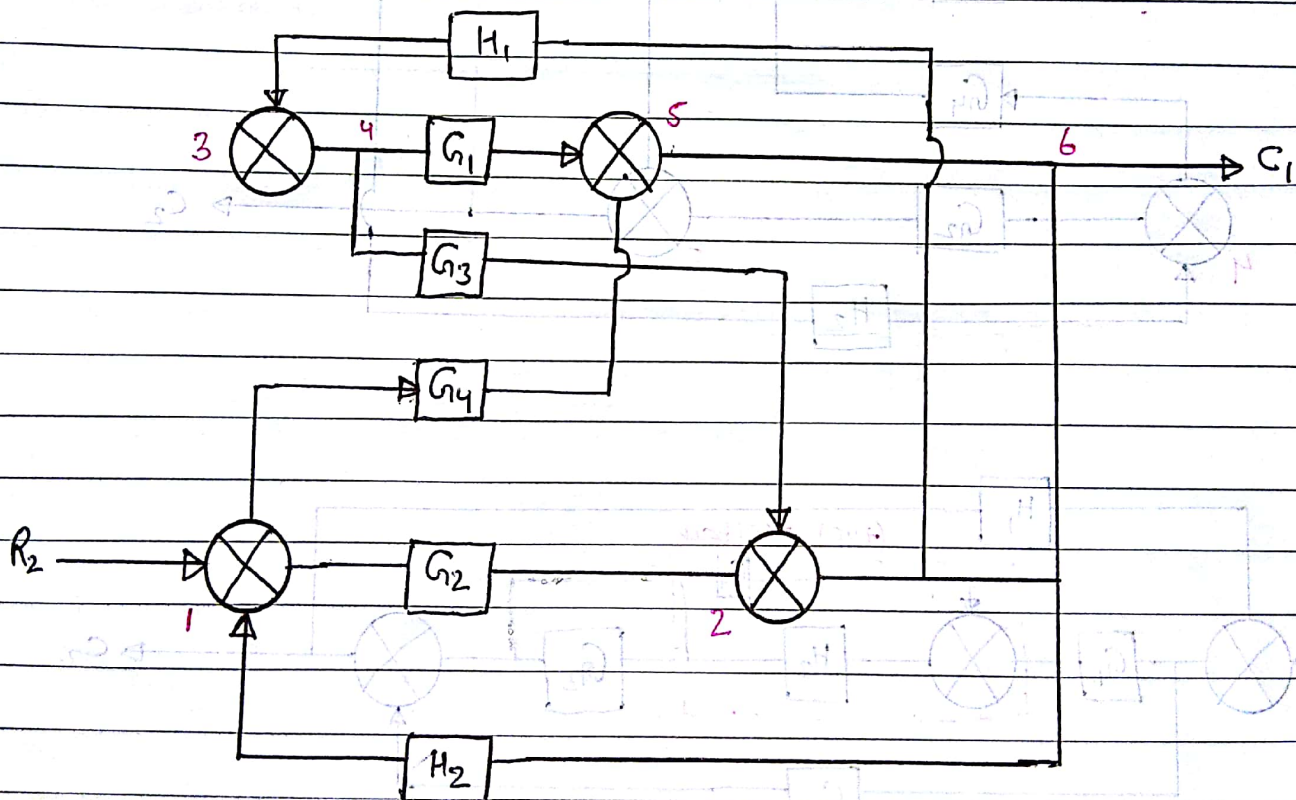
Case:-I $C_1=0$ & $R_2=0$





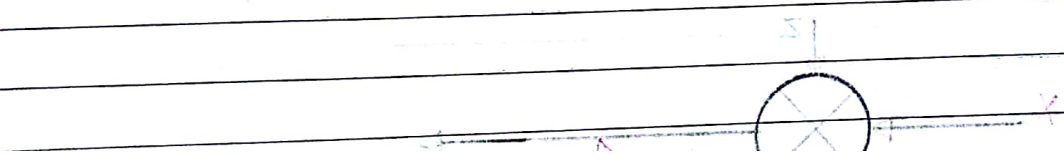
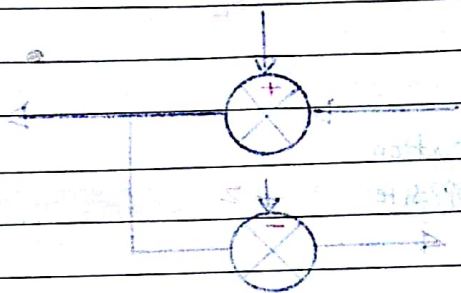
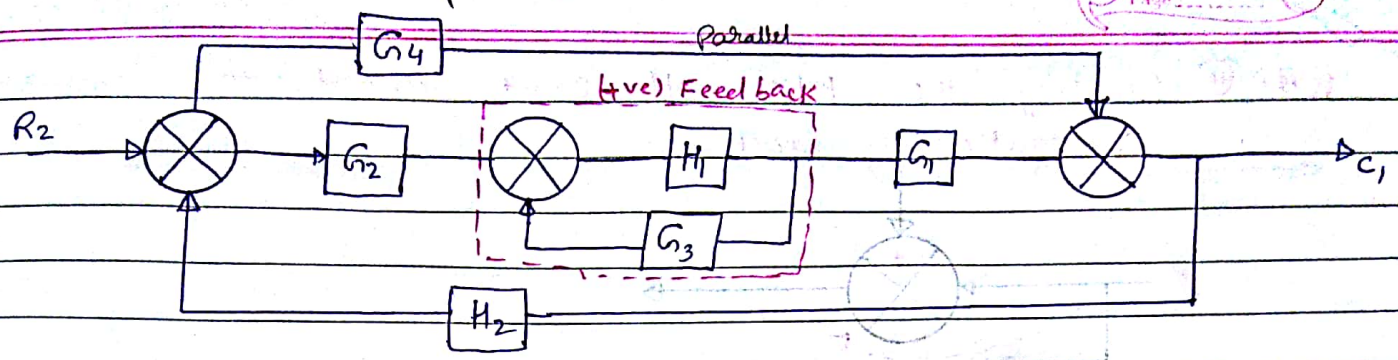
$$\frac{C_2}{R_1} = \frac{\left(\frac{G_1 G_2 H_2}{1 - G_4 H_2} \right) + G_3}{1 - H_1 \left[\left(\frac{G_1 G_2 G_4}{1 - G_4 H_2} \right) + G_3 \right]}$$

Case-II Find:- $\frac{C_1}{R_2}$ $C_2=0, R_1=0$

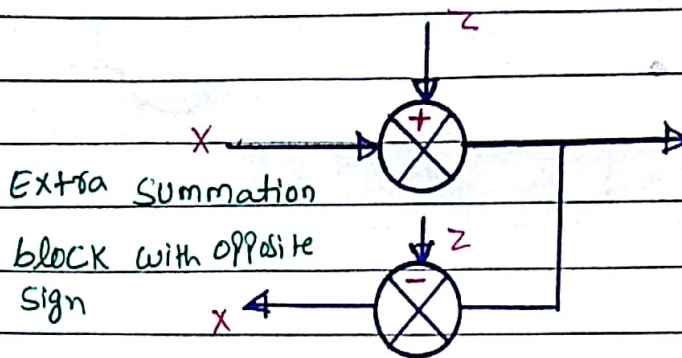
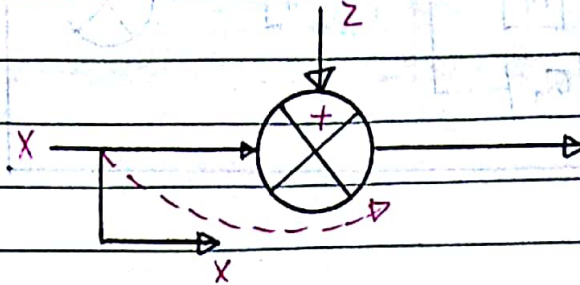


$$\frac{\left(\frac{H_1 G_1 G_2}{1 + H_1 G_1}\right) + G_4}{1 - \left(\frac{G_3}{1 + H_1 G_1}\right) H_2}$$

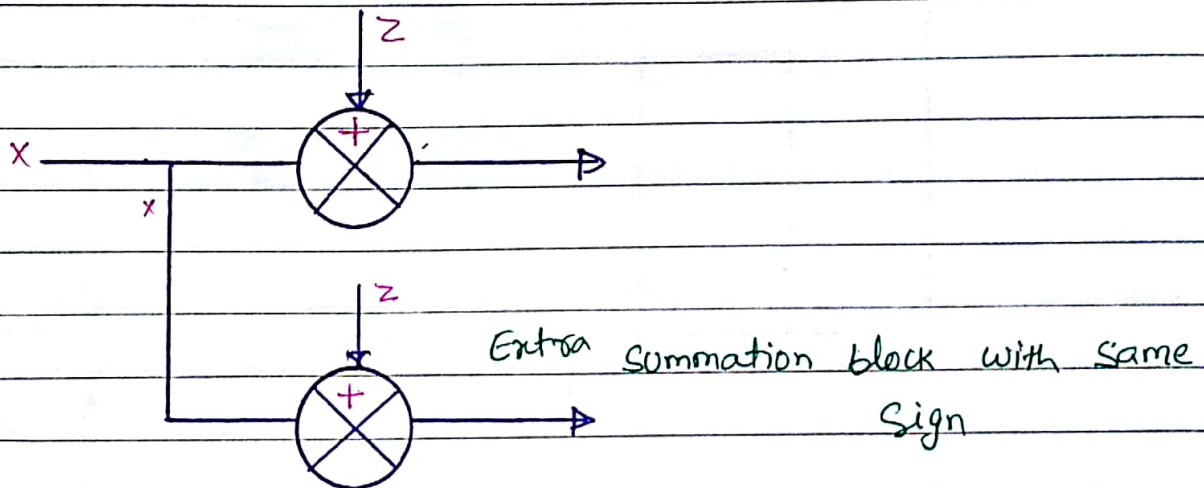
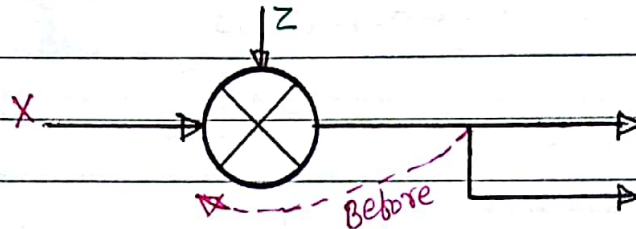
Date _____
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Rule (I) Shifting the take off point ahead of summation block.



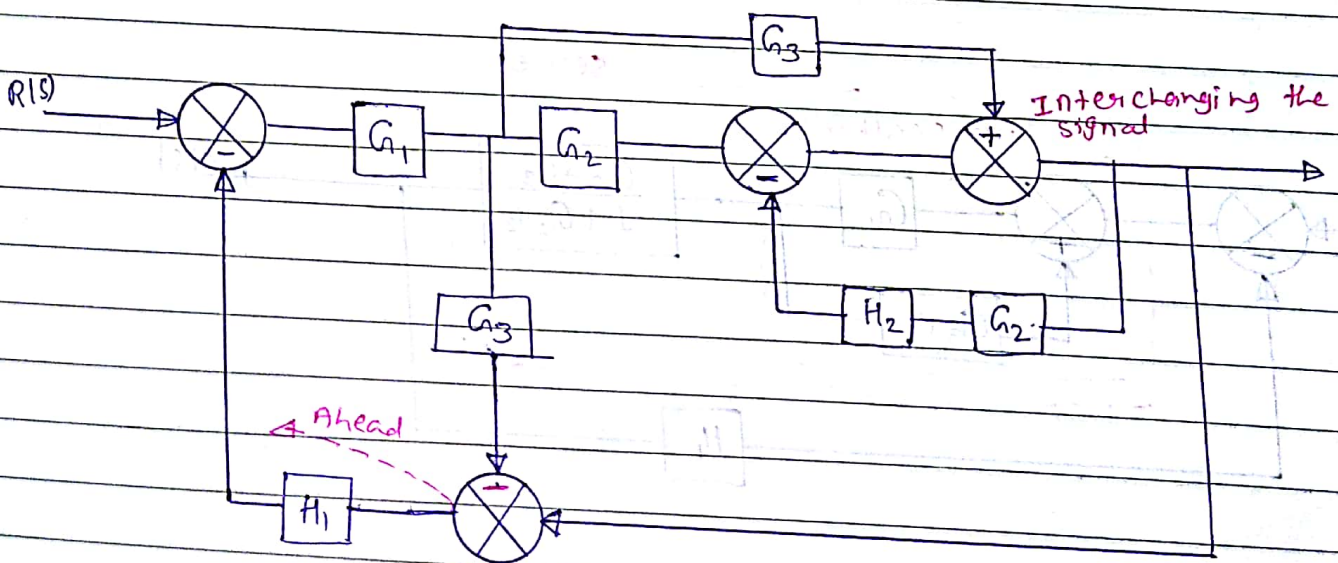
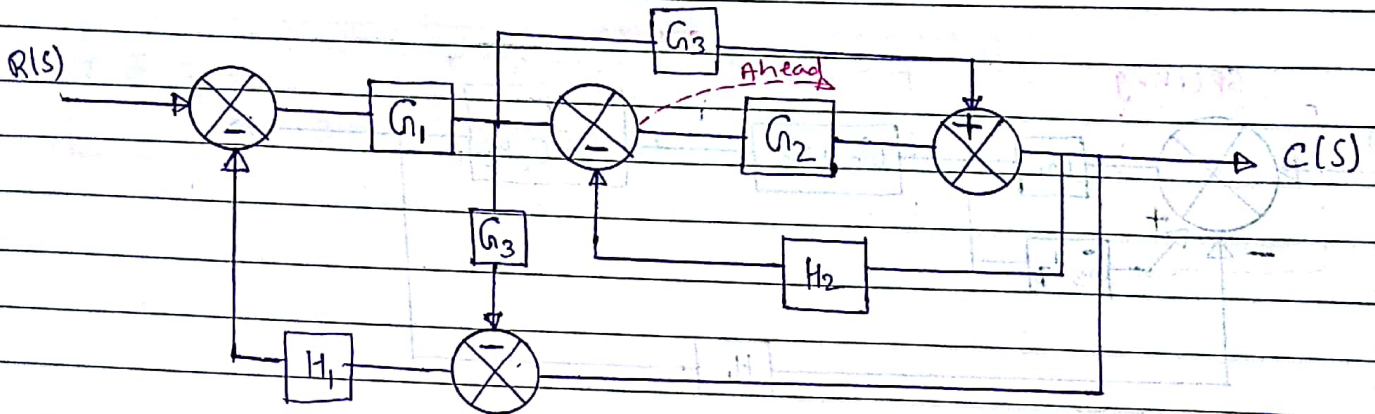
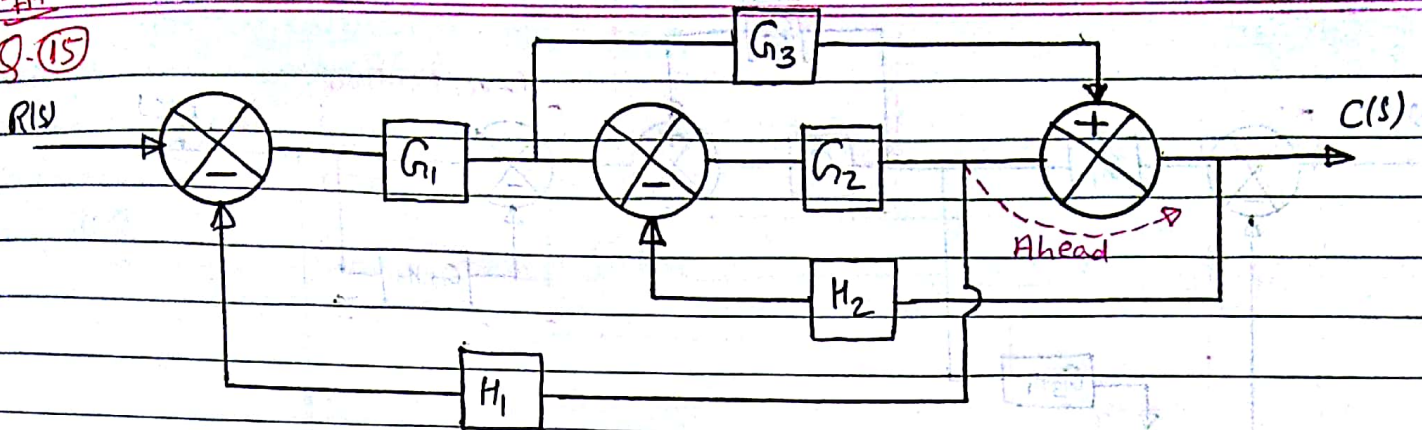
Rule (II) Shifting the take off point before the summation block.



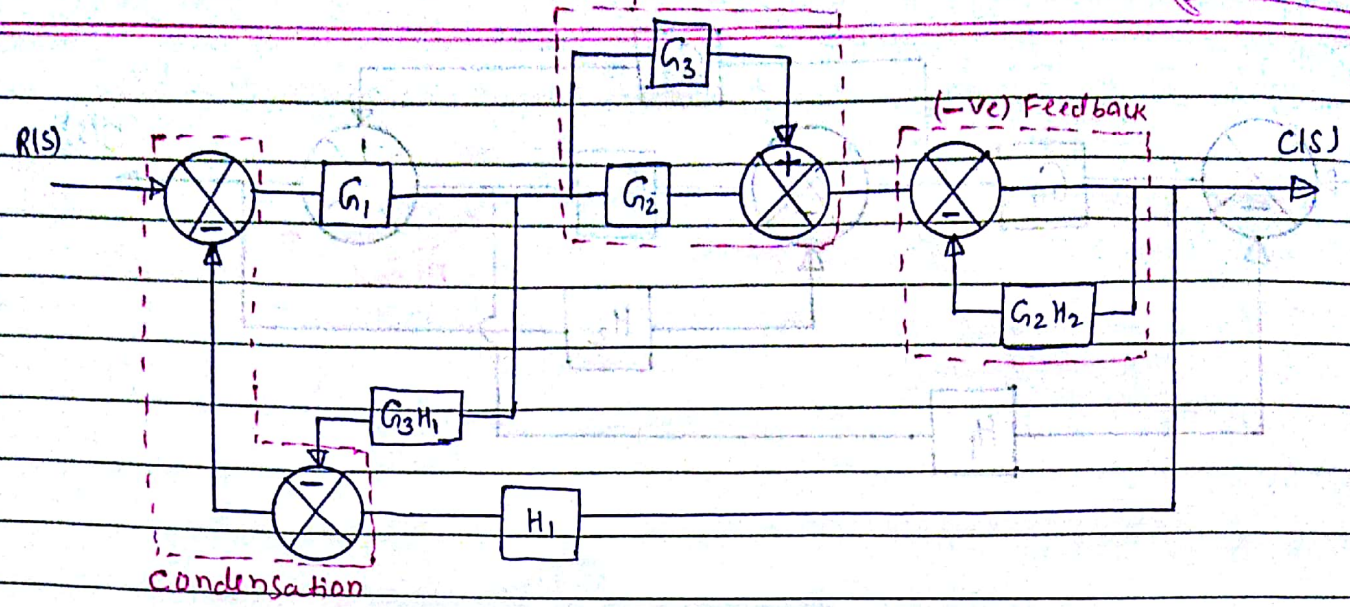
W-16
FM

Q-15

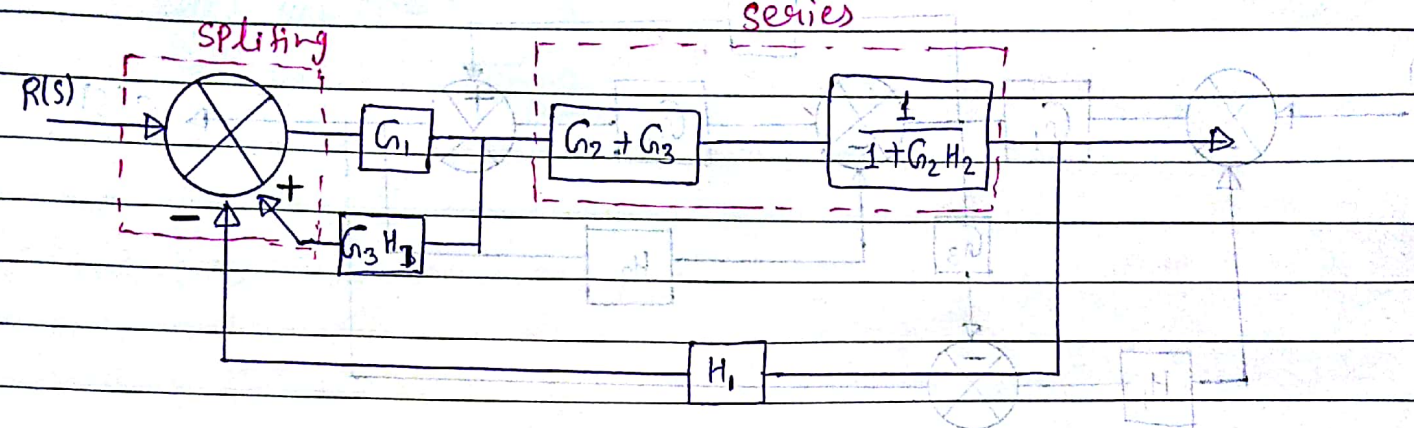
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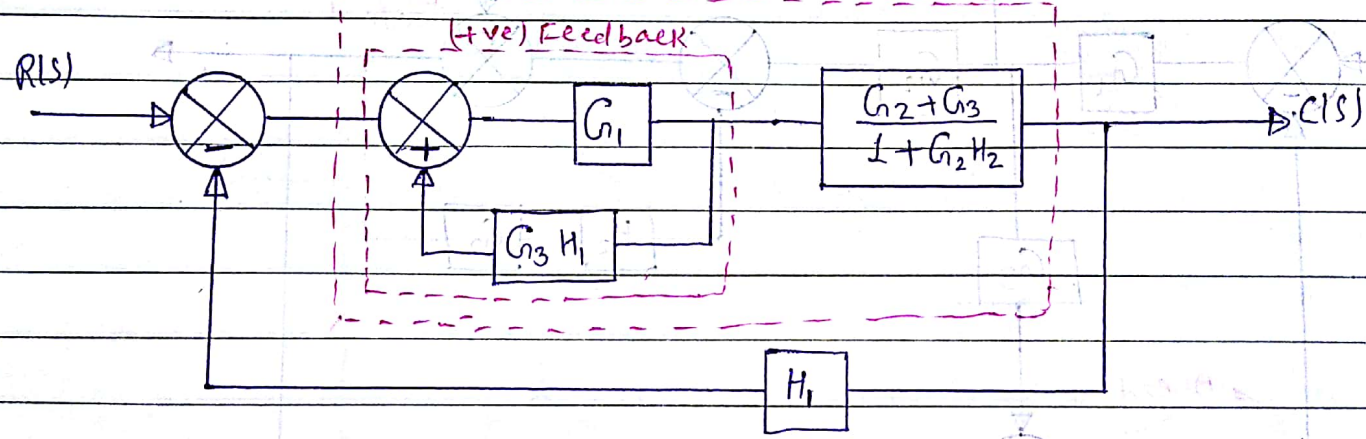
parallel

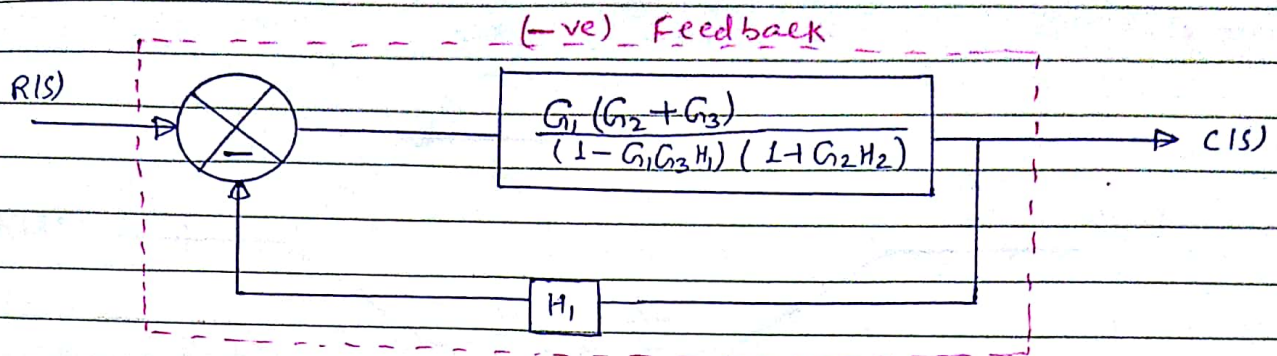


series



series



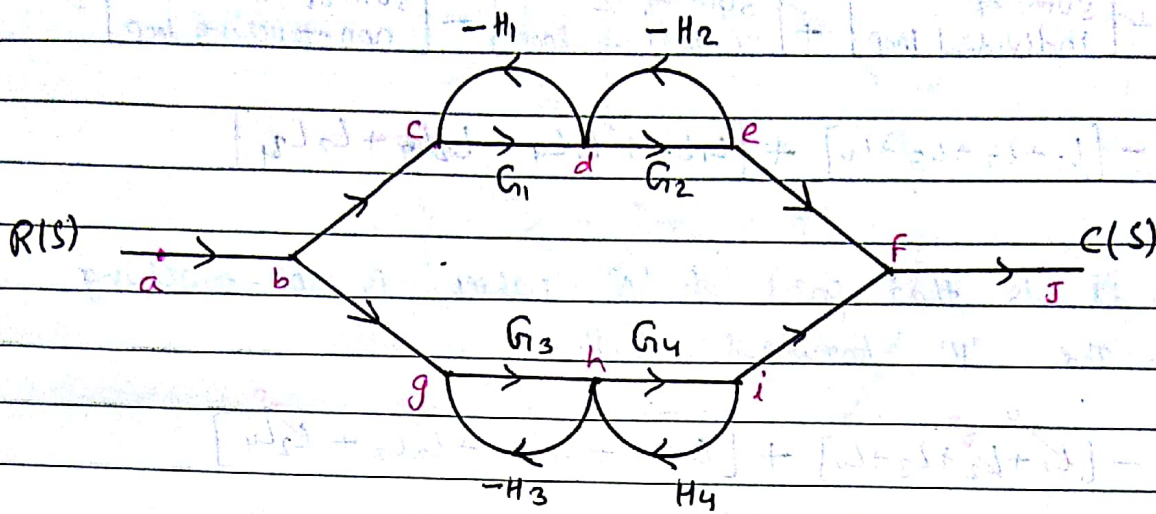


$$T.F. = \frac{G_1 (G_2 + G_3)}{(1 - G_1 G_3 H_1) (1 + G_2 H_2) + G_1 (G_2 + G_3) H_1}$$

SIGNAL Flow Graph

Date 16/01/18

Q-1 Find The Transfer function by Mason's Gain Formula,



- (1) Forward path (P) :-
- $$P_1 = a-b-c-d-e-f-j = G_1 G_2$$
- $$P_2 = a-b-g-h-i-f-j = G_3 G_4$$

(2) Individual loop (L) :-

$$L_1 = c-d-c = -G_1 H_1$$

$$L_2 = d-e-d = -G_2 H_2$$

$$L_3 = g-h-g = -G_3 H_3$$

$$L_4 = h-i-h = G_4 H_4$$

(3) 2-Non-touching loop:-

$$L_1 L_3 = -G_1 G_3 H_1 H_3$$

$$L_1 L_4 = -G_1 G_4 H_1 H_4$$

$$L_2 L_3 = G_2 G_3 H_2 H_3$$

$$L_2 L_4 = -G_2 G_4 H_2 H_4$$

(4) 3 Non touching loop.

Not given

Δ = Determinant of graph

$$= 1 - [\text{Sum of Individual loop}] + [\text{Sum of 2 non-touching loop}] - [\text{Sum of 3 non-touching loop}] + \dots$$

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_3 + L_1 L_4 + L_2 L_3 + L_2 L_4]$$

$\Delta_{K=1} \rightarrow$ it is that part of " Δ " which is non-touching the " K " forward path.

$$\Delta_1 = 1 - [\cancel{L_1} + \cancel{L_2} + L_3 + L_4] + [\cancel{L_1 L_3} + \cancel{L_1 L_4} + L_2 L_3 + L_2 L_4]$$

$$\Delta_1 = 1 - [L_3 + L_4] = 1 + G_3 H_3 - G_4 H_4$$

$$\Delta_2 = 1 - [L_1 + L_2] = 1 + G_1 H_1 + G_2 H_2$$

By manson's gain formula's,

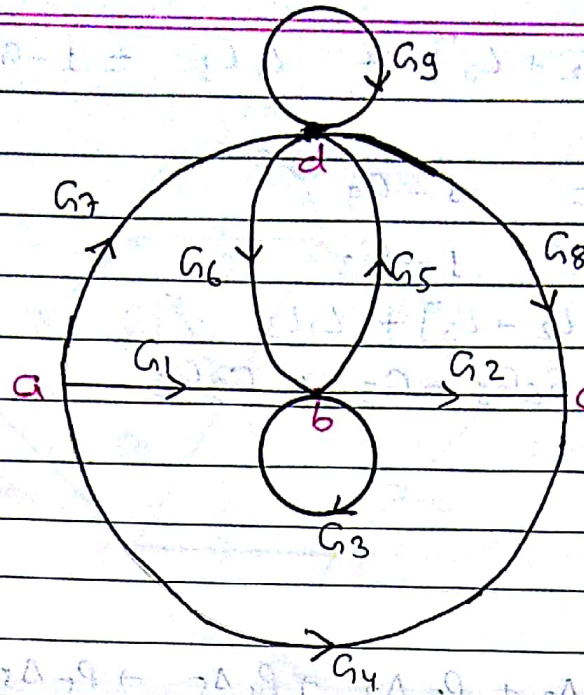
$$T.F. = \frac{\sum P_K \Delta_K}{\Delta}$$

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$T.F. = \frac{G_1 G_2 [1 + G_3 H_3 - G_4 H_4] + G_3 G_4 [1 + G_1 H_1 + G_2 H_2]}{1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_3 + L_1 L_4 + L_2 L_3 + L_2 L_4]}$$

$$T.F. = \frac{G_1 G_2 [1 + G_3 H_3 - G_4 H_4] + G_3 G_4 [1 + G_1 H_1 + G_2 H_2]}{1 - [G_1 H_1 + G_2 H_2 + G_3 H_3 - G_4 H_4] + [G_1 G_3 H_1 H_3 + G_1 G_4 H_1 H_4 + G_2 G_3 H_2 H_3 - G_2 G_4 H_2 H_4]}$$

Q-2



* Forward path (P)

$$P_1 = a-b-c = G_1 G_2$$

$$P_2 = a-d-c = G_7 G_8$$

$$P_3 = a \rightarrow c = G_4$$

$$P_4 = a \rightarrow b \rightarrow d = G_1 G_5 G_8$$

$$P_5 = a \rightarrow d \rightarrow c = G_7 G_6 G_2$$

* Individual loop

$$L_1 = d-d = G_9$$

$$L_2 = d-b-d = G_6 G_5$$

$$L_3 = b-b = G_3$$

* 2 - Non-touching loop

$$L_1 L_3 = G_9 G_3$$

$$\Delta = 1 - [L_1 + L_2 + L_3] + L_1 L_3 = 1 - G_9 - G_6 G_5 - G_3 + G_9 G_3$$

$$\Delta_1 = 1 - L_1 = 1 - G_9$$

$$\Delta_2 = 1 - L_3 = 1 - G_3$$

$$\begin{aligned} \Delta_3 &= 1 - [L_1 + L_2 + L_3] + L_1 L_3 \\ &= 1 - G_9 - G_6 G_5 - G_3 + G_9 G_3 \end{aligned}$$

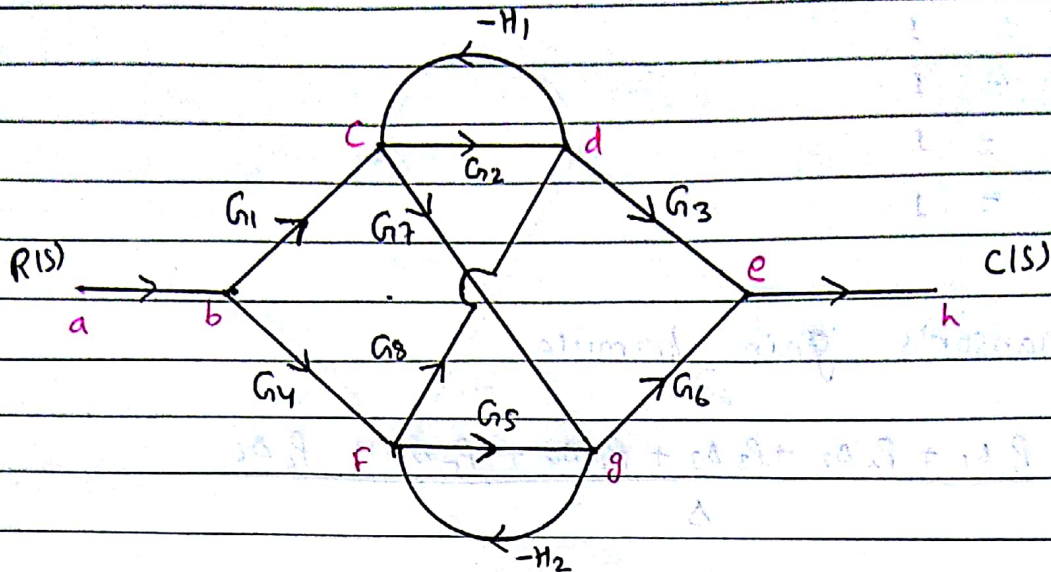
$$\Delta_4 = 1$$

$$\Delta_5 = 1$$

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4 + P_5 \Delta_5}{\Delta}$$

$$\begin{aligned} T.F. &= \frac{G_1 G_2 (1 - G_9) + G_7 G_8 (1 - G_3) + G_4 (1 - G_9 - G_6 G_5 - G_3 + G_9 G_3) \\ &\quad + G_1 G_5 G_8 + G_7 G_6 G_2}{1 - G_9 - G_6 G_5 - G_3 + G_9 G_3} \end{aligned}$$

Q-3



① Forward path

$$P_1 = a-b-c-d-e-h = G_1 G_2 G_3$$

$$P_2 = a-b-f-g-e-h = G_4 G_5 G_6$$

$$P_3 = a-b-c-g-e-h = G_1 G_7 G_6$$

$$P_4 = a-b-f-d-e-h = G_4 G_8 G_3$$

$$P_5 = a-b-c-g-b-d-e-h = -G_1 G_7 H_2 G_8 G_3$$

$$P_6 = a-b-b-d-c-g-e-h = -G_4 G_8 H_1 G_7 G_6$$

② Individual loop (L)

$$L_1 = c-d-c = -G_2 H_1$$

$$L_2 = -G_5 H_2$$

$$L_3 = G_7 H_2 G_8 H_1$$

③ Non-touching loop

$$L_1 L_2 = G_2 G_5 H_1 H_2$$

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_2]$$

$$\Delta_1 = 1 - L_2 = 1 + G_5 H_2$$

$$\Delta_2 = 1 - L_1 = 1 + G_2 H_1$$

$$\Delta_3 = 1$$

$$\Delta_4 = 1$$

$$\Delta_5 = 1$$

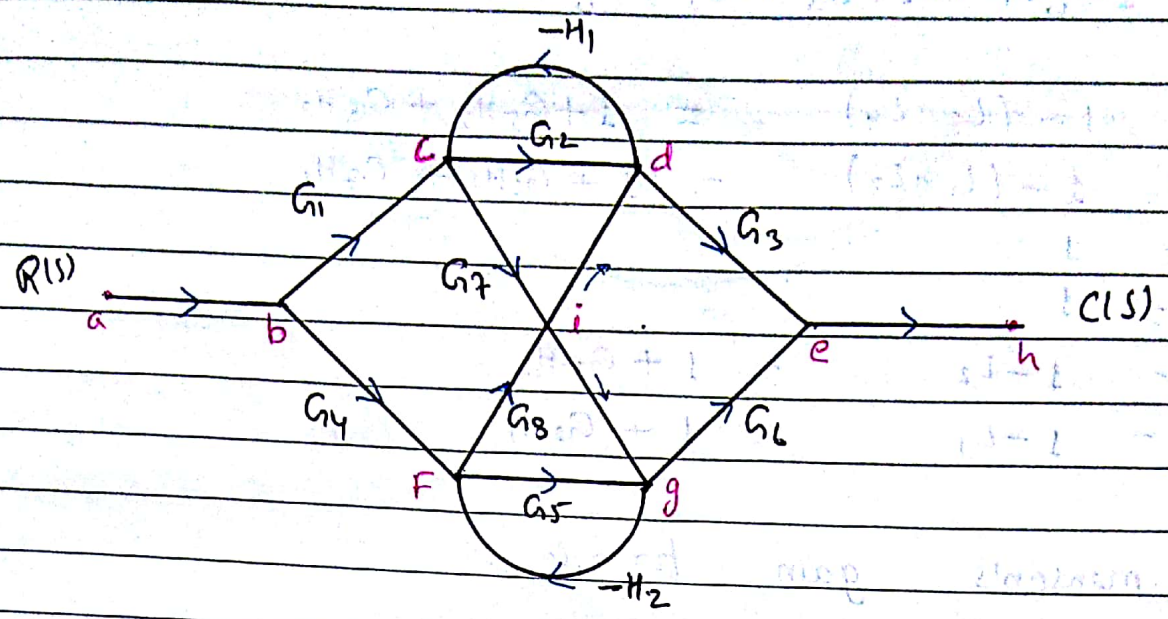
$$\Delta_6 = 1$$

By manson's gain formula,

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4 + P_5 \Delta_5 + P_6 \Delta_6}{\Delta}$$

$$T.F. = \frac{G_1 G_2 G_3 (1 + G_5 H_2) + G_4 G_5 G_6 (1 + G_2 H_1) + G_1 G_6 G_7 + G_4 G_8 G_3 - G_1 G_7 H_2 G_8 G_3 - G_4 G_8 H_1 G_7 G_6}{1 - [-G_2 H_1 - G_5 H_2 + G_7 H_2 G_8 H_1] + G_2 G_5 H_1 H_2}$$

Q-4)



① Forward path

- $P_1 = a-b-c-d-e-h = G_1 G_2 G_3$
- $P_2 = a-b-f-g-e-h = G_4 G_5 G_6$
- $P_3 = a-b-c-i-g-e-h = G_1 G_7 G_6$
- $P_4 = a-b-f-i-d-e-h = G_4 G_8 G_3$
- $P_5 = a-b-c-i-d-e-h = G_1 G_7 G_3$
- $P_6 = a-b-f-i-g-e-h = G_4 G_8 G_6$

② Individual loop

- $L_1 = c-d-c = -G_2 H_1$
- $L_2 = f-g-f = -G_5 H_2$
- $L_3 = c-i-d-c = -G_7 H_1$
- $L_4 = f-i-g-f = -G_8 H_2$

③ Non touching loop

- $L_1 L_2 = G_2 G_5 H_1 H_2$
- $L_1 L_4 = G_2 G_8 H_1 H_2$
- $L_2 L_3 = G_5 G_7 H_1 H_2$

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_2 + L_1 L_4 + L_2 L_3]$$

$$\Delta_1 = 1 - (L_2 + L_4) = 1 + G_5 H_2 + G_8 H_2$$

$$\Delta_2 = 1 - (L_1 + L_3) = 1 + G_2 H_1 + G_7 H_1$$

$$\Delta_3 = 1$$

$$\Delta_4 = 1$$

$$\Delta_5 = 1 - L_2 = 1 + G_5 H_2$$

$$\Delta_6 = 1 - L_1 = 1 + G_2 H_1$$

By mason's gain formula,

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4 + P_5 \Delta_5 + P_6 \Delta_6}{\Delta}$$

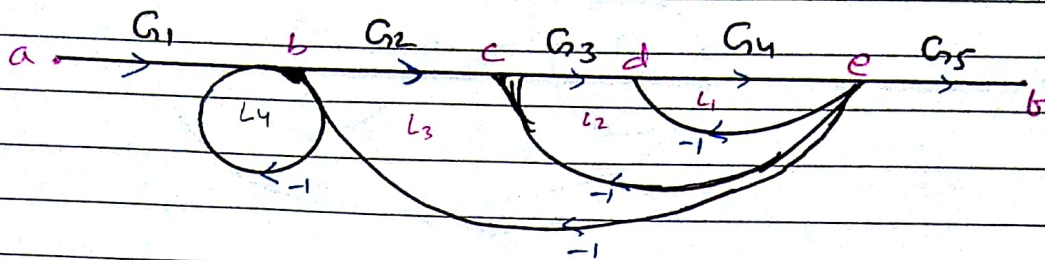
$$\Delta = 1 + G_2 H_1 + G_5 H_2 + G_7 H_1 + G_8 H_2 + G_2 G_5 H_1 H_2 + G_2 G_8 H_1 H_2 + G_5 H_2 G_7 H_1$$

$$T.F. = G_1 G_2 G_3 (1 + G_5 H_2 + G_8 H_2) + G_4 G_5 G_6 (1 + G_2 H_1 + G_7 H_1) + G_1 G_7 G_6 + G_4 G_8 G_3 + G_1 G_7 G_3 (1 + G_5 H_2) + G_4 G_8 G_6 (1 + G_2 H_1) + 1 + G_2 H_1 + G_5 H_2 + G_7 H_1 + G_8 H_2 + G_2 H_1 G_5 H_2 + G_2 G_8 H_1 H_2 + G_5 G_7 H_2 H_1$$

$FIR = P_1$
 $2P = L_1, L_2, L_3, L_4$
 $2N - 10 = L_1, L_4, L_2, L_3$
 $D_1 = 1$

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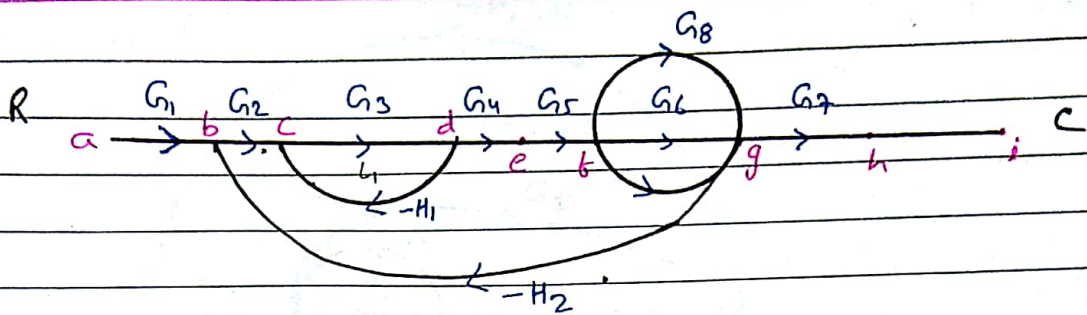
Q-5



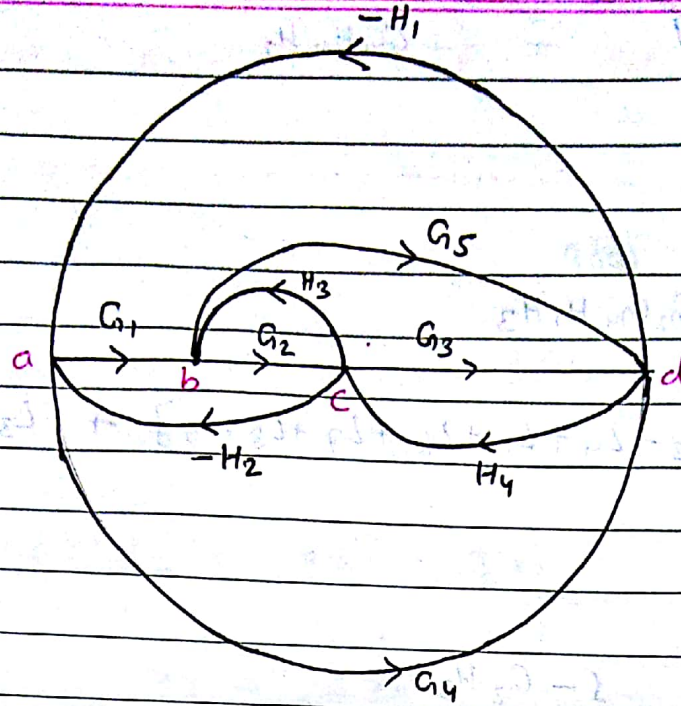
$FP = P_1, P_2, P_3$
 $2L = L_1, L_2, L_3, L_4$
 $D_1 = D_2 = D_3 = 1$

Date
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Q-6



Q-7



Forward path

$$P_1 = a-b-c-d = G_1 G_2 G_3$$

$$P_2 = a-b-d = G_1 G_5$$

$$P_3 = a-d = G_4$$

Individual loop

$$L_1 = a-b-c-a = -G_1 G_2 H_2$$

$$L_2 = c-d-c = G_3 H_4$$

$$L_3 = b-c-b = G_2 H_3$$

$$L_4 = a-d-a = -G_4 H_1$$

$$L_5 = a-b-c-d-a = -G_1 G_2 G_3 H_1$$

$$L_6 = a-b-d-a = -G_1 G_5 H_1$$

$$L_7 = b-c-d-b = G_5 H_4 H_3$$

$$L_8 = a-b-c-d-b-a = -G_1 G_5 H_4 H_2$$

$$L_9 = \begin{array}{c} a \quad c \quad d \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} = -G_4 H_4 H_2$$

2 - non touching loop

$$L_3 L_4 = -G_2 G_4 H_1 H_3$$

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8 + L_9] + L_3 L_4$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\Delta_3 = 1 - L_3 = 1 - G_2 H_3$$

By Mason's gain formula

$$T.F = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

$$\Delta = 1 + G_1 G_2 H_2 - G_3 H_4 - G_2 H_3 + G_4 H_1 + G_1 G_2 G_3 H_1 + G_1 G_5 H_1 - G_5 H_4 H_3 + G_1 G_5 H_4 H_2 + G_4 H_4 H_2 - G_2 G_4 H_1 H_3$$

$$T.F = \frac{G_1 G_2 G_3 + G_1 G_5 + G_4 (1 - G_2 H_3)}{1 + G_1 G_2 H_2 - G_3 H_4 - G_2 H_3 + G_4 H_1 + G_1 G_2 G_3 H_1 + G_1 G_5 H_1 - G_5 H_4 H_3 + G_1 G_5 H_4 H_2 + G_4 H_4 H_2 - G_2 G_4 H_1 H_3}$$

S-16
6/11

Q-8) Construct the SFG for following eqⁿ

$$x_2 + 3x_3 - 3x_1 = 0$$

$$x_3 + 4x_4 - 4x_2 = 0$$

$$x_4 - 5x_3 = 0$$

Solⁿ:-

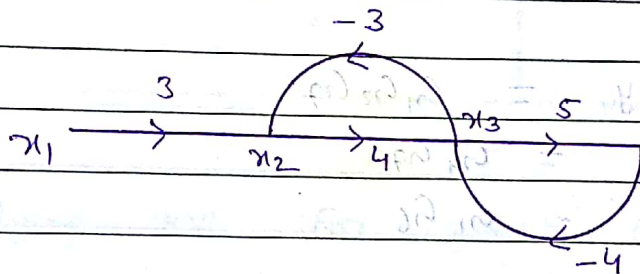
$x_1 \rightarrow$ Input

$x_4 \rightarrow$ output

$$x_2 = 3x_1 - 3x_3$$

$$x_3 = 4x_2 - 4x_4$$

$$x_4 = 5x_3$$



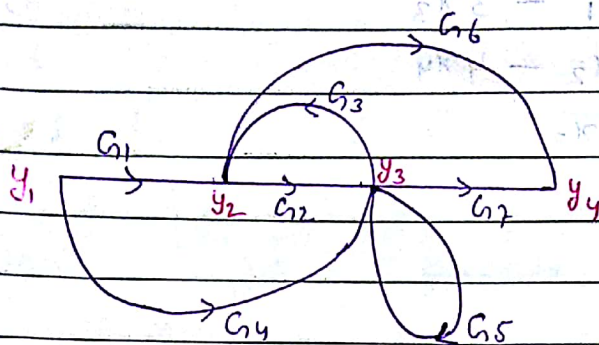
Q-9

Convert algebraic equation in SFG and find TF

$$y_2 = G_1 y_1 + G_3 y_3$$

$$y_3 = G_4 y_1 + G_2 y_2 + G_5 y_3$$

$$y_4 = G_6 y_2 + G_7 y_3$$

 $y_1 \rightarrow$ Input $y_4 \rightarrow$ output

Forward path

$$P_1 = y_1 - y_2 - y_3 - y_4 = G_1 G_2 G_7$$

$$P_2 = y_1 - y_3 - y_4 = G_4 G_7$$

$$P_3 = y_1 - y_2 - y_4 = G_1 G_6$$

$$P_4 = y_1 - y_2 - y_3 - y_4 = G_4 G_3 G_6$$

Individual loop

$$L_1 = y_2 - y_3 - y_2 = G_2 G_3$$

$$L_2 = y_3 - y_3 = G_5$$

$$\Delta = 1 - (L_1 + L_2) = 1 - G_2 G_3 - G_5$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

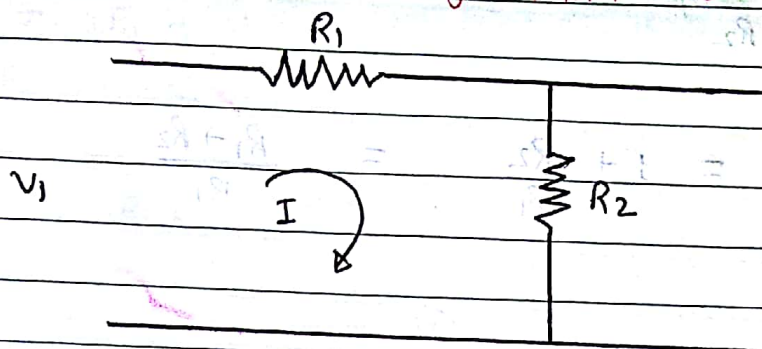
$$\Delta_3 = 1 - L_2 = 1 - G_5$$

$$\Delta_4 = 1$$

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3 + P_4 \Delta_4}{\Delta}$$

$$T.F. = \frac{G_1 G_2 G_7 + G_4 G_7 + G_1 G_6 (1 - G_5) + G_4 G_3 G_6}{1 - G_2 G_3 - G_5}$$

Q-10) Find T.F. = $\frac{V_o(s)}{V_i(s)}$ by manson's gain formula.



(1) Applying KVL for given circuit

$$V_i = IR_1 + IR_2$$

$$\text{but } V_o = IR_2$$

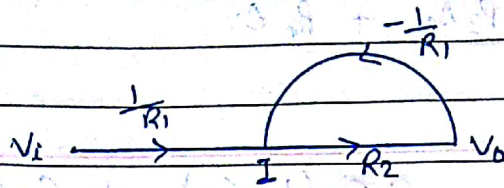
$$V_i = IR_1 + V_o$$

$$\therefore IR_1 = V_i - V_o$$

$$I = \frac{1}{R_1} V_i - \frac{1}{R_2} V_o \quad \text{--- (1)}$$

$$\textcircled{2} \quad V_o = IR_2 \quad \text{---} \quad \textcircled{2}$$

From eqⁿ ① & ②, plot SFG.



* Forward path

$$P_1 = \frac{1}{R_1} \times R_2$$

Individual loop

$$L_1 = -\frac{1}{R_1} \times R_2$$

$$\Delta = 1 - L_1 = 1 + \frac{R_2}{R_1} = \frac{R_1 + R_2}{R_1}$$

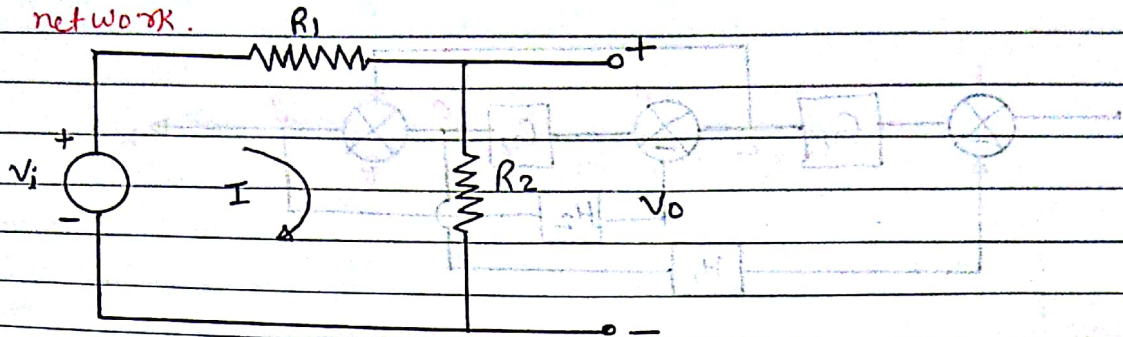
$$\Delta_1 = 1$$

By manson's gain formula,

$$T.F. = \frac{P_1 \Delta_1}{\Delta} = \frac{R_2/R_1}{\frac{R_1 + R_2}{R_1}} = \frac{R_2}{R_1 + R_2}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_1 + R_2}}$$

Q 11) Draw close loop block diagram of voltage divider network.



① Applying KVL for given circuit

$$V_i = IR_1 + IR_2$$

$$\text{but, } V_o = IR_2$$

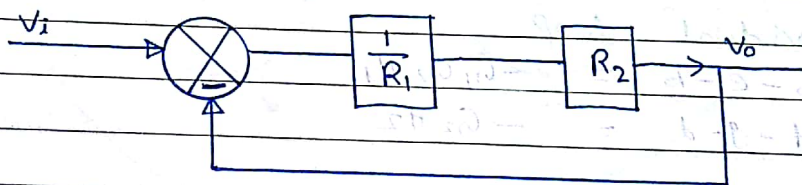
$$\therefore V_i = IR_1 + V_o$$

$$IR_1 = V_i - V_o$$

$$I = \frac{1}{R_1} [V_i - V_o] \quad \text{--- (1)}$$

$$\text{② } V_o = IR_2 \quad \text{--- (2)}$$

From eqn (1) & (2) draw Block diagram.



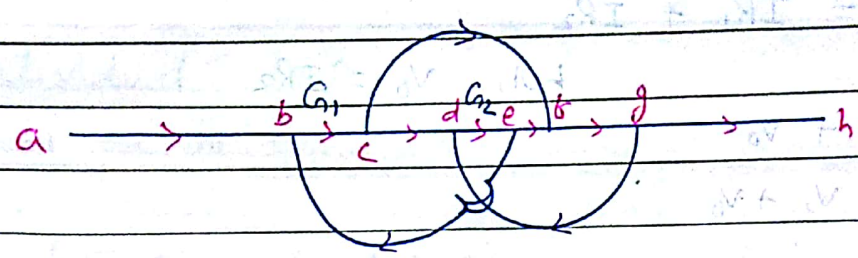
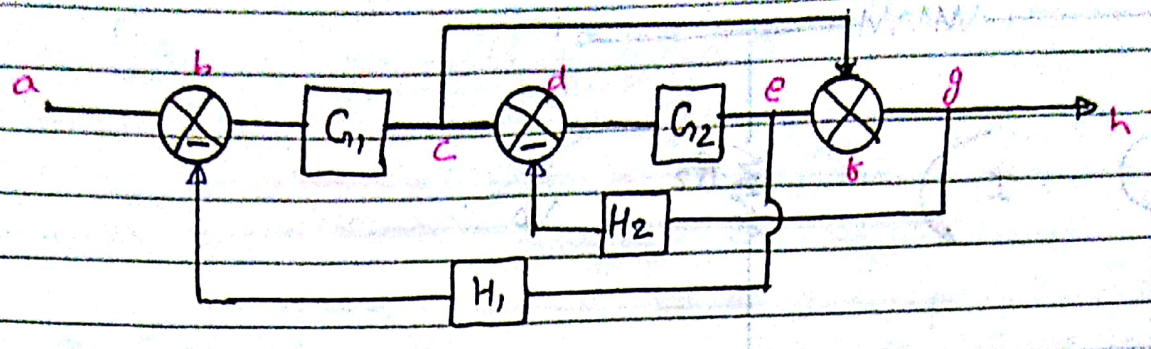
Series & (-ve) Feedback

$$\text{T.f} = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

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7M

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Q-12) Convert The block diagram in SFG and find T.F.



* Forward Path

$$P_1 = a \rightarrow h = G_1 G_2$$

$$P_2 = a-b-c-f-g-h = G_1$$

* Individual loop

$$L_1 = b-e-b = -G_1 G_2 H_1$$

$$L_2 = d-g-d = -G_2 H_2$$

$$L_3 = \text{[Diagram of loop b-d-e-f-g-b]} = G_1 H_2 G_2 H_1$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 + G_1 G_2 H_1 + G_2 H_2 - G_1 H_2 G_2 H_1$$

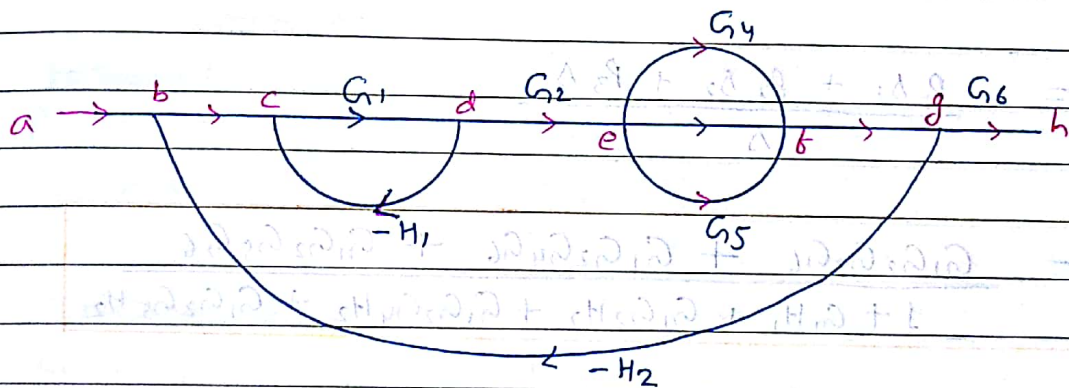
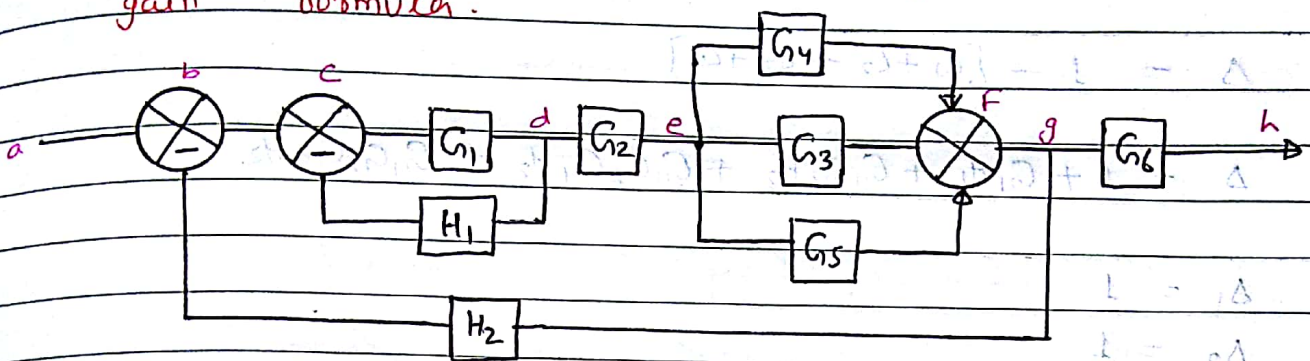
$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$T.F. = \frac{G_1 G_2 + G_1}{1 + G_1 G_2 H_1 + G_2 H_2 - G_1 H_2 G_2 H_1}$$

Q-13) Obtain SFG and Find T.F by using mason's gain formula.



* Forward path

$$P_1 = a \rightarrow h = G_1 G_2 G_3 G_6$$

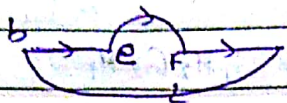
$$P_2 = a \rightarrow e \rightarrow h = G_1 G_2 G_4 G_6$$

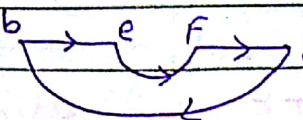
$$P_3 = a \rightarrow e \rightarrow h = G_1 G_2 G_5 G_6$$

* Individual loop

$$L_1 = c-d-c = -G_1 H_1$$

$$L_2 = b-g-b = -G_1 G_2 H_2$$

$$L_3 = \text{b} \rightarrow \text{e} \rightarrow \text{f} \rightarrow \text{g} \rightarrow \text{b} = -G_1 G_2 G_4 H_2$$


$$L_4 = \text{b} \rightarrow \text{e} \rightarrow \text{f} \rightarrow \text{g} \rightarrow \text{b} = -G_1 G_2 G_5 H_2$$


$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4]$$

$$\Delta = 1 + G_1 H_1 + G_1 G_2 H_2 + G_1 G_2 G_4 H_2 + G_1 G_2 G_5 H_2$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

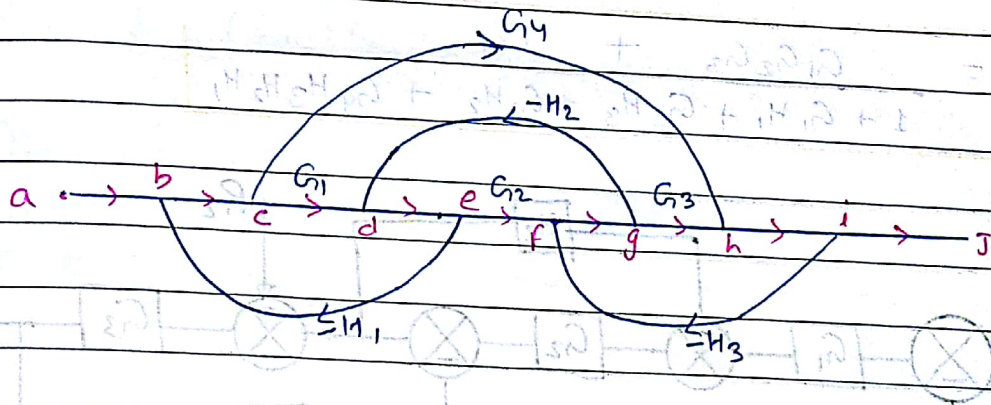
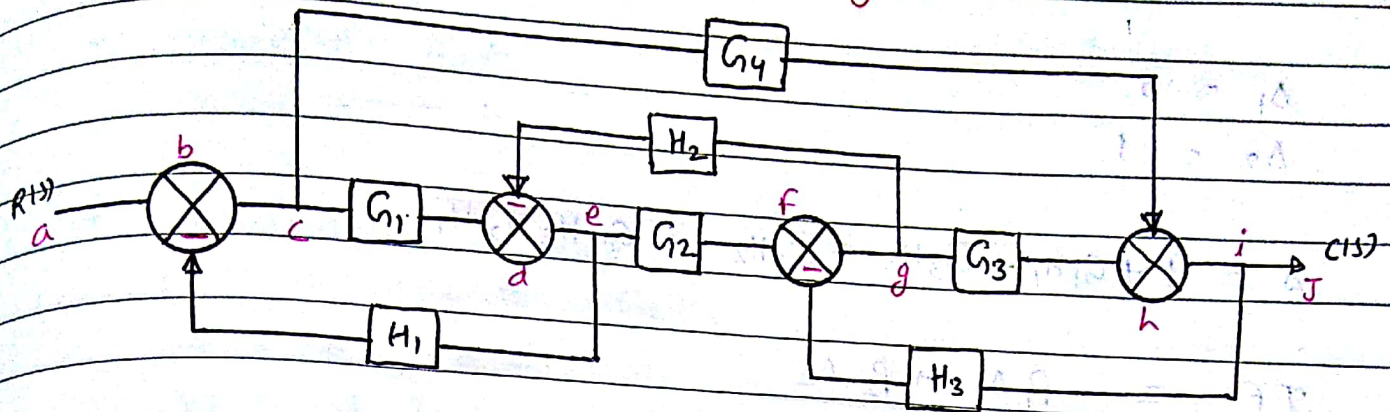
$$\Delta_3 = 1$$

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

$$T.F. = \frac{G_1 G_2 G_3 G_6 + G_1 G_2 G_4 G_6 + G_1 G_2 G_5 G_6}{1 + G_1 H_1 + G_1 G_2 H_2 + G_1 G_2 G_4 H_2 + G_1 G_2 G_5 H_2}$$

Q14

Find T.F. by manson's gain formula.

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* Forward path

$$P_1 = a \rightarrow j = G_1 G_2 G_3$$

$$P_2 = a \rightarrow c \rightarrow h \rightarrow j = G_4$$

* Individual loop

$$L_1 = b-e-b = -G_1 H_1$$

$$L_2 = f-i-f = -G_3 H_3$$

$$L_3 = d-g-d = -G_2 H_2$$

$$L_4 = b-c-h-i-f-g-d-e-b = -G_4 H_3 H_2 H_1$$

* 2 - Non touching loop

$$L_1 L_2 = G_1 G_3 H_1 H_3$$

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + L_1 L_2$$

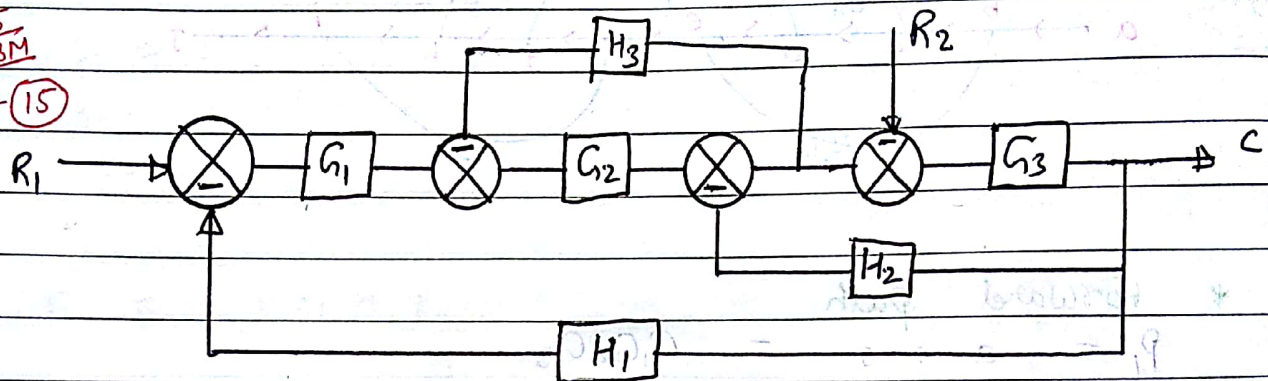
$$\Delta_1 = 1$$

$$\Delta_2 = 1$$

$$\Delta = 1 + G_1 H_1 + G_3 H_3 + G_2 H_2 + G_4 H_3 H_2 H_1$$

$$T.F. = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$T.F. = \frac{G_1 G_2 G_3 + G_4}{1 + G_1 H_1 + G_3 H_3 + G_2 H_2 + G_4 H_3 H_2 H_1}$$

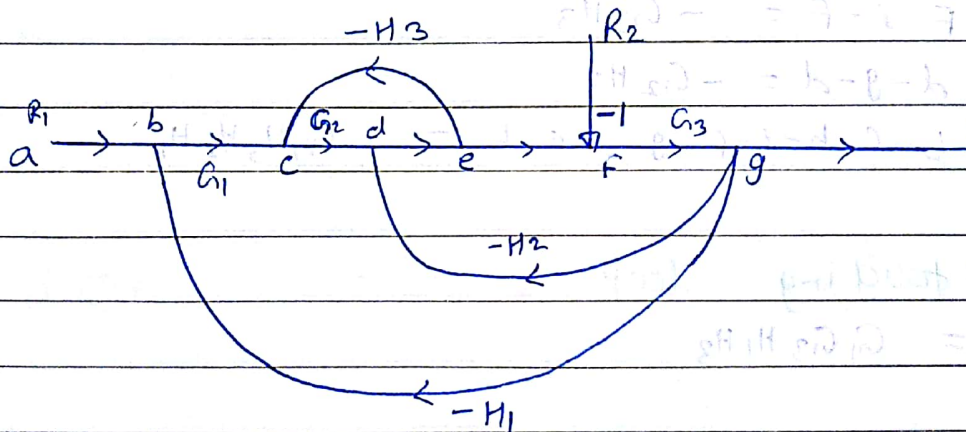


Obtain an expression for C/R_1 & C/R_2

$R_1 \rightarrow$ Reference input signal

$R_2 \rightarrow$ noise signal

use signal flow graph



Case-I Find:- $\frac{C}{R_1}$

Take, $R_2 = 0$

* Forward path

$$P_1 = a \rightarrow h = G_1 G_2 G_3$$

* Individual loop

$$L_1 = c - e - c = -G_2 H_3$$

$$L_2 = b - g - b = -G_1 G_2 G_3 H_1 - 1$$

$$L_3 = d - g - d = -G_3 H_2$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta_1 = 1$$

$$T.F. = \frac{C}{R_1} = \frac{P_1 \Delta_1}{\Delta}$$

$$\frac{C}{R_1} = \frac{G_1 G_2 G_3}{1 + G_2 H_3 + G_1 G_2 G_3 H_1 + G_3 H_2}$$

Case-II Find:- $\frac{C}{R_2}$

Take, $R_1 = 0$

* Forward path

$$P_1 = f \rightarrow g \rightarrow h = G_3$$

* Individual loop

$$L_1 = c - e - c = -G_2 H_3$$

$$L_2 = b - g - b = -G_1 G_2 G_3 H_1$$

$$L_3 = d - g - d = -G_3 H_2$$

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta_1 = 1 - L_1 = 1 + G_2 H_3$$

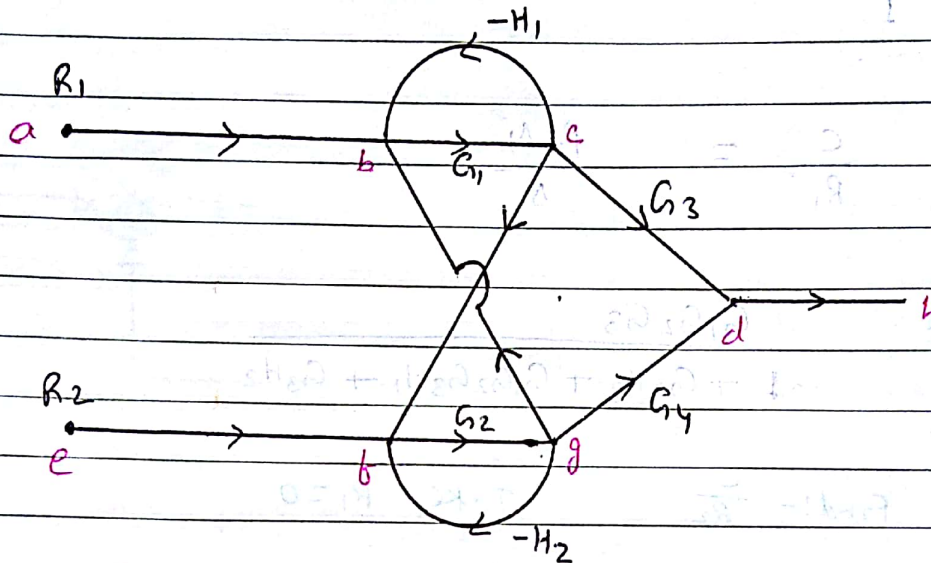
$$T.F. = \frac{C}{R_2} = \frac{P_1 \Delta_1}{\Delta}$$

$$\frac{C}{R_2} = \frac{G_3 (1 + G_2 H_3)}{1 + G_2 H_3 + G_1 G_2 G_3 H_1 + G_3 H_2}$$

W-15
8M

Q-16

Find overall T.F.



* Forward path

$$P_1 = a-b-c-d-h = G_1 G_3$$

$$P_2 = a-b-c-f-g-d-h = G_1 G_2 G_4$$

* Individual loop

$$L_1 = b-c-b = -G_1 H_1$$

$$L_2 = f-g-f = -G_2 H_2$$

$$L_3 = b-c-f-g-b = G_1 G_2$$

2 - Non touching loop

$$L_1 L_2 = G_1 G_2 H_1 H_2$$

$$\Delta = 1 - [L_1 + L_2 + L_3] + L_1 L_2$$

$$\Delta_1 = 1 - L_2 = 1 + G_2 H_2$$

$$\Delta_2 = 1$$

$$T.F_1 = \frac{C}{R_1} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\Delta = 1 + G_1 H_1 + G_2 H_2 - G_1 G_2 + G_1 G_2 H_1 H_2$$

$$T.F_1 = \frac{G_1 G_3 (1 + G_2 H_2) + G_1 G_2 G_4}{1 + G_1 H_1 + G_2 H_2 - G_1 G_2 + G_1 G_2 H_1 H_2}$$

Case-II

$$\frac{C}{R_2}$$

Take, $R_1 = 0$

* Forward path

$$* P_1 = e-f-g-d-h = G_2 G_4 H_1 H_2$$

$$P_2 = e-f-g-b-c-d-h = G_2 G_1 G_3$$

* Individual loop

$$L_1 = F-g-F = -G_2 H_2$$

$$L_2 = b-c-b = -G_1 H_1$$

$$L_3 = F-g-b-c-F = G_2 G_1$$

* 2 - Non touching loop

$$L_1 L_2 = G_1 G_2 H_1 H_2$$

$$\Delta = 1 - (L_1 + L_2 + L_3) + L_1 L_2$$

$$\Delta_1 = 1 - L_2 = 1 + G_1 H_1$$

$$\Delta_2 = 1$$

$$T.F_2 = \frac{C}{R_2} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\Delta = 1 + G_2 H_2 + G_1 H_1 - G_2 G_1 + G_1 G_2 H_1 H_2$$

$$T.F_2 = \frac{G_2 G_4 (1 + G_1 H_1) + G_1 G_2 G_3}{1 + G_2 H_2 + G_1 H_1 - G_2 G_1 + G_1 G_2 H_1 H_2}$$

* Overall T.F.

$$T.F. = T.F_1 + T.F_2$$

$$T.F. = \frac{G_1 G_3 (1 + G_2 H_2) + G_1 G_2 G_4}{1 + G_1 H_1 + G_2 H_2 - G_1 G_2 + G_1 G_2 H_1 H_2} + \frac{G_2 G_4 (1 + G_1 H_1) + G_1 G_2 G_3}{1 + G_2 H_2 + G_1 H_1 - G_2 G_1 + G_1 G_2 H_1 H_2}$$

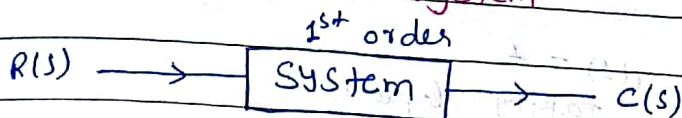
Time Response and System ResponseDate 24/01/18
Page

* When a System is given an input then we get a response (output) with time. Then it is called as Time response.

* There are two types of System:-

- (1) First order System
- (2) Second order System

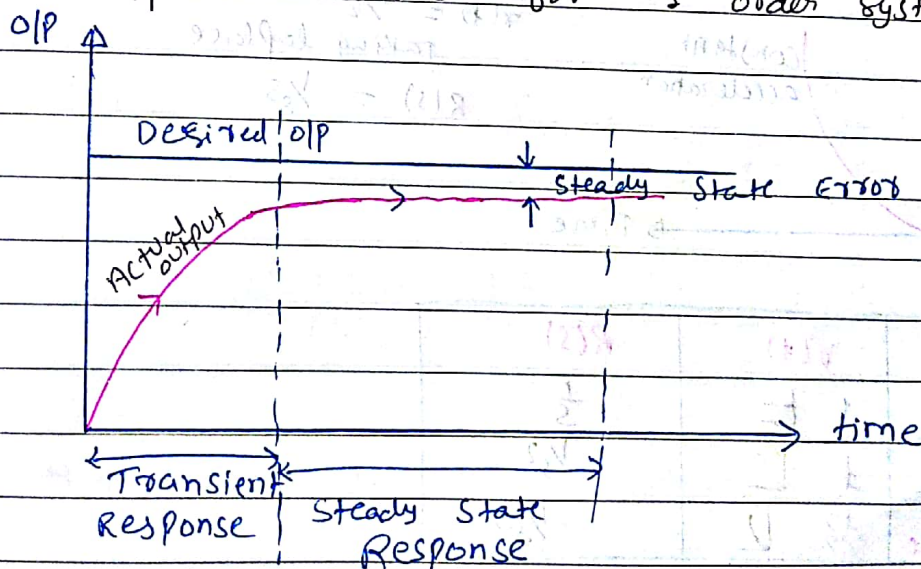
(1) First order System



$$T.F = \frac{C(s)}{R(s)} = \frac{1}{1 + \tau s}$$

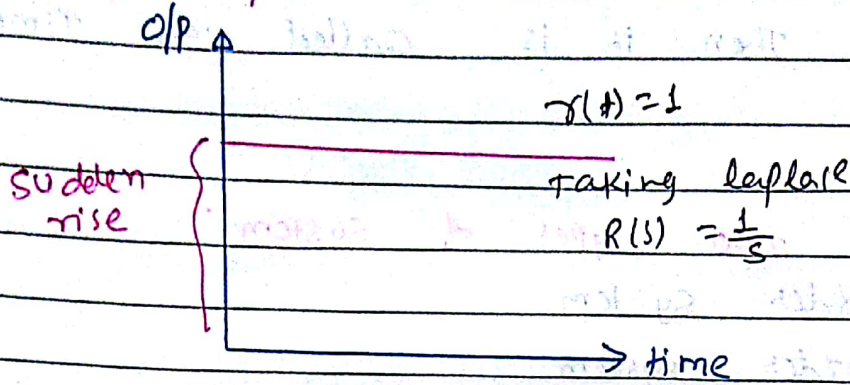
$\tau \rightarrow$ Time constant

* Time Response curve for 1st order system,

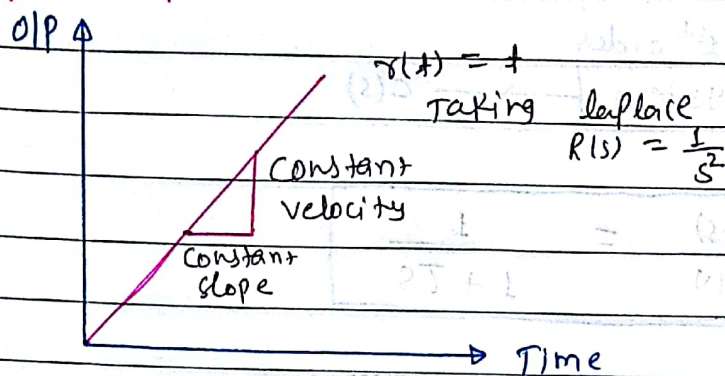


There are Three types of input :-

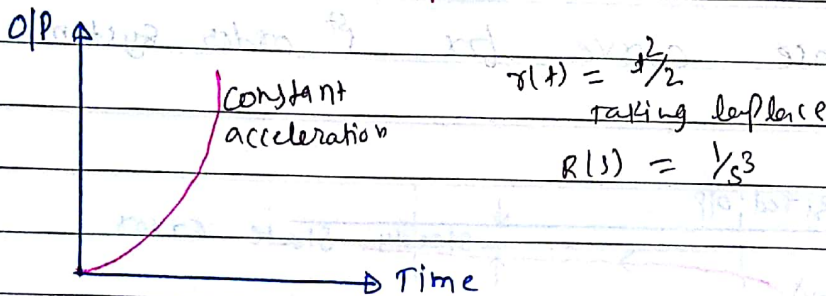
(1) Step input



(2) Ramp input



(3) parabolic Input



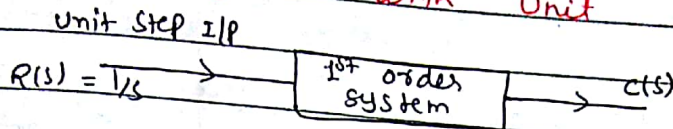
I/P	$x(t)$	$R(s)$
Step	1 $\sqcap$	$\frac{1}{s}$
Ramp	t $\searrow$	$\frac{1}{s^2}$
Parabolic	$\frac{t^2}{2}$ $\searrow$	$\frac{1}{s^3}$

Derivation

Show That the Response on standard 1st order System is 63.2% for unit step input.

W-17
SM

* Derive an expression for response of the first order system with unit step input.



For 1st order system,

$$T.F = \frac{C(s)}{R(s)} = \frac{1}{1 + \tau s}$$

For step I/P, $R(s) = \frac{1}{s}$

$$\therefore \frac{C(s)}{1/s} = \frac{1}{1 + \tau s}$$

$$C(s) = \frac{1}{s} \times \frac{1}{1 + \tau s}$$

* By partial fraction method,

$$C(s) = \frac{1}{s} \times \frac{1}{1 + \tau s} = \frac{A}{s} + \frac{B}{s + 1/\tau} \quad \text{--- (1)}$$

$$\frac{1}{s(s + 1/\tau)} = \frac{A(s + 1/\tau) + Bs}{s(s + 1/\tau)}$$

$$1 = A + A\tau s + Bs$$

$$1 = A + (A\tau + B)s$$

Compare LHS & RHS,

$$\boxed{A=1}$$

$$\S \quad A\tau + B = 0$$

$$\boxed{B = -\tau}$$

put in eqn ①

$$C(s) = \frac{1}{s} - \frac{\tau}{1+\tau s}$$

$$C(s) = \frac{1}{s} - \frac{\tau}{\tau(s + \frac{1}{\tau})}$$

taking inverse laplace,

$$C(t) = 1 - e^{-\frac{1}{\tau}t}$$

$$= 1 - e^{-\frac{t}{\tau}}$$

$$= 1 - e^{-1}$$

$$= 1 - 0.3678$$

$$= 0.632$$

$$\boxed{C(t) = 63.2\%}$$

NOTE:-

Function

Laplace

$$1 \rightarrow \frac{1}{s}$$

$$t \rightarrow \frac{1}{s^2}$$

$$t^2 \rightarrow \frac{1}{s^3}$$

$$t^2 \rightarrow \frac{2}{s^3}$$

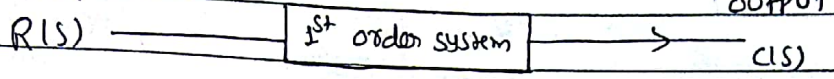
$$e^{at} \rightarrow \frac{1}{s-a}$$

$$e^{-at} \rightarrow \frac{1}{s+a}$$

W-16
FM

Derive an expression for Unit Ramp Response of first order system.

Ramp Input,



Now,

for 1st order,

$$T.F. = \frac{C(s)}{R(s)} = \frac{1}{1 + \tau s}$$

for Ramp Input, $R(s) = \frac{1}{s^2}$

put in above eqn,

$$\therefore \frac{C(s)}{\frac{1}{s^2}} = \frac{1}{1 + \tau s}$$

$$\therefore C(s) = \frac{1}{s^2} \cdot \frac{1}{1 + \tau s}$$

By partial fraction method,

$$C(s) = \frac{1}{s^2} \cdot \frac{1}{1 + \tau s} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{1 + \tau s} \quad \text{--- (1)}$$

$$\frac{1}{s^2(1 + \tau s)} = \frac{A s(1 + \tau s) + B(1 + \tau s) + C s^2}{s^2(1 + \tau s)}$$

$$1 = A s + A \tau s^2 + B + B \tau s + C s^2$$

$$1 = B + (A + B \tau) s + (A \tau + C) s^2$$

Compare LHS and RHS

$$\therefore \boxed{B = 1}$$

$$\therefore A + B\tau = 0$$

$$A = -\tau$$

$$\therefore A\tau + C = 0$$

$$C = +\tau^2$$

put A, B, C in eqn (1)

$$\therefore C(s) = \frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{s+1}$$

$$\therefore C(s) = -\tau\left(\frac{1}{s}\right) + \frac{1}{s^2} + \frac{\tau^2}{s+1}$$

Taking Inverse Laplace

$$\therefore C(t) = -\tau(1) + t + \tau e^{-t}$$

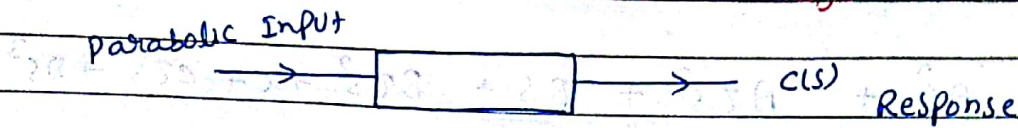
$$\text{put } t = \tau$$

$$\therefore C(t) = -\tau + \tau + \tau e^{-\tau}$$

$$\therefore C(t) = \tau e^{-1}$$

$$C(t) = 0.3678 \tau$$

* Derive an expression for unit parabolic Response of 1st order System.



For 1st order

$$T.F = \frac{C(s)}{R(s)} = \frac{1}{1 + \tau s}$$

For parabolic Input, $R(s) = \frac{1}{s^3}$

put in above eqⁿ,

$$\frac{C(s)}{1/s^3} = \frac{1}{1 + \tau s}$$

$$C(s) = \frac{1}{s^3} \times \frac{1}{1 + \tau s}$$

By partial Fraction method,

$$\therefore C(s) = \frac{1}{s^3} \cdot \frac{1}{1 + \tau s}$$

$$\therefore C(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s^3} + \frac{D}{1 + \tau s} \quad \text{--- (1)}$$

$$\frac{1}{s^3(1+\tau s)} = \frac{As^2(1+\tau s) + Bs(1+\tau s) + C(1+\tau s) + Ds^3}{s^3(1+\tau s)}$$

$$\therefore 1 = As^2 + A\tau s^3 + Bs + B\tau s^2 + C + C\tau s + Ds^3$$

$$1 = C + (B+C\tau)s + (A+B\tau)s^2 + (A\tau+D)s^3$$

Compare LHS and RHS.

$$\therefore \boxed{C=1}$$

$$* \quad B + C\tau = 0$$

$$\boxed{B = -\tau}$$

$$* \quad A + B\tau = 0$$

$$\boxed{A = \tau^2}$$

$$* \quad A\tau + D = 0$$

$$\boxed{D = -\tau^3}$$

Put A, B, C, D in eqn (1)

$$\therefore G(s) = \frac{\tau^2}{s} - \frac{\tau}{s^2} + \frac{1}{s^3} - \frac{\tau^3}{1+\tau s}$$

$$\therefore C(s) = \tau^2 \left(\frac{1}{s} \right) - \tau \left(\frac{1}{s^2} \right) + \frac{1}{s^3} - \frac{\tau^3}{\tau(s + \frac{1}{2})}$$

Take inverse Laplace,

$$C(t) = \tau^2(1) - \tau(t) + \frac{t^2}{2} - \tau^2 e^{-\frac{1}{2}t}$$

put $t = \tau$

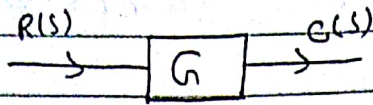
$$\therefore C(t) = \tau^2 - \tau(\tau) + \frac{\tau^2}{2} - \tau^2 e^{-\frac{\tau}{2}}$$

$$= \frac{\tau^2}{2} - \tau^2 e^{-1}$$

$$C(t) = 0.5 \tau^2 - 0.3678 \tau^2$$

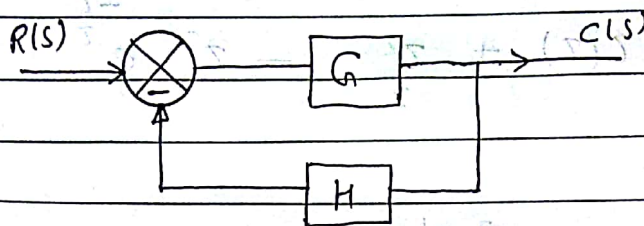
$$\therefore \boxed{C(t) = 0.1322 \tau^2}$$

open loop Transfer function



$$OLTF = \frac{C(s)}{R(s)} = G$$

* Closed loop Transfer function



$$CLTF = \frac{C(s)}{R(s)} = \frac{G}{1 + GH}$$

* The denominator of transfer function is called as characteristic eqⁿ.

Here,

$$Ch \cdot eq^n \rightarrow 1 + GH$$

* 2nd order system standard Transfer function:-

$$T.F. = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

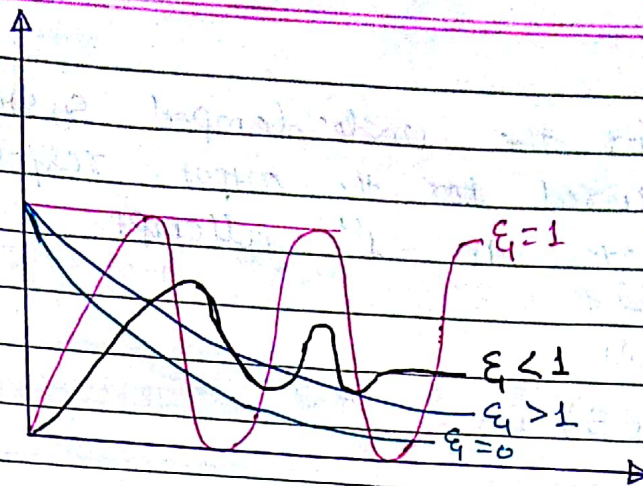
Here,

$$Ch \cdot eq^n \rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$$

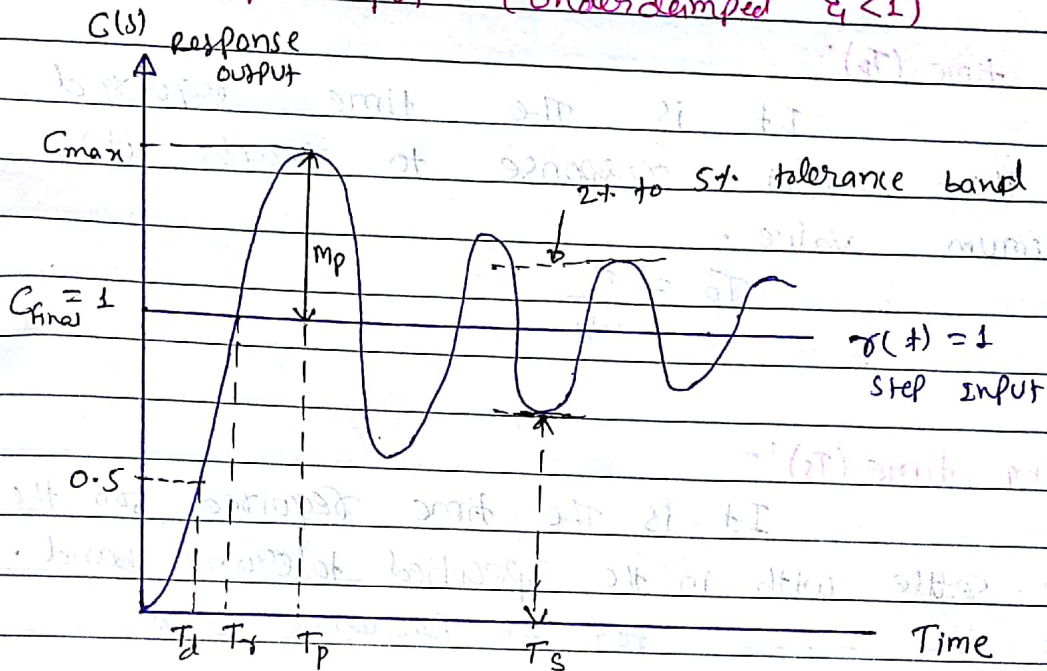
where,

$\omega_n \rightarrow$ undamped natural frequency

$\zeta \rightarrow$ damping factor (ratio)



Time Response curve for 2nd order system with step input (underdamped $\xi < 1$)



① Delay time (T_d):-

It is the time required for the output curve to reach 50% of final value in 1st attempt.

$$T_d = \frac{1 + 0.7 \xi}{\omega_n}$$

② Rise Time (T_r):-

For the underdamped system it is the time required for the output response to rise from 0 to 100% in 1st attempt.

$$T_r = \frac{\pi - \phi}{\omega_d}$$

where, $\phi = (\cos^{-1} \xi) \frac{\pi}{180} \text{ rad}$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$\omega_d \rightarrow$ damped natural frequency.

③ Peak time (T_p):-

It is the time required for the output response to reach its maximum value.

$$T_p = \frac{\pi}{\omega_d}$$

④ Settling time (T_s):-

It is the time required for the final output to settle within the specified tolerance band.

$$T_s = \frac{4}{\xi \omega_n} \text{ --- for 2% Tolerance band}$$

$$T_s = \frac{3}{\xi \omega_n} \text{ --- for 5% Tolerance band}$$

⑤ Peak overshoot (m_p):-

It is the maximum overshoot above the 100% output line.

$$m_p = \left(\frac{C_{\max} - C_{\text{final}}}{C_{\text{final}}} \right) \times 100$$

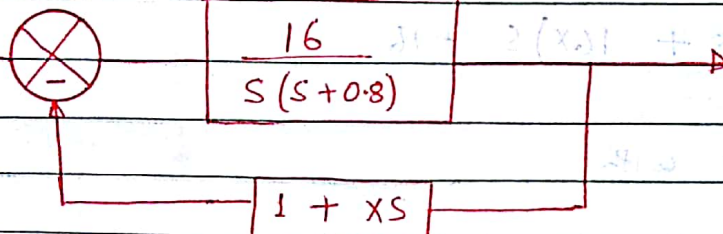
$$* \quad m_p = \exp \left[\frac{-\pi \xi}{\sqrt{1-\xi^2}} \right] \times 100$$

✓✓ NOTE: - Characteristic eqⁿ $\rightarrow 1 + GH$

Std. Characteristic eqⁿ $\rightarrow s^2 + 2\xi\omega_n s + \omega_n^2$

S-15
Q-7M

R(s)



Find "X" for $\xi = 0.5$ and peak overshoot.

$$\xi = 0.5$$

$$\therefore M_p = \exp \left[\frac{-\pi \xi}{\sqrt{1-\xi^2}} \right] \times 100$$

$$= \exp \left[\frac{-\pi \times 0.5}{\sqrt{1-0.5^2}} \right] \times 100$$

$$M_p = 0.163 \times 100$$

$$M_p = 16.3\%$$

Here, $G = \frac{16}{s(s+0.8)}$

$$H = 1 + xs$$

As ch. eqⁿ $\rightarrow 1 + GH$

$$1 + \frac{16}{s(s+0.8)} (1 + xs) = 0$$

(cross multiply & transfer to RHS)

$$s^2 + 0.8s + 16 + 16xs = 0$$

$$s^2 + (0.8 + 16x)s + 16$$

compare with.

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

$\therefore$ we get.

$$\omega_n^2 = 16$$

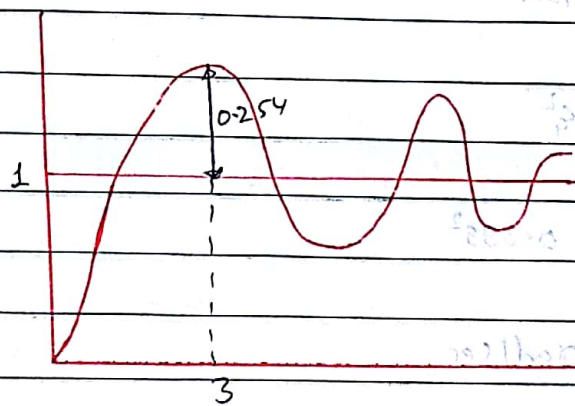
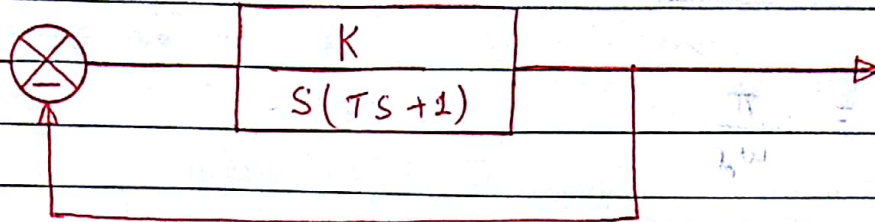
$$\boxed{\omega_n = 4} \text{ rad/sec.}$$

$$2\xi\omega_n = 0.8 + 16x$$

$$2 \times 0.5 \times 4 = 0.8 + 16x$$

$$\boxed{x = 0.2}$$

Q2) When the system is subjected to step input the system response is shown in fig. Find value of "K" and "T"



From block diagram,

$$G = \frac{K}{s(Ts + 1)}$$

$$H = 1$$

From time response curve,

$$T_p = 3 \text{ sec}$$

$$M_p = 0.254$$

$$M_p = \exp \left[\frac{-\pi \xi}{\sqrt{1 - \xi^2}} \right]$$

$$0.254 = \exp \left[\frac{-\pi \times \xi}{\sqrt{1-\xi^2}} \right]$$

$$\xi = 0.399$$

$$* T_p = \frac{\pi}{\omega_d}$$

$$\omega_d = 1.04 \text{ rad/sec}$$

$$* \omega_d = \omega_n \sqrt{1-\xi^2}$$

$$1.04 = \omega_n \sqrt{1-0.399^2}$$

$$\omega_n = 1.134 \text{ rad/sec}$$

$$\text{Ch. } e^{i\omega t} \rightarrow 1 + G.H$$

$$\therefore 1 + \frac{K}{s(s+1)} = 0$$

$$\therefore Ts^2 + s + K = 0$$

$$s^2 + \frac{1}{T}s + \frac{K}{T} = 0$$

$$\text{Compare with, } s^2 + 2\xi\omega_n s + \omega_n^2$$

$$* 2\xi\omega_n = \frac{1}{T}$$

$$2 \times 0.399 \times 1.134 = \frac{1}{T}$$

$$T = 1.094$$

$$* \omega_n^2 = \frac{K}{T}$$

$$1.134^2 = \frac{K}{1.094}$$

$$K = 1.428$$

W-15
7M
Q-(3)

Consider the close loop system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Determine the value of ζ and ω_n so that the system response to a step input approximately 5% overshoot with a settling time of 2 sec. use 2% criteria

Find:- ζ , ω_n

$$* M_p = 0.05 = \exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

$$\zeta = 0.6901$$

Now, for 2% tolerance band,

$$T_s = 2 = \frac{4}{\zeta\omega_n}$$

$$2 = \frac{4}{0.6901 \times \omega_n}$$

$$\omega_n = 2.89 \text{ rad/sec.}$$

Q-4) OLTF of a servo system with using feedback $G(s) = \frac{10}{s(0.5s+1)}$

find damping ratio, damped and undamped natural frequency, find max. overshoot for step input.

Here, $G = \frac{10}{s(0.5s+1)}$

$H = 1$

Char. eqⁿ = $1 + GH$

$= 1 + \frac{10}{s(0.5s+1)} = 0$

$0.5s^2 + s + 10 = 0$

$s^2 + \frac{1}{0.5}s + \frac{10}{0.5} = 0$

$s^2 + 2s + 20 = 0$

Compare with, $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$\therefore 2\zeta\omega_n = 2$ — (1)

$\omega_n^2 = 20$

$\omega_n = 4.47$

$\therefore$ from eqⁿ ①

$$\therefore 2 \xi / 4.47 = 2$$

$$\boxed{\xi = 0.223}$$

$$\star \omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 4.47 \sqrt{1 - 0.223^2}$$

$$\boxed{\omega_d = 3.94 \text{ rad/sec}}$$

$$\star M_p = \exp \left[\frac{-\pi \xi}{\sqrt{1 - \xi^2}} \right]$$

$$= \exp \left[\frac{-\pi \times 0.223}{\sqrt{1 - 0.223^2}} \right]$$

$$\boxed{M_p = 0.492}$$

$$\boxed{M_p = 49.2 \%}$$

Q-5) For step input $2.103s + 2$: find $C(t)$

$$C(t) = 1 + 0.2 e^{-60t} - 1.2 e^{-10t}$$

Obtain: 1) CLTF

2) ξ & ω_n

Solⁿ:- NOTE:- $CLTF = \frac{C(s)}{R(s)} = \frac{1}{1+GH} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$C(t) = 1 + 0.2 e^{-60t} - 1.2 e^{-10t}$$

Taking Laplace,

$$C(s) = \frac{1}{s} + \frac{0.2}{s+60} - \frac{1.2}{s+10}$$

$$= \frac{(s+60)(s+10) + 0.2s(s+10) - 1.2s(s+60)}{s(s+60)(s+10)}$$

$$= \frac{s^2 + 70s + 600 + 0.2s^2 + 2s - 1.2s^2 - 72s}{s(s^2 + 70s + 600)}$$

$$C(s) = \frac{600}{s(s^2 + 70s + 600)}$$

For step input, $R(s) = \frac{1}{s}$

$$CLTF = \frac{C(s)}{R(s)} = \frac{600}{s(s^2 + 70s + 600)} \cdot \frac{1}{s}$$

$$CLTF = \frac{600}{s^2 + 70s + 600}$$

Here,

$$\text{Ch. eq}^n \rightarrow s^2 + 70s + 600$$

Compare with : $s^2 + 2\zeta\omega_n s + \omega_n^2$

We get,

$$\ast \omega_n^2 = 600$$

$$\therefore \omega_n = 24.49 \text{ rad/s}$$

$$\ast 2\zeta\omega_n = 70$$

$$2 \times \zeta \times 24.49 = 70$$

$$\zeta = 1.42 > 1 \text{ (overdamped)}.$$

W-15 FM

Q-6 A Unity feedback system has OLF,

$$G(s) = \frac{49}{s(s+6)}$$

The system is subjected to unit step input.

Find domain specification as:-

- (1) Natural Frequency (ω_n)
- (2) damping ratio (ζ)
- (3) Damped Natural Frequency (ω_d)
- (4) % peak overshoot (M_p)
- (5) Settling time (T_s) (2% error band)

$$H = 1$$

$$G = \frac{49}{s(s+6)}$$

Char. eqn $\rightarrow 1 + GH$

$$1 + \frac{49}{s(s+6)} = 0$$

$$\frac{s^2 + 6s + 49}{s(s+6)} = 0$$

$$s^2 + 6s + 49$$

Compare, $s^2 + 2\xi\omega_n s + \omega_n^2$

we get,

$$\ast \omega_n^2 = 49$$

$$\boxed{\omega_n = 7} \text{ rad/s}$$

$$\ast 2\xi\omega_n = 6$$

$$2\xi \times 7 = 6$$

$$\boxed{\xi = 0.428}$$

$$\ast \omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 7 \sqrt{1 - 0.428^2}$$

$$= 6.326 \text{ rad/s}$$

$$* m_p = \exp \left[\frac{-\pi \xi}{\sqrt{1-\xi^2}} \right] \times 100$$

$$= \exp \left[\frac{-\pi \times 0.428}{\sqrt{1-0.428^2}} \right] \times 100$$

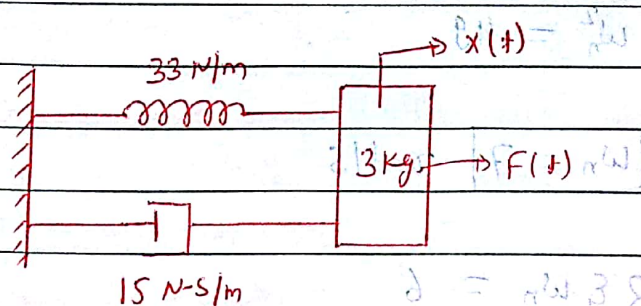
$$= 22.54\%$$

For 2% error band,

$$T_s = \frac{4}{\xi \omega_n} = \frac{4}{0.428 \times 7} = 1.335 \text{ sec, (approx)}$$

S-16,17

Q-7



Find domain specification. (T_d, T_r, T_p, T_s, m_p)

Solⁿ: - mass, $m = 3 \text{ kg}$

Damping Coeff., $B = 15 \text{ N-s/m}$

Stiffness, $K = 33 \text{ N/m}$

Input $\rightarrow F(s)$

Output $\rightarrow X(s)$

$$T.F = \frac{X(s)}{F(s)} = \frac{1}{ms^2 + Bs + K}$$

$$\text{Ch. } e^{st} \rightarrow ms^2 + Bs + K = 0$$

$$s^2 + \frac{B}{m} s + \frac{K}{m}$$

Compare, $s^2 + 2\zeta \omega_n s + \omega_n^2$

We get,

$$\omega_n^2 = \frac{K}{m} = \frac{33}{3} = 11$$

$$\omega_n = 3.31 \text{ rad/s}$$

$$2\zeta \omega_n = \frac{B}{m} = \frac{15}{3} = 5$$

$$2\zeta \times 3.31 = 5$$

$$\zeta = 0.75$$

* Delay time,

$$T_d = \frac{1 + 0.7\zeta}{\omega_n} = \frac{1 + 0.7 \times 0.75}{3.31} = 0.461 \text{ sec.}$$

* Rise Time,

$$\phi = (\cos^{-1} \zeta) \frac{\pi}{180} = (\cos^{-1} 0.75) \frac{\pi}{180} = 0.718 \text{ rad}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 2.017 \text{ rad/sec.}$$

$$T_r = \frac{\pi - \phi}{\omega_d} = 1.07 \text{ sec.}$$

* Peak time

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.17} = 1.44 \text{ sec}$$

* Settling time

Assume, 2% error band,

$$T_s = \frac{4}{\xi \omega_n} = 1.6 \text{ sec}$$

* Peak overshoot

$$m_p = \exp\left[\frac{-\pi \xi}{\sqrt{1-\xi^2}}\right]$$

=

$$= 0.0268$$

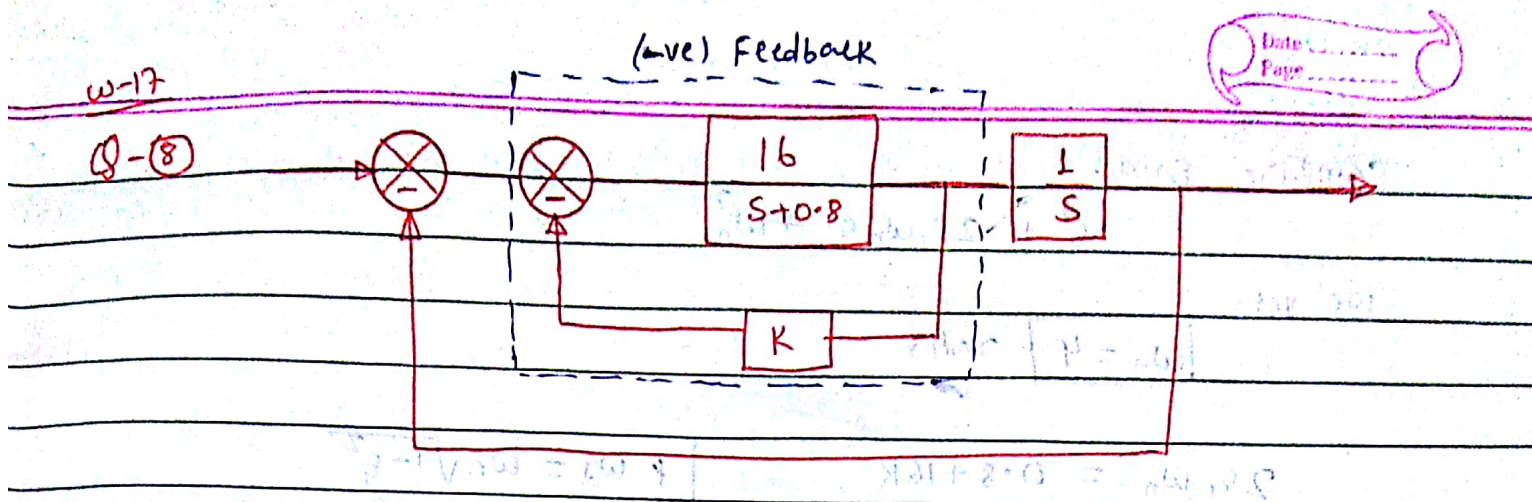
$$\Rightarrow \cos^{-1} 0.0268 = \frac{2\pi \times 2.17 \times T + 1}{2\pi \times 2.17 \times T} \times \pi$$

$$= 2.681 \times 10^{-5}$$

$$\Rightarrow \cos^{-1} 0.0268 = \frac{\pi}{2} (2\pi \times 2.17 \times T) = \frac{\pi}{2} (3.702) = \phi$$

$$\Rightarrow \cos^{-1} 0.0268 = \phi = \cos^{-1} \sqrt{1 - \omega_d^2 / \omega_n^2} = \cos^{-1} \sqrt{1 - 0.85^2}$$

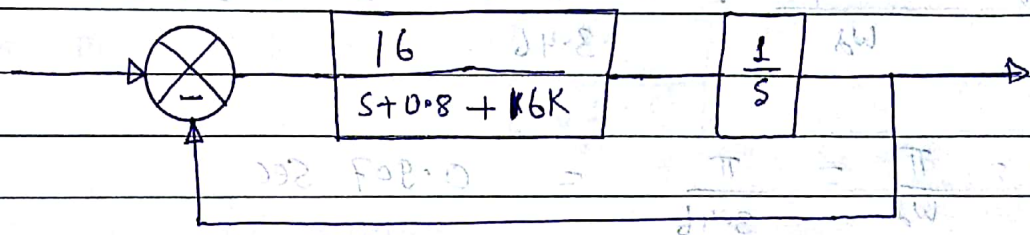
$$\Rightarrow \cos^{-1} 0.0268 = \phi = \cos^{-1} \sqrt{1 - 0.85^2} = \cos^{-1} 0.525 = \phi$$



Find the value of "K" such that damping ratio is 0.5. Then obtain rise time, peak time, max. overshoot, & settling time for unit step response.

Solⁿ:- Find:- K, T_r , T_p , m_p , T_s

$$\zeta = 0.5$$



Here,

$$G = \frac{16}{s(s+0.8+16K)}$$

$$\text{Ch. } e_a^n \rightarrow 1 + GH$$

$$1 + \frac{16}{s(s+0.8+16K)} = 0$$

$$s^2 + 0.8s + 16Ks + 16$$

Compare with,

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

we get,

$$\omega_n = 4 \text{ rad/s}$$

$$2\zeta\omega_n = 0.8 + 16K$$

$$2 \times 0.5 \times 4 = 0.8 + 16K$$

$$K = 0.2$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

$$= 3.46 \text{ rad/s}$$

$$\phi = (\cos^{-1} \zeta) \frac{\pi}{180} = (\cos^{-1} 0.5) \frac{\pi}{180} = 1.047 \text{ rad}$$

$$T_d = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 1.047}{3.46} = 0.605 \text{ Sec.}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.46} = 0.907 \text{ Sec}$$

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5 \times 4} = 2 \text{ Sec.}$$

$$M_p = \exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right] \times 100 = \exp\left[\frac{-\pi \times 0.5}{\sqrt{1-0.5^2}}\right] \times 100$$

$$M_p = 0.163 \times 100$$

$$M_p = 16.3\%$$

5-05
Q-9

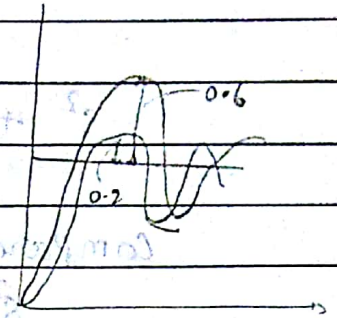
A unity feedback system has, $G(s) = \frac{K}{s(sT+1)}$

overshoot reduce from 0.6 to 0.2.

Show that, $\frac{TK_1 - 1}{TK_2 - 1} \approx 4.3$

Solⁿ: - $H = 1$

$$G(s) = \frac{K}{s(sT+1)}$$



$$m_{p_1} = 0.6$$

$$m_{p_2} = 0.2$$

$$* m_{p_1} = 0.6 = \exp \left[\frac{-\pi \xi_1}{\sqrt{1-\xi_1^2}} \right]$$

$$\xi_1 = 0.16$$

$$* m_{p_2} = 0.2 = \exp \left[\frac{-\pi \xi_2}{\sqrt{1-\xi_2^2}} \right]$$

$$\xi_2 = 0.455$$

$$\text{Ch. } e^{\bar{q}^n} \rightarrow 1 + GH$$

$$1 + \frac{K}{s(s+1)} x_1 = 0$$

$$\frac{Ts^2 + s + K}{s(sT + 1)} = 0$$

$$Ts^2 + s + K = 0$$

Divide by "T"

$$s^2 + \frac{s}{T} + \frac{K}{T}$$

Compare with,

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

we get,

$$\omega_n^2 = \frac{K}{T}$$

$$\omega_n = \sqrt{\frac{K}{T}}$$

$$2\zeta\omega_n = \frac{1}{T}$$

$$2\zeta\sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$T\sqrt{\frac{K}{T}} = \frac{1}{2\zeta}$$

$$\sqrt{\frac{T^2 K}{T}} = \frac{1}{2\zeta}$$

$$\sqrt{TK} = \frac{1}{2\zeta}$$

$$TK = \frac{1}{4\zeta^2}$$

$$TK_1 = \frac{1}{4\zeta_1} = \frac{1}{4(0.16)^2} = 9.765$$

$$TK_2 = \frac{1}{4\zeta_2} = \frac{1}{4(0.455)^2} = 1.207$$

Now,

$$\frac{TK_1 - 1}{TK_2 - 1} = \frac{9.765 - 1}{1.207 - 1} = 43$$

$$\boxed{\frac{TK_1 - 1}{TK_2 - 1} \approx 43}$$

Imp-

Q-10

A Unity feedback system has, $G(s) = \frac{K}{s(sT+1)}$

By what factor the amplifier gain "K" must be multiplied in order to reduce the overshoot from 75% to 25%.

Solⁿ:-

$$H = 1$$

$$G(s) = \frac{K}{s(sT+1)}$$

$$m_p = 0.75$$

$$m_p = 0.25$$

$$m_p = 0.75 = \exp\left[\frac{-\pi\zeta_1}{\sqrt{1-\zeta_1^2}}\right]$$

$$\therefore \boxed{\xi_1 = 0.0911}$$

$$\ast m_{p2} = 0.25 = \exp \left[\frac{-\pi \xi_2}{\sqrt{1 - \xi_2^2}} \right]$$

$$\boxed{\xi_2 = 0.4037}$$

$$\text{Ch. eqn} \rightarrow 1 + GH L - 70F \cdot E = 1 - \pi T$$

$$1 + \frac{K}{S(ST+1)} = 0$$

$$\frac{TS^2 + S + K}{S(ST+1)} = 0$$

$$TS^2 + S + K = 0$$

Divide by 'T'

$$S^2 + \frac{S}{T} + \frac{K}{T}$$

Compare with,

$$S^2 + 2\xi\omega_n S + \omega_n^2$$

we get,

$$\ast \omega_n^2 = \frac{K}{T}$$

$$\omega_n = \sqrt{\frac{K}{T}}$$

$$\ast 2\xi\omega_n = \frac{1}{T}$$

$$\therefore 2\xi \sqrt{\frac{K}{T}} = \frac{1}{T}$$

$$T \sqrt{\frac{K}{T}} = \frac{1}{2\epsilon}$$

$$\sqrt{\frac{T^2 K}{T}} = \frac{1}{2\epsilon}$$

$$\sqrt{TK} = \frac{1}{2\epsilon}$$

$$TK = \frac{1}{4\epsilon^2}$$

$$* TK_1 = \frac{1}{4\epsilon_1^2} = \frac{1}{4(0.0911)^2} = 30.123$$

$$* TK_2 = \frac{1}{4\epsilon_2^2} = \frac{1}{4(0.4037)^2} = 1.533$$

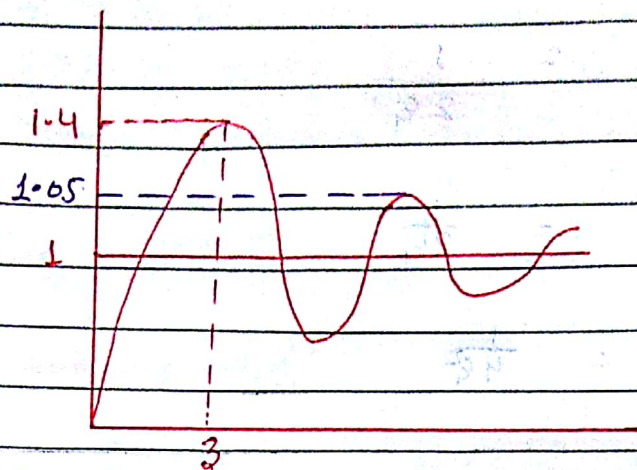
Divide above eqⁿ,

$$\frac{TK_2}{TK_1} = \frac{1.533}{30.123}$$

$$K_2 = 0.05 K_1$$

* The multiplying factor is 0.05.

Q-11) Find rise time and settling time.



Find:- T_r , T_s

From time response curve,

$$T_p = 3 \text{ sec.}$$

$$C_{\text{man}} = 1.4$$

$$C_{\text{final}} = 1$$

1.05 $\rightarrow$ represent 5% tolerance band.

$$* T_p = 3 = \frac{\pi}{\omega_d}$$

$$\boxed{\omega_d = 1.047} \text{ rad/s}$$

$$* M_p = \frac{C_{\text{man}} - C_{\text{final}}}{C_{\text{final}}} = \frac{1.4 - 1}{1} = 0.4$$

$$* m_p = 0.4 \Rightarrow \exp \left[\frac{-\pi \xi}{\sqrt{1-\xi^2}} \right]$$

$$\xi = 0.279$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$1.047 = \omega_n \sqrt{1 - 0.279^2}$$

$$\omega_n = 1.09 \text{ rad/s}$$

$$\phi = (\cos^{-1} \xi) \frac{\pi}{180} = (\cos^{-1} 0.279) \frac{\pi}{180} = 1.288 \text{ rad}$$

$$T_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 1.288}{1.047} = 1.7 \text{ sec}$$

for 5% band,

$$T_s = \frac{3}{\xi \omega_n} = \frac{3}{0.279 \times 1.09} = 9.86 \text{ sec}$$

Q-12) Find Transient Response Specification for a Step of 10 Unit $G(s) H(s) = \frac{16}{s(s+16)}$

Find:- Transient Response Specification (T_d, T_r, T_p, T_s, M_p)

$$\text{Ch. } e_q^n \rightarrow 1 + GH$$

$$1 + \frac{16}{s(s+16)} \times 1 = 0$$

$$s^2 + 16s + 16 = 0$$

$\gamma(t) = 10$ (Extra data)

Compare,

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

we get,

$$\omega_n = 4 \text{ rad/s}$$

$$* 2\zeta\omega_n = 6$$

$$2 \times \zeta \times 4 = 6$$

$$\zeta = 0.75$$

$$* T_d = \frac{1 + 0.7\zeta}{\omega_n} = \frac{1 + 0.7 \times 0.75}{4} = 0.381 \text{ sec.}$$

$$* \phi = (\cos^{-1} \zeta) \frac{\pi}{180} = (\cos^{-1} 0.75) \frac{\pi}{180} = 0.723 \text{ rad.}$$

$$* \omega_d = \omega_n \sqrt{1 - \zeta^2} = 4 \sqrt{1 - 0.75^2} = 2.645 \text{ rad/s}$$

$$* T_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 0.723}{2.645} = 0.914 \text{ sec.}$$

$$* T_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.645} = 1.187 \text{ sec.}$$

* Assume 2% error band,

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.75 \times 4} = 1.33 \text{ sec.}$$

$$* M_p = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = \exp\left(\frac{-\pi \times 0.75}{\sqrt{1-0.75^2}}\right) \times 100 = 2.83\%$$

* No. of Oscillation,

$$N = \frac{2\sqrt{1-\xi^2}}{\pi \xi}$$

$$K = 434$$

$$\omega_n = 20.833$$

$$s^2 + 25s + K$$



Q-(13) A Unity Feedback C.S. has forward path T.F.

$$G(s) = \frac{K}{s(s+25)}$$

(1) Find :- 'K' for $\xi = 0.6$

(2) Find :- T_r , T_s , T_p - no. of Oscillation

Soln :- $H = 1$

$$G(s) = \frac{K}{s(s+25)}$$

$$s(s+25)$$

$$Ch. e^{i\omega t} \rightarrow 1 + GH = 0$$

$$1 + \frac{K}{s(s+25)} = 0$$

$$s^2 + 25s + K = 0$$

Compare with,

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

we get,

$$2\xi\omega_n = 25$$

$$[8 \times 0.6 = 4]$$

$$2 \times 0.6 \times \omega_n = 25$$

$$\omega_n = 20.83 \text{ rad/s}$$

$$\omega_n^2 = K$$

$$K = 20.83^2$$

$$K = 434.02$$

$$\phi = (\cos^{-1} \xi) \frac{\pi}{180} = (\cos^{-1} 0.6) \frac{\pi}{180} = 0.927 \text{ rad}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2} = 20.83 \sqrt{1 - 0.6^2} = 16.664 \text{ rad/s}$$

$$T_d = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 0.927}{16.664} = 0.132 \text{ sec.}$$

* Assume 2% error band.

$$T_s = \frac{4}{\xi \omega_n} = \frac{4}{0.6 \times 20.83} = 0.32 \text{ sec.}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{16.664} = 0.188 \text{ sec.}$$

* No. of oscillation,

$$N = \frac{2\sqrt{1 - \xi^2}}{\pi \xi}$$

$$= \frac{2\sqrt{1 - 0.6^2}}{\pi \times 0.6}$$

$$N = 0.848$$

$$K = 134.05$$

Note:- $\mathcal{L}^{-1} \left[\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \right] = 1 - \frac{\exp(-\zeta\omega_n t) \sin(\omega_d t + \phi)}{\sqrt{1-\zeta^2}}$

imp w-9,16
Q-14

A system is described by

$$\frac{d^2 y}{dt^2} + 10 \frac{dy}{dt} + 49 y(t) = 100 x(t)$$

Find Response, maximum output and all domain specification for a step of 2.5 unit.

Soln:- $\frac{d^2 y}{dt^2} + 10 \frac{dy}{dt} + 49 y(t) = 100 x(t)$

Taking Laplace: $(s^2 + 10s + 49) Y(s) = 100 X(s)$

$$s^2 Y(s) + 10s Y(s) + 49 Y(s) = 100 X(s)$$

$$[s^2 + 10s + 49] Y(s) = 100 X(s)$$

T.F = $\frac{Y(s)}{X(s)} = \frac{100}{s^2 + 10s + 49}$

Here, Ch. eqn $\rightarrow s^2 + 10s + 49$

Compare with,

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

we get,

$\omega_n = 7$ rad/s

$$* 2 \xi \omega_n = 10$$

$$2 \xi \times 7 = 10$$

$$\boxed{\xi = 0.71}$$

$$* \omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 7 \sqrt{1 - 0.71^2}$$

$$\omega_d = 4.92 \text{ rad/s}$$

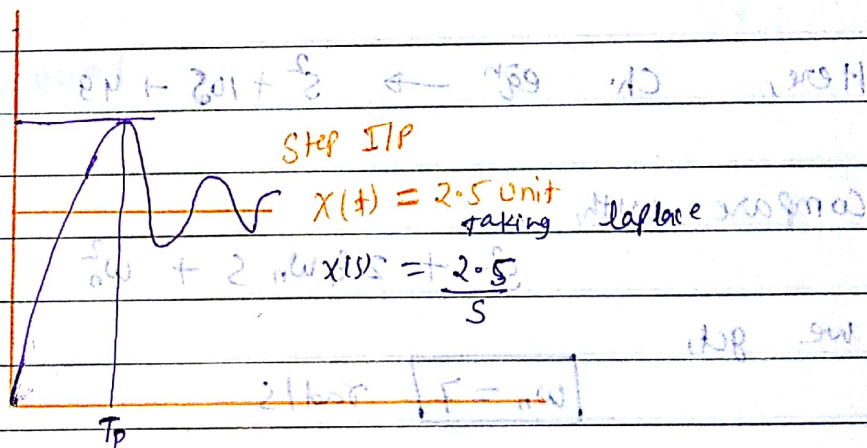
$$* \phi = (\cos^{-1} \xi) \frac{\pi}{180} \text{ rad}$$

$$= (\cos^{-1} 0.71) \frac{\pi}{180}$$

$$\phi = 0.78 \text{ rad} \times 0.01 = 0.0078 \text{ rad}$$

Find:- Response $y(t)$, max Response $y(t)_{\text{max}}$

Now,



Now,

$$\frac{y(s)}{x(s)} = \frac{100}{s^2 + 10s + 49}$$

$$\frac{y(s)}{2.5/s} = \frac{100}{s^2 + 10s + 49}$$

$$y(s) = \frac{250}{s(s^2 + 10s + 49)}$$

$$y(s) = \frac{250}{49} \left[\frac{49}{s(s^2 + 10s + 49)} \right]$$

Taking inverse Laplace,

$$y(t) = \frac{250}{49} \left[1 - \frac{\exp(-\xi \omega_n t)}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \phi) \right]$$

$$= \frac{250}{49} \left[1 - \frac{\exp(-0.71 \times 7t)}{\sqrt{1 - 0.71^2}} \sin(4.92t + 0.78) \right]$$

$$y(t) = 5.1 \left[1 - 1.41 \exp(-4.97t) \sin(4.92t + 0.78) \right]$$

This is the response.

TD get maximum response,

$$\text{put, } t = T_p = \frac{\pi}{\omega_d} = \frac{\pi}{4.92} = 0.638 \text{ sec.}$$

$$y(t)_{\max} = 5.1 \left[1 - 1.41 \exp(-4.97 \times 0.638) \sin(4.92 \times 0.638 + 0.78) \right]$$

here $\frac{180}{\pi}$
or
calc is in radian

$$= 5.1 \left[1 - 1.41 \times 0.0419 \right] (= 0.78)$$

$$y(t)_{\max} = 5.31$$

T_d, η, T_p, T_s, M_p

Find domain specification.

AS, $\xi = 0.71$, $\omega_n = 7 \text{ rad/sec}$

$$* T_d = \frac{1 + 0.7\xi}{\omega_n} = \frac{1 + 0.7 \times 0.71}{7} = 0.21 \text{ sec.}$$

$$* T_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 0.78}{4.92} = 0.479 \text{ sec.}$$

$$* T_p = \frac{\pi}{\omega_d} = \frac{\pi}{4.92} = 0.638 \text{ sec.}$$

$$* T_s = \frac{4}{\xi \omega_n} = \frac{4}{0.71 \times 7} = 0.804 \text{ sec.}$$

$$* M_p = \exp \left[\frac{-\pi \xi}{\sqrt{1 - \xi^2}} \right] \times 100 = \exp \left[\frac{-\pi \times 0.71}{\sqrt{1 - 0.71^2}} \right] \times 100$$

$$M_p = 4.21\%$$

S-12, W-12

Q-(15) A system is described by

$$\frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 25 y(t) = 50 x(t)$$

Find:- Response, maximum Response for a step of 2.5 unit.

Soln:- $\frac{d^2 y}{dt^2} + 8 \frac{dy}{dt} + 25 y(t) = 50 x(t)$

Taking Laplace,

$$s^2 y(s) + 8s y(s) + 25 y(s) = 50 x(s)$$

$$(s^2 + 8s + 25) y(s) = 50 x(s)$$

$$T.F = \frac{y(s)}{x(s)} = \frac{50}{s^2 + 8s + 25}$$

Here, ch. eqn $\rightarrow s^2 + 8s + 25$

Compare with,

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

we get,

$$\boxed{\omega_n = 5} \text{ rad/s}$$

$$* 2\zeta\omega_n = 8$$

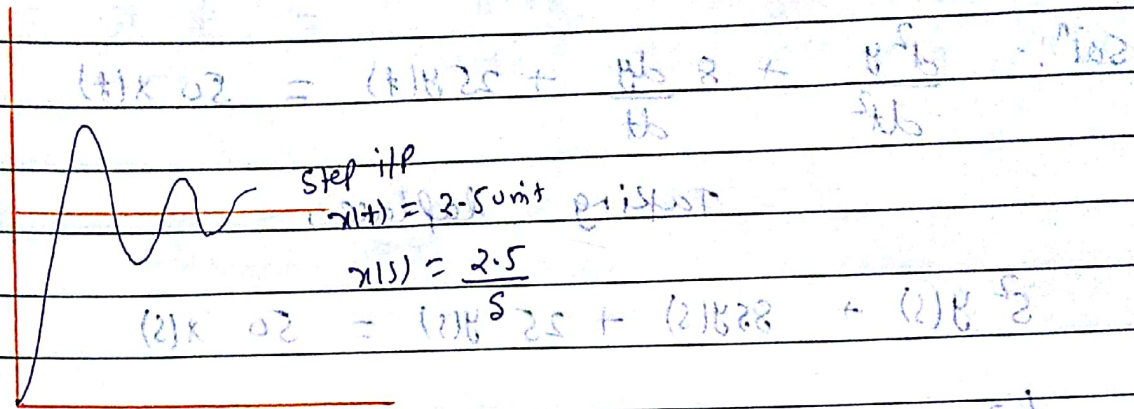
$$2\zeta \times 5 = 8$$

$$\boxed{\zeta = 0.8}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2} = 5 \sqrt{1 - 0.8^2} = 3 \text{ rad/s}$$

$$\phi = (\cos^{-1} \xi) \frac{\pi}{180} = (\cos^{-1} 0.8) \frac{\pi}{180} = 0.643 \text{ rad}$$

Find:- Response $y(t)$, Maximum Response $y(t)_{\max}$



$$(2) \times 0.2 = (2) \times (2.5 + 2.5 + 2.5)$$

$$\frac{y(s)}{x(s)} = \frac{50}{s^2 + 8s + 25} = \frac{(2) \times 25}{(s + 4)^2 + 9}$$

$$\frac{y(s)}{\frac{2.5}{s}} = \frac{50}{s^2 + 8s + 25} \Rightarrow y(s) = \frac{125}{s(s^2 + 8s + 25)}$$

$$\therefore y(s) = \frac{125}{s(s^2 + 8s + 25)}$$

$$\therefore y(s) = \frac{125}{25} \left[\frac{2.5 \times 5 [2 - \dots]}{s(s^2 + 8s + 25)} \right]$$

Taking inverse Laplace

$$\therefore y(t) = 5 \left[1 - \frac{\exp(-\xi \omega_n t)}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \phi) \right]$$

$$[0.8 = \xi]$$

$$\therefore y(t) = 5 \left[1 - 1.667 \exp(-4t) \sin(3t + 0.643) \right]$$

This is response. [Eqn of curve]

Now, max. response,

$$\text{put, } t = t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = 1.0471 \text{ sec.}$$

$$\begin{aligned} \therefore y(t)_{\max} &= 5 \left[1 - 1.667 \exp(-4 \times 1.0471) \sin(3 \times 1.0471 + 0.643) \right] \\ &= 5 \left[1 - 0.0253 (-0.5994) \right] \end{aligned}$$

$$y(t)_{\max} = 5.075$$

Now, Domain specification,

$$* T_d = \frac{1 + 0.7 \xi}{\omega_n} = \frac{1 + 0.7 \times 0.8}{5} = 0.312 \text{ sec.}$$

$$* \phi \approx 0 \quad T_s = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 0.643}{(2.2)3} = 0.832 \text{ sec.}$$

$$* T_s = \frac{4}{\xi \omega_n} = \frac{4}{0.8 \times 5} = 1.0 \text{ sec.}$$

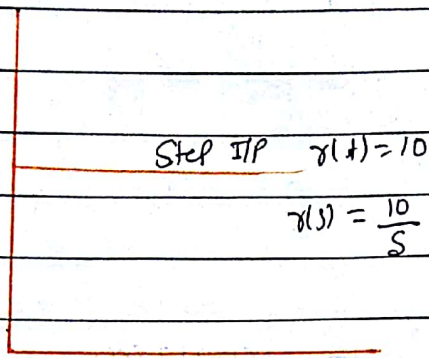
$$\begin{aligned} * M_p &= \exp \left[\frac{-\pi \xi}{\sqrt{1 - \xi^2}} \right] = \exp \left[\frac{-\pi \times 0.8}{\sqrt{1 - 0.8^2}} \right] \times 100 \\ &= 0.015 \times 100 \end{aligned}$$

$$M_p = 1.576 \%$$

Q-16) The unity feedback system has $G = \frac{16}{s(s+6)}$

Find Response and max. Response for a Step of 10 unit.

$H = 1$, $G = \frac{16}{s(s+6)}$



Ch. eqn $= 1 + GH$

$= 1 + \frac{16}{s(s+6)}$

$\therefore s^2 + 6s + 16 = 0$

$\therefore$ by comparing, we get.

$s^2 + 2\zeta\omega_n s + \omega_n^2$

$\omega_n = 4$ rad/sec.

$2\zeta\omega_n = 6$

$2 \times \zeta \times 4 = 6$

$\zeta = 0.75$

$$\omega_d = \omega_n \sqrt{1 - \xi^2} = 2.645 \text{ rad/sec.}$$

$$\phi = (\cos^{-1} \xi) \frac{\pi}{180} = 0.72 \text{ rad.}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{2.645} = 1.187 \text{ sec}$$

now,

$$T.F. = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\text{put, } R(s) = \frac{10}{s}$$

$$\therefore \frac{C(s)}{\frac{10}{s}} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = 10 \left[\frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} \right]$$

By taking inverse Laplace we get,

$$C(t) = 10 \left[1 - \frac{e^{-\xi\omega_n t} \sin(\omega_d t + \phi)}{\sqrt{1 - \xi^2}} \right]$$

$$C(t) = 10 \left[1 - \frac{\exp(-0.75 \times 4t)}{\sqrt{1 - 0.75^2}} \times \sin(2.645t + 0.721) \right]$$

$$C(t) = 10 \left[1 - 1.511 e^{-3t} \sin(2.645t + 0.721) \right]$$

To get max. response, put, $t = t_p = 1.18 \text{ sec.}$

$$C(t)_{\max} = 10 \left[1 - 1.511 e^{-3 \times 1.18} \times \sin(2.645 \times 1.18 + 0.721) \right]$$

$$C(t)_{\max} = 10.3$$

Q-17) The Unity feedback system has;
 $G = \frac{100}{s(s+5)}$. find the maximum response $C(t)_{\max}$

Soln:- $\gamma(t) = 1$

$$\therefore R(s) = \frac{1}{s}$$

$$\omega_n = 10$$

$$\xi = 0.23$$

$$\omega_d = 9.6824 \text{ rad/sec.}$$

$$\phi = 1.318 \text{ rad}$$

$$T_p = 0.32$$

$$\therefore \text{char eqn} = 1 + GH$$

$$= 1 + \frac{100}{s(s+5)}$$

$$\therefore s^2 + 5s + 100 = 0$$

by comparing, we get.

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

$$\omega_n = 10 \text{ rad/sec.}$$

$$2\xi\omega_n = 5$$

$$\xi = 0.25$$

$$* \quad \omega_d = \omega_n \sqrt{1 - \xi^2} = 10 \times \sqrt{1 - 0.25^2} = 9.68 \text{ rad/sec.}$$

$$* \quad \phi = (\cos^{-1} \xi) \times \frac{\pi}{180} = (\cos^{-1} 0.25) \times \frac{\pi}{180} = 1.318 \text{ rad.}$$

$$* \quad T_p = \frac{\pi}{\omega_d} = \frac{\pi}{9.68} = 0.32$$

now, $T.F = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

put $R(s) = \frac{1}{s}$

$$\therefore \frac{C(s)}{1/s} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

by taking inverse laplace.

$$C(t)_{max} = \left[1 - \frac{\exp(-\xi\omega_n t)}{\sqrt{1 - \xi^2}} \times \sin(\omega_d t + \phi) \right]$$

$$= 1 - \frac{\exp(-0.25 \times 10 t)}{\sqrt{1 - 0.25^2}} \sin(9.68 t + 1.318)$$

$$= [1 - 1.032 e^{-2.5t} \times \sin(9.68 \times 0.32 + 1.318)]$$

$$C(t)_{max} = 1.44$$

II topic

Steady State Response

* Steady State error,

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$$

Q- Derive The relationship between the steady state error and error constant for step, Ramp and parabolic Input.

Soln:- ① In step IP,

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] \frac{1}{s}$$

$$= \frac{1}{1 + \lim_{s \rightarrow 0} (GH)}$$

$$e_{ss} = \frac{1}{1 + K_p}$$

where,

K_p = position error constant

$$K_p = \lim_{s \rightarrow 0} [GH]$$

② Ramp input $[R(s) = \frac{1}{s^2}]$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] \times \frac{1}{s^2}$$

$$= \lim_{s \rightarrow 0} \left[\frac{1}{s+SGH} \right]$$

$$= \frac{1}{0 + \lim_{s \rightarrow 0} SGH}$$

$$e_{ss} = \frac{1}{K_v}$$

where,

K_v = velocity error constant.

$$K_v = \lim_{s \rightarrow 0} [sGH]$$

③ parabolic input

$$R(s) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] \times \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{1}{s^2 + s^2 GH} \right]$$

$$= \frac{1}{0 + \lim_{s \rightarrow 0} (s^2 GH)}$$

$$e_{ss} = \frac{1}{K_a}$$

where,

K_a = accⁿ error constant

$$K_a = \lim_{s \rightarrow 0} [s^2 GH]$$

"formula"

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1 + GH} \right] R(s)$$

* Step I/P $[x(t) = 1]$, $x(s) = \frac{1}{s}$

$$K_p = \lim_{s \rightarrow 0} [GH]$$

$$e_{ss} = \frac{1}{1 + K_p}$$

* Ramp I/P $[x(t) = t]$, $x(s) = \frac{1}{s^2}$

$$K_v = \lim_{s \rightarrow 0} [sGH]$$

$$e_{ss} = \frac{1}{K_a}$$

* parabolic

T/P

$$[r(t) = \frac{t^2}{2}]$$

$$r(s) = \frac{1}{s^3}$$

$$K_a = \lim_{s \rightarrow 0} [s^2 GH]$$

$$e_{ss} = \frac{1}{K_a}$$

$$\left[\begin{array}{c|c} (2+3)01 & \\ \hline (1+2)(1+2)s^2 & \end{array} \right] \begin{array}{l} m1 \\ 002 \end{array}$$

Q-

$$[H02] \begin{array}{l} m1 \\ 002 \end{array} = \sqrt{1}$$

$$\left[\begin{array}{c|c} (2+3)01 & 2 \\ \hline (1+2)(1+2)s^2 & \end{array} \right] \begin{array}{l} m1 \\ 002 \end{array}$$

$$2 \times 01 = \sqrt{1}$$

$$100 = \sqrt{1}$$

S-14
10M

Q1

A unity feedback system has,

$$G(s) = \frac{10(s+5)}{s^2(s+2)(s+7)}$$

Find Steady State error and error constant.

$$H = 1$$

Find:- e_{ss} , K_p , K_v , K_a

* Step I/P

$$K_p = \lim_{s \rightarrow 0} \left[\frac{10(s+5)}{s^2(s+2)(s+7)} \right]$$

$$= \frac{10 \times 5}{0} = \infty$$

$$e_{ss} = \frac{1}{1+K} = \frac{1}{1+\infty} = 0$$

* Ramp I/P.

$$K_v = \lim_{s \rightarrow 0} [sGH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{s \cdot 10(s+5)}{s^2(s+2)(s+7)} \right]$$

$$K_v = \frac{10 \times 5}{0}$$

$$K_v = \infty$$

$$* e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0.$$

* parabolic I/P

$$K_a = \lim_{s \rightarrow 0} [s^2 G H]$$

$$= \lim_{s \rightarrow 0} \left[s^2 \frac{10 \times (s+5)}{s^2 (s+2)(s+7)} \right]$$

$$= \frac{10 \times 5}{2 \times 7}$$

$$= \frac{50}{14}$$

$$K_a = 3.57$$

$$* e_{ss} = \frac{1}{K_a} = \frac{1}{3.57} = 0.28.$$

Q-2) Find Steady State error and error Constant for U.F.B has

$$G(s) = \frac{25}{s(s+7)}$$

where, step, ramp and parabolic input of 5 units are applied.

Soln:- Here, 5 unit represent
ie, $r(t) = 5$

$\therefore$ Step I/P $[r(t) = 5]$

$$K_p = \lim_{s \rightarrow 0} [GH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{25}{s(s+7)} \right]$$

$$= \frac{25}{0}$$

$$= \infty$$

$$\therefore e_{ss} = \frac{\overset{5 \text{ units}}{5}}{1 + K_p} = \frac{5}{1 + \infty} = 0$$

$\therefore$ Ramp I/P $[r(t) = 5t]$

$$\therefore r(s) = \frac{5}{s^2}$$

$$\therefore K_v = \lim_{s \rightarrow 0} [s + GH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{s \times 25}{s(s+7)} \right]$$

$$= \frac{25}{7}$$

$$= 3.57$$

$$\therefore e_{ss} = \frac{5}{K_v} = \frac{5}{3.57} = 1.4$$

* parabolic i/p $\left[r(t) = 5 \times \frac{t^2}{2} \right]$

$$\therefore K_a = \lim_{s \rightarrow 0} [s^2 GH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2 \times 25}{s(s+7)} \right]$$

$$K_a = 0$$

$$e_{ss} = \frac{5}{K_a} = \frac{5}{0} = \infty$$

Q-3 For a system, $G(s)H(s) = \frac{K}{s^2(s+2)(s+3)}$

Find The value of "K" to limit the Steady State error to 10 when I/P to system $1 + 10t + \frac{40}{2}t^2$.

Find:-
 $e_{ss} = 10$

$$\text{or } r(t) = 1 + 10t + \frac{40}{2}t^2$$

Taking Laplace.

$$r(s) = \frac{1}{s} + \frac{10}{s^2} + \frac{40}{s^3}$$

$$\therefore R(s) = \frac{s^2 + 10s + 40}{s^3}$$

Now,

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$$

$$10 = \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{K}{s^2(s+2)(s+3)}} \right] \cdot \frac{s^2 + 10s + 40}{s^3}$$

$$10 = \lim_{s \rightarrow 0} \left[\frac{s^3(s+2)(s+3)}{s^2(s+2)(s+3) + K} \right] \left[\frac{s^2 + 10s + 40}{s^3} \right]$$

$$10 = \frac{2 \times 3 \times 40}{K}$$

$$K = 24$$

Q-4) For $G(s) = \frac{K(1+2s)}{s(s+1)(1+0.4s)^2}$

Find: - 'K' to limit the steady state error 10% for ramp input.

Assume, $H = 1$

$e_{ss} = 10\% = 0.1$

Ramp input $R(s) = \frac{1}{s^2}$

Soln:- As, $e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$

$0.1 = \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{K(1+2s)}{s(s+1)(1+0.4s)^2}} \right] \cdot \frac{1}{s^2}$

$0.1 = \lim_{s \rightarrow 0} \left[\frac{s^2(s+1)(1+0.4s)^2}{s(s+1)(1+0.4s)^2 + K(1+2s)} \right] \cdot \frac{1}{s^2}$

$\therefore 0.1 = \frac{(0+1)(1+0)^2}{0(0+1)(1+0)^2 + K(1+2 \times 0)}$

$K = 10$

v.v. imp

Q.5) Determine error constant and steady state error for a system.

$$G(s)H(s) = \frac{16}{s(s+8)(s+5)}$$

For input,

(a) $r(t) = 10$

(b) $r(t) = 5t$

(c) $r(t) = 10 + 5t + \frac{6}{2}t^2$

Find:- K_p, K_v, K_a, e_{ss} (for all 3 inputs)

* $K_p = \lim_{s \rightarrow 0} [GH]$

$$= \lim_{s \rightarrow 0} \left[\frac{16}{s(s+8)(s+5)} \right]$$

$$= \frac{16}{0}$$

$$K_p = \infty$$

* $K_v = \lim_{s \rightarrow 0} [sGH]$

$$= \lim_{s \rightarrow 0} \left[s \times \frac{16}{s(s+8)(s+5)} \right]$$

$$= \frac{16}{8 \times 5}$$

$K_v = 0.4$

$$* K_a = \lim_{s \rightarrow 0} [s^2 G_H]$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2 \times 16}{s(s+8)(s+5)} \right]$$

$K_a = 0$

Now, $* e_{ss}$

For (a) $r(s) = 10$

$$r(s) = \frac{10}{s}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+G_H} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{16}{s(s+8)(s+5)}} \right] \frac{10}{s}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s(s+8)(s+5)}{s(s+8)(s+5) + 16} \right] 10$$

By applying limit

$$= 0 \times \frac{8 \times 5 \times 10}{16}$$

$e_{ss} = 0$

⑥ $x(t) = 5t$

$\therefore R(s) = \frac{5}{s^2}$

$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$

$= \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{16}{s(s+8)(s+5)}} \right] \frac{5}{s^2}$

$\therefore e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s^2 (s+8)(s+5)}{s(s+8)(s+5) + 16} \right] \frac{5}{s^2}$

$= \lim_{s \rightarrow 0} \left[\frac{(s+8)(s+5)}{s(s+8)(s+5) + 16} \right] 5$

Apply limit

$e_{ss} = \frac{8 \times 5 \times 5}{0 + 16}$

$e_{ss} = \frac{200}{16}$

$e_{ss} = 12.5$

$$(c) \quad r(t) = 10 + 5t + \frac{6}{2} t^2$$

$$\therefore r(s) = \frac{10}{s} + \frac{5}{s^2} + \frac{6}{s^3}$$

Take L.C.M.

$$\therefore R(s) = \frac{10s^2 + 5s + 6}{s^3}$$

$$\therefore e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+G(s)} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{16}{s(s+8)(s+5)}} \right] \frac{10s^2 + 5s + 6}{s^3}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2(s+8)(s+5)}{s(s+8)(s+5) + 16} \right] \frac{10s^2 + 5s + 6}{s^3}$$

$$= \frac{8 \times 5 \times 6}{16 \times 0} = \infty$$

$$e_{ss} = \infty$$

$$0 = \infty$$

W-17
6M

Q-6 For unity feedback C.S. The Forward path

T.F. is,

$$G(s) = \frac{20}{s(s+2)(s^2+2s+20)}$$

Find Steady State Error (e_{ss}) when input are,

(a) 5

(b) $5t$

(c) $\frac{3}{2}t^2$

Find:- e_{ss} for input 5 , $5t$, $\frac{3}{2}t^2$ $\therefore R(s) = 5$

$\therefore$ ① $e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$ $R(s) = \frac{5}{s}$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{20}{s(s+2)(s^2+2s+20)}} \right] \frac{5}{s}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s(s+2)(s^2+2s+20)}{s(s+2)(s^2+2s+20) + 20} \right] 5 =$$

$$e_{ss} = 0$$

② e_{ss} For input $5t$

$$r(t) = 5t$$

$$r(s) = \frac{5}{s^2}$$

$$\therefore e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s^2 (s+2) (s^2 + 2s + 20)}{s (s+2) (s^2 + 2s + 20) + 20} \right] \frac{5}{s^2}$$

$$\therefore e_{ss} = \frac{2 \times 20 \times 5}{20}$$

$$e_{ss} = 10$$

$$(3) \quad r(t) = \frac{3}{2} t^2$$

$$r(s) = \frac{3}{s^3}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s \cdot 3}{1 + \frac{20}{s (s+2) (s^2 + 2s + 20)}} \right] \frac{3}{s^3}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2 (s+2) (s^2 + 2s + 20)}{s (s+2) (s^2 + 2s + 20) + 20} \right] \frac{3}{s^2 s}$$

$$= \frac{2 \times 20 \times 3}{20 \times 0}$$

$$e_{ss} = \infty$$

W-10

Q-7) Find error co-efficient and Steady State error for system having,

$$G(s) = \frac{K}{s(s^2 + 2s + 5)}$$

$$H(s) = \frac{10}{s+4}$$

If input $x(t) = 5 + 10t + \frac{t^2}{2}$

Assume, $K = 20$

$$G(s) \cdot H(s) = \frac{10 \times 20}{s(s^2 + 2s + 5)(s+4)}$$

$$G(s) \cdot H(s) = \frac{200}{s(s+4)(s^2 + 2s + 5)}$$

Find!- K_p, K_v, K_a, e_{ss}

$$* K_p = \lim_{s \rightarrow 0} [GH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{200}{s(s+4)(s^2 + 2s + 5)} \right]$$

$$= \frac{200}{0}$$

$$K_p = \infty$$

$$\# \quad K_v = \lim_{s \rightarrow 0} [sGH]$$

$$= \lim_{s \rightarrow 0} \left[\cancel{s} \times \frac{200}{\cancel{s}(s+4)(s^2+2s+5)} \right]$$

$$= \frac{200}{4 \times 5}$$

$$K_v = 10$$

$$\# \quad K_a = \lim_{s \rightarrow 0} [s^2 GH]$$

$$= \lim_{s \rightarrow 0} \left[s^2 \times \frac{200}{s(s+4)(s^2+2s+5)} \right]$$

$$K_a = 0$$

$$\# \quad \gamma(t) = 5 + 10t + \frac{t^2}{2}$$

$$\gamma(s) = \frac{5}{s} + \frac{10}{s^2} + \frac{1}{s^3}$$

Take L.C.M.

$$\gamma(s) = \frac{5s^2 + 10s + 1}{s^3}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] r(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{200}{s(s+4)(s^2+2s+5)}} \right] \frac{5s^2+10s+1}{s^3}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2(s+4)(s^2+2s+5)}{s(s+4)(s^2+2s+5)+200} \right] \frac{5s^2+10s+1}{s^3}$$

$$= \frac{4 \times 5 \times 1}{200 \times 0}$$

$$e_{ss} = \infty$$

S-09
7M

Q-8 Find the steady state error for

$$GH = \frac{200}{S(S+8)}$$

for $r(t) = 2t$

Find 'K' to reduce the error by 5% = 100.0

Find:- e_{ss}

$r(t) = 2t$

$$R(s) = \frac{2}{s^2}$$

Soln:-
$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{S}{1+GH} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{S}{1 + \frac{200}{S(S+8)}} \right] \frac{2}{s^2}$$

$$e_{ss} = 0.08$$

2nd part,

find 'K'.

Error is reduced by 5%.

$$e'_{ss} = 95\% \times 0.08$$

$$e'_{ss} = 0.076$$

Here, Now,

Consider, $GH = \frac{K}{S(S+8)}$

$$e_{ss}' = \lim_{S \rightarrow 0} \left[\frac{S}{1+GH} \right] R(S)$$

$$0.076 = \lim_{S \rightarrow 0} \left[\frac{S}{1 + \frac{K}{S(S+8)}} \right] \frac{2}{S^2}$$

$$0.076 = \lim_{S \rightarrow 0} \left[\frac{S^2(S+8)}{S(S+8) + K} \right] \frac{2}{S^2}$$

$$0.076 = \lim_{S \rightarrow 0} \left[\frac{S+8}{S(S+8) + K} \right] 2$$

$$0.076 = \frac{8 \times 2}{K}$$

$$K = 210.5$$

S-12, W-12
7M

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Page

Q-9

A Unity Feedback System

Las = 0.2

$$G(s) = \frac{K}{s(s+1)(1+0.4s)}$$

① If $x(t) = 4t$ & $K = 2$

Find:- ① Steady State error

② If Steady State error is 0.2

Find value of 'K'.

③ Find Steady state error if Input is change to $2 + 6t$ & $K = 10$

$$H = 1$$

$$G(s) = \frac{K}{s(s+1)(1+0.4s)}$$

① If $x(t) = 4t$ & $K = 2$

Find:- e_{ss}

$$x(t) = 4t$$

Taking Laplace

$$R(s) = \frac{4}{s^2}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1 + GH} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{K}{s(s+1)(1+0.4s)}} \right] \cdot \frac{4}{s^2}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s^2 (s+1) (1+0.4s)}{s (s+1) (1+0.4s) + K} \right] \frac{4}{s^2}$$

$$= \frac{1 \times 1 \times 4}{K}$$

$$e_{ss} = \frac{4}{K} \quad \text{--- ①}$$

put $K=2$

$$e_{ss} = 2$$

② If $e_{ss} = 0.2$

find!- K

from eqn ①

$$e_{ss} = \frac{4}{K}$$

$$0.2 = \frac{4}{K}$$

$$K = 20$$

③ find!- e_{ss} if $K=10^2$

$$r(t) = 2 + 6t$$

Taking Laplace

$$R(s) = \frac{2}{s} + \frac{6}{s^2}$$

$$R(s) = \frac{2s + 6}{s^2}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1 + GH} \right] R(s)$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2 (s+1) (1+0.4s)}{s(s+1)(1+0.4s) + K} \right] \frac{2s+6}{s^2}$$

$$e_{ss} = \frac{1 \times 1 \times 6}{K} \quad \text{put } K = 10$$

$$e_{ss} = \frac{6}{10}$$

$$e_{ss} = 0.6$$

Q-10 A CS. having unity feedback

$$G(s) = \frac{K}{s(s+2)(s^2+2s+5)}$$

For ramp input,
 $e_s \leq 0.2$

$$e_{ss} = \frac{10}{K}$$

$$K \geq 50$$

Find:- K

$$H = 1$$

$$G(s) = \frac{K}{s(s+2)(s^2+2s+5)}$$

For ramp input, $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] R(s)$$

$$0.2 = \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{K}{s(s+2)(s^2+2s+5)}} \right] \frac{1}{s^2}$$

$$0.2 = \lim_{s \rightarrow 0} \left[\frac{s^2 (s+2)(s^2+2s+5)}{s(s+2)(s^2+2s+5) + K} \right] = \frac{1}{s^2}$$

$$0.2 = \frac{2 \times 5}{K}$$

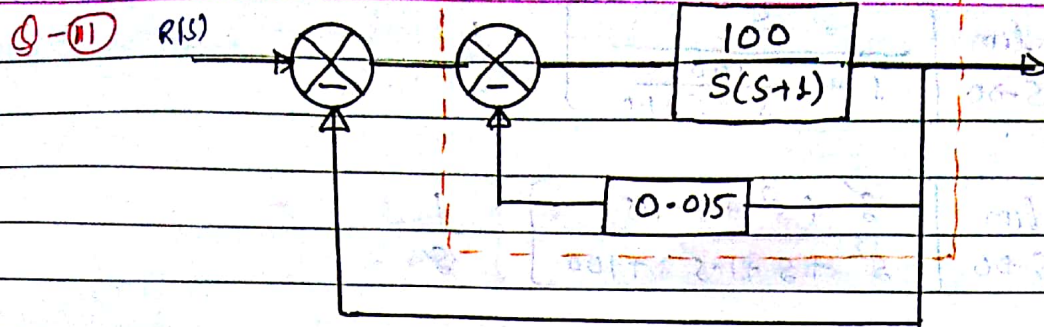
$$K \geq \frac{2 \times 5}{0.2}$$

$$\boxed{K \geq 50}$$

S-16
7M

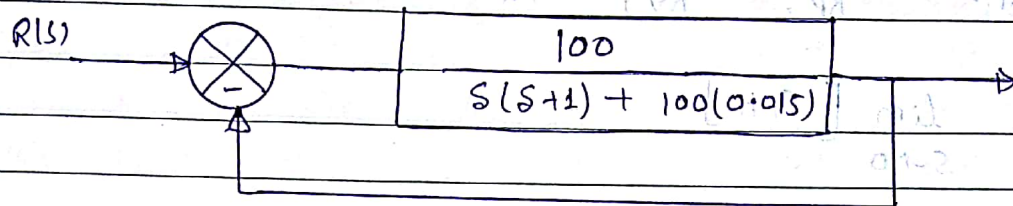
(-ve) Feedback

Date _____
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A position control system is damped with velocity feedback as shown.

- (1) For step input find E_{ss}
- (2) Determine system static error co-efficient (K_p, K_v, K_a)



Here, $H = 1$

$$G = \frac{100}{s(s+1) + 100(0.015)}$$

$$G = \frac{100}{s^2 + s + 1.5}$$

- (1) Find:- E_{ss}
For step input, $R(s) = \frac{1}{s}$

$$E_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1 + GH} \right] R(s)$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{100}{s^2 + s + 1.5}} \right] \cdot \frac{1}{s}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s (s^2 + s + 1.5)}{s^2 + s + 1.5 + 100} \right] \cdot \frac{1}{s}$$

$$= \frac{1.5}{1.5 + 100}$$

$$e_{ss} = 0.0149$$

② Find - K_p , K_v , K_a

$$K_p = \lim_{s \rightarrow 0} [GH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{100}{s^2 + s + 1.5} \right]$$

$$= \frac{100}{1.5}$$

$$K_p = 66.67$$

$$K_v = \lim_{s \rightarrow 0} [s GH]$$

$$= \lim_{s \rightarrow 0} \left[s \times \frac{100}{s^2 + s + 1.5} \right]$$

$$K_v = 0$$

$$* K_a = \lim_{s \rightarrow 0} [s^2 G H]$$

$$= \lim_{s \rightarrow 0} \left[s^2 \times \frac{100}{s^2 + s + 1.5} \right]$$

$$K_a = 0$$

W-05

Q-12

A C.S. having Unity feedback has

$$G(s) = \frac{K(s+2)}{s(s^3 + 7s^2 + 12s)}$$

(1) State Type & order of system

(2) Find error constant $K_p =$

(3) Find e_{ss} due to step, ramp & parabolic input.

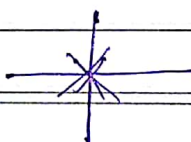
(4) Find e_{ss} if $x(t) = 1 + 2t + \frac{3}{2}t^2$

$$(1) GH = \frac{K(s+2)}{s(s^3 + 7s^2 + 12s)}$$

$$= \frac{K(s+2)}{s^2(s^2 + 7s + 12)}$$

Order:- Highest power of "s" in denominator
 $\therefore$ It is 4th order.

Type:- Roots of denominator is called as poles & denoted by (x)



$$s^2 = 0$$

$$s = 0, 0$$

No. of roots at origin define its type.

$\therefore$ It is Type-II

(2) Find:- K_p, K_v, K_a

$$* K_p = \lim_{s \rightarrow 0} [GH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{K(s+2)}{s^2(s^2+7s+12)} \right]$$

$$= \frac{2K}{0}$$

$$K_p = \infty$$

$$* K_v = \lim_{s \rightarrow 0} [sGH]$$

$$= \lim_{s \rightarrow 0} \left[\frac{s K(s+2)}{s^2(s^2+7s+12)} \right]$$

$$= \frac{2K}{0}$$

$$K_v = \infty$$

$$K_a = \lim_{s \rightarrow 0} [s^2 G H]$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2 \times K(s+2)}{s^2 (s^2 + 7s + 12)} \right]$$

$$= \frac{2K}{12}$$

$$K_a = \frac{K}{6}$$

$$\textcircled{3} \quad e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{K/6} = \frac{6}{K}$$

$$\textcircled{4} \quad \text{Final:- } e_{ss}$$

$$r(t) = 1 + 2t + \frac{3}{2}t^2$$

$$R(s) = \frac{1}{s} + \frac{2}{s^2} + \frac{3}{s^3} = \frac{s^2 + 2s + 3}{s^3}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+G H} \right] R(s)$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{K(s+2)}{s^2(s^2+7s+12)}} \right] \frac{s^2+2s+3}{s^3}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^3(s^2+7s+12)}{s^2(s^2+7s+12) + K(s+2)} \right] \frac{s^2+2s+3}{s^3}$$

$$e_{ss} = \frac{12 \times 3}{2K}$$

$$e_{ss} = \frac{18}{K}$$

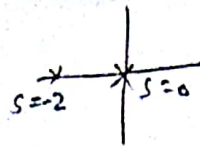
Q-13) The forward path T.F. of Unity feedback 2nd order, Type-I has pole at -2. The nature of gain "K" is so adjusted that damping ratio is 0.4. Find:- e_{ss} if $r(t) = 1 + 4t$

$$R(s) = \frac{1}{s} + \frac{4}{s^2} = \frac{s+4}{s^2}$$

Solⁿ:- $GH = \frac{K}{s(s+2)}$

$$\zeta = 0.4$$

Ch. $e_{ss} \rightarrow 1 + GH$



$$1 + \frac{K}{s(s+2)} = 0$$

$$s^2 + 2s + K$$

Compare, $s^2 + 2\zeta\omega_n s + \omega_n^2$

we get,

$$2\zeta\omega_n = 2$$

$$2 \times 0.4 \times \omega_n = 2$$

$$\boxed{\omega_n = 2.5} \text{ rad/s}$$

$$\& \omega_n^2 = K$$

$$\boxed{K = 6.25}$$

$$\& GH = \frac{6.25}{s(s+2)}$$

Now, find e_{ss}

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1+GH} \right] \text{ (RIS)}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{s}{1 + \frac{6.25}{s(s+2)}} \right] \cdot \frac{s+4}{s^2}$$

$$= \lim_{s \rightarrow 0} \left[\frac{s^2(s+2)}{s(s+2) + 6.25} \right] \cdot \frac{s+4}{s^2}$$

$$e_{ss} = \frac{2 \times 4}{6.25}$$

$$e_{ss} = 1.28$$

Unit-6 State Variable Analysis

Date 9/4/18
Page

✓✓
Type I:- State space Model (SSM)

* State Equation:-

$$\dot{x} = [A] \{x\} + [B] u$$

$$\overset{\uparrow}{s} = \frac{d}{dt} = \dot{y}$$

+ output equation:-

$$y = [C] \{x\}$$

$$s^2 = \frac{d^2}{dt^2} = \ddot{y}$$

$$s^3 = \frac{d^3}{dt^3} = \dddot{y}$$

y → output

u → input

W-8,11 S-17

Q ① Construct the state model for a system characterized by differential equation:

$$\frac{d^3 y}{dt^3} + 6 \frac{d^2 y}{dt^2} + 11 \frac{dy}{dt} + 6y = u$$

So Iⁿ:- $\ddot{y} + 6\ddot{y} + 11\dot{y} + 6y = u$

$$\ddot{y} = -6\ddot{y} - 11\dot{y} - 6y + u \quad \text{--- ①}$$

Let, $x_1 = y$ $x_2 = \dot{y}$ $x_3 = \ddot{y}$ --- ②

$\dot{x}_1 = \dot{y}$ $\dot{x}_2 = \ddot{y}$ $\dot{x}_3 = \dddot{y}$ --- ③

From eqⁿ ①, ②, ③ we get,

$$\dot{x}_1 = x_2 \quad \text{--- ④}$$

$$\dot{x}_2 = x_3 \quad \text{--- ⑤}$$

$$\dot{x}_3 = -6x_3 - 11x_2 - 6x_1 + u \quad \text{--- ⑥}$$

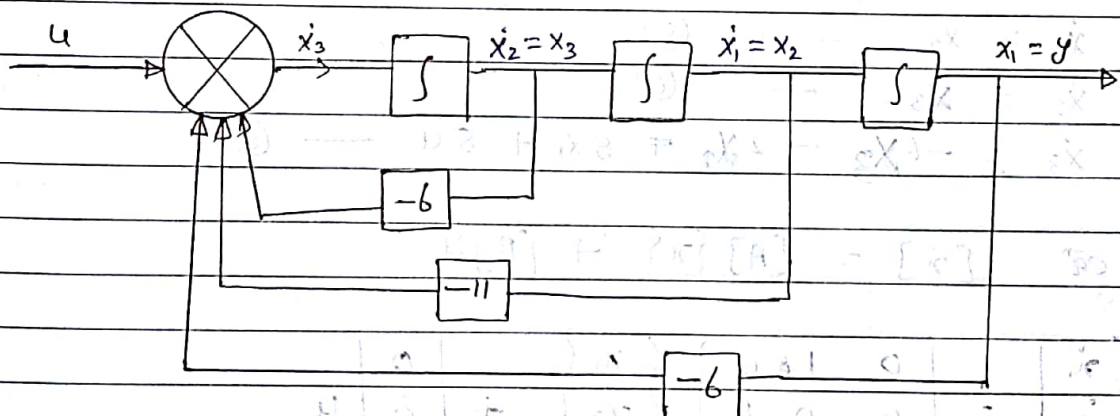
State eqⁿ:- $\dot{x} = [A] \{x\} + [B] u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

output eqⁿ:- $y = [C] \{x\}$
($y = x_1$)

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

* Block diagram (From eqⁿ 6)



Q-2 Construct the state model for a system characteristics by differential eqⁿ.

$$\frac{d^3 y}{dt^3} + 6 \frac{d^2 y}{dt^2} + 2 \frac{dy}{dt} + 9y = 5u$$

Solⁿ:- $\ddot{y} + 6\ddot{y} + 2\dot{y} + 9y = 5u$

$$\ddot{y} = -6\ddot{y} - 2\dot{y} - 9y + 5u \quad \text{--- (1)}$$

Let, $x_1 = y$ $x_2 = \dot{y}$ $x_3 = \ddot{y}$ --- (2)

$$\dot{x}_1 = \dot{y} \quad \dot{x}_2 = \ddot{y} \quad \dot{x}_3 = \ddot{\ddot{y}} \quad \text{--- (3)}$$

From eqⁿ (1), (2), (3) we get.

$$\dot{x}_1 = x_2 \quad \text{--- (5)}$$

$$\dot{x}_2 = x_3 \quad \text{--- (6)}$$

$$\dot{x}_3 = -6x_3 - 2x_2 + 9x_1 + 5u \quad \text{--- (6)}$$

State eqⁿ $[\dot{x}] = [A]\{x\} + [B]u$

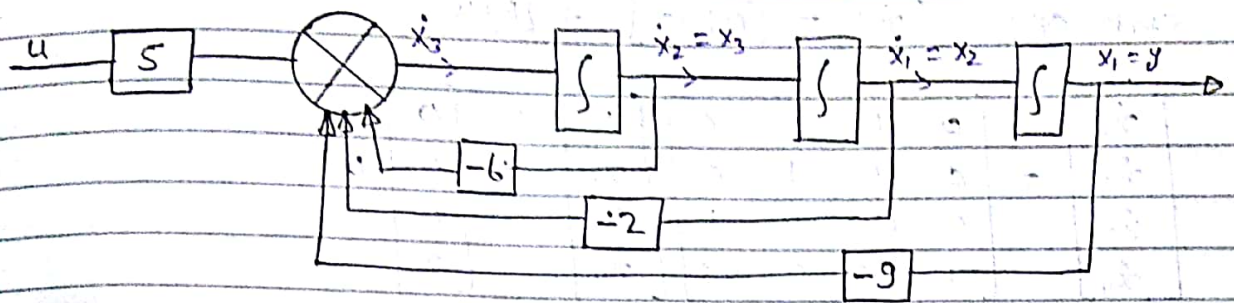
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9 & -2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u$$

output eqⁿ:- $y = [C]\{x\}$

($y=x_1$)

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

* Block diagram:-



W16
7M

Q-3) Give the state space representation for the system whose T.F. is.

$$\frac{y(s)}{u(s)} = \frac{2}{s^4 + 1.5s^3 + 2.5s + 1}$$

$$s^4 y(s) + 1.5s^3 y(s) + 2.5s y(s) + y(s) = 2u(s)$$

Taking Inverse Laplace

$$\ddot{\ddot{y}} + 1.5\ddot{\ddot{y}} + 0\ddot{\ddot{y}} + 2.5\dot{y} + y = 2u$$

$$\ddot{\ddot{y}} = -1.5\ddot{\ddot{y}} - 0\ddot{\ddot{y}} - 2.5\dot{y} - y + 2u \quad \text{--- (1)}$$

$$\text{Let, } x_1 = y \quad x_2 = \dot{y} \quad x_3 = \ddot{y} \quad x_4 = \ddot{\ddot{y}} \quad \text{--- (2)}$$

$$\dot{x}_1 = \dot{y} \quad \dot{x}_2 = \ddot{y} \quad \dot{x}_3 = \ddot{\ddot{y}} \quad \dot{x}_4 = \ddot{\ddot{\ddot{y}}} \quad \text{--- (3)}$$

From eqn (1) (2) (3) we get

$$\dot{x}_1 = x_2 \quad \text{--- (4)}$$

$$\dot{x}_2 = x_3 \quad \text{--- (5)}$$

$$\dot{x}_3 = x_4 \quad \text{--- (6)}$$

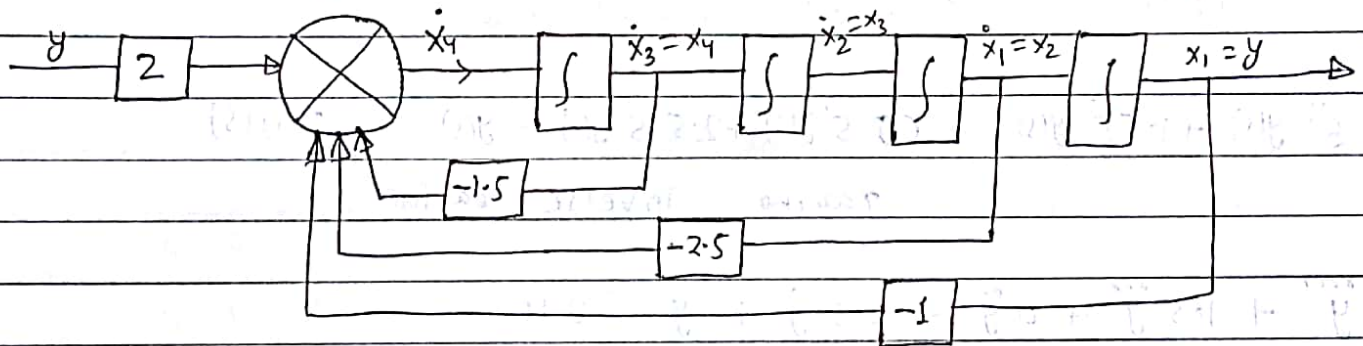
$$\dot{x}_4 = -1.5x_4 - 0x_3 - 2.5x_2 - x_1 + 2u \quad \text{--- (7)}$$

State eqⁿ:- $\dot{x} = [A] \{x\} + [B]u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -2.5 & 0 & -1.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} u$$

Output eqⁿ:- $y = [C] \{x\}$

Block diagram (From eqⁿ 6)



✓ $\frac{s-1.5}{s^4}$

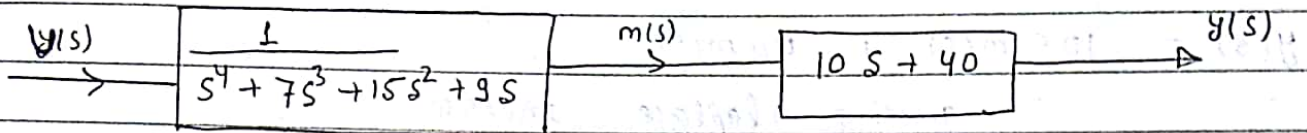
(4) Give the State space representation for the system whose T.F. is, $\frac{y(s)}{u(s)} = \frac{10(s+4)}{s(s+1)(s+3)^2}$

$$\frac{y(s)}{u(s)} = \frac{10s + 40}{(s^2 + s)(s^2 + 6s + 9)}$$

$$= \frac{10s + 40}{s^4 + 6s^3 + 9s^2 + s^3 + 6s^2 + 9s}$$

$$\frac{y(s)}{u(s)} = \frac{10s + 40}{s^4 + 7s^3 + 15s^2 + 9s}$$

$$\frac{y(s)}{u(s)} = \frac{1}{s^4 + 7s^3 + 15s^2 + 9s} \times 10s + 40$$



$$\textcircled{1} \frac{m(s)}{u(s)} = \frac{1}{s^4 + 7s^3 + 15s^2 + 9s}$$

$$s^4 m(s) + 7s^3 m(s) + 15s^2 m(s) + 9s m(s) + 0 \cdot m = u(s)$$

Taking inverse Laplace.

$$\ddot{m}'' + 7\ddot{m}' + 15\ddot{m} + 9\dot{m} + 0m = u$$

$$\ddot{m}'' = -7\ddot{m}' - 15\ddot{m} - 9\dot{m} - 0m + u$$

Let,

$$x_1 = m \quad x_2 = \dot{m} \quad x_3 = \ddot{m} \quad x_4 = \ddot{m}'$$

$$\dot{x}_1 = \dot{m} \quad \dot{x}_2 = \ddot{m} \quad \dot{x}_3 = \ddot{m}' \quad \dot{x}_4 = \ddot{m}''$$

$$\therefore \dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = -7x_4 - 15x_3 - 9x_2 - 0x_1 + u$$

State eqn:-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -9 & -15 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$\textcircled{2} \quad \frac{y(s)}{m(s)} = 10s + 40$$

$$y(s) = 10s m(s) + 40 m(s)$$

Taking Laplace Inverse

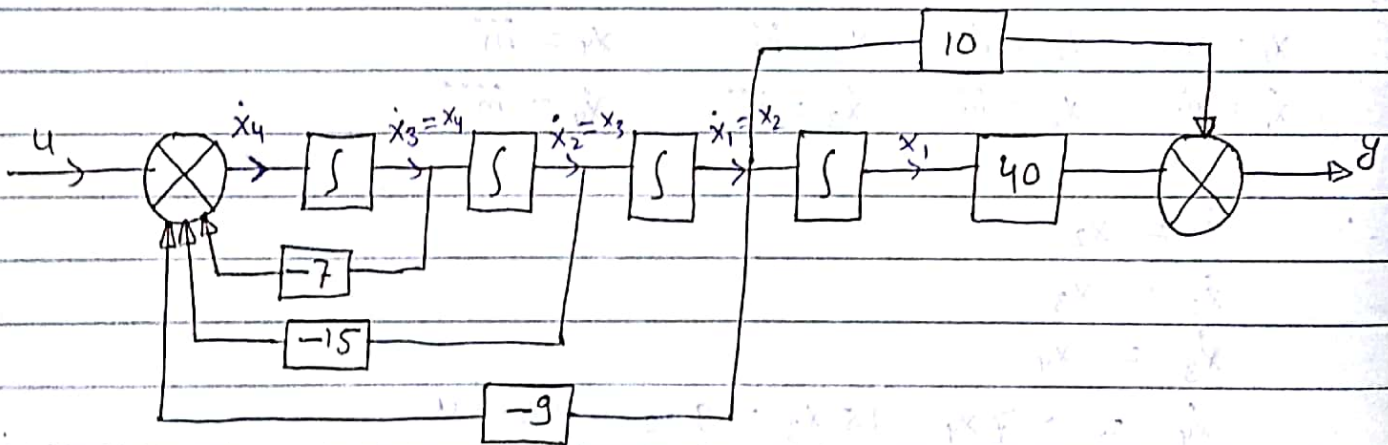
$$y = 10 \dot{m} + 40m$$

$$y = 10 x_2 + 40 x_1$$

output eqⁿ :- $y = [C] \{x\}$

$$y = \begin{bmatrix} 40 & 10 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix}$$

Block diagram:-



Type-II: Controllability & observability

→ Imp. Query →

(Refer notes)

* Controllable matrix,

$$S_c = \begin{bmatrix} B \\ AB \\ A^2B \\ \vdots \end{bmatrix} \quad [B \quad AB \quad A^2B \quad \dots]$$

If $|S_c| = 0$ ----- uncontrollable

If $|S_c| \neq 0$ ----- controllable

* observable matrix,

$$S_o = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \end{bmatrix} \quad [C \quad CA \quad CA^2 \quad \dots]$$

If $|S_o| = 0$ ----- unobservable

If $|S_o| \neq 0$ ----- observable

Q-1) $\frac{y(s)}{u(s)} = \frac{2}{s^3 + 6s^2 + 11s + 6}$

Find the state and output equation in matrix form and also test controllability & observability.

$$s^3 y(s) + 6s^2 y(s) + 11s y(s) + 6y(s) = 2u(s)$$

TAKING Inverse Laplace

$$\ddot{y} + 6\dot{y} + 11y + 6y = 2u$$

$$\ddot{y} = -6\dot{y} - 11y - 6y + 2u$$

Let,

$$\begin{aligned} x_1 &= y & x_2 &= \dot{y} & x_3 &= \ddot{y} \\ \dot{x}_1 &= \dot{y} & \dot{x}_2 &= \ddot{y} & \dot{x}_3 &= \dddot{y} \end{aligned}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -6x_3 - 11x_2 - 6x_1 + 2u$$

State eqⁿ :-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u$$

Output eqⁿ :-

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

* Controllable matrix

$$Q_c = [B \quad AB \quad A^2B]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -12 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & 6 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -12 \end{bmatrix} = \begin{bmatrix} 2 \\ -12 \\ 50 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & -12 \\ 2 & -12 & 50 \end{bmatrix}$$

$$|Q_c| = 0 - 0 + 2[0 - (2 \times 2)] = -8 \neq 0 \text{ --- Controllable matrix.}$$

* observable matrix, $Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$CA^2 = C(A)A = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_0| = 1(1 \times 1) - 0 + 0 = 1 \neq 0 \text{ — observable matrix}$$

W-15
 3M

② Find the state space representation in matrix form for T.F.

$$\frac{C(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

Also test controllability & observability.

$$s^3 C(s) + 9s^2 C(s) + 26s C(s) + 24 C(s) = 24 R(s)$$

Taking inverse Laplace

$$\ddot{c} + 9\dot{c} + 26c + 24c = 24R$$

$$\ddot{c} = -9\dot{c} - 26c - 24c + 24R$$

Let,

$$x_1 = c \quad x_2 = \dot{c} \quad \dot{x}_2 = \ddot{c}$$

$$\dot{x}_1 = \dot{c} \quad \dot{x}_2 = \ddot{c} \quad \dot{x}_3 = \ddot{c}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = -9x_3 - 26x_2 - 24x_1 + 24R$$

State eqn :-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} R$$

output eqn:-

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

Here,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

* Controllable matrix

$$Q_c = [B \quad AB \quad A^2B]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} = \begin{bmatrix} 0 \\ 24 \\ -216 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} 0 \\ 24 \\ -216 \end{bmatrix} = \begin{bmatrix} 24 \\ -216 \\ 1320 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 0 & 24 \\ 0 & 24 & -216 \\ 24 & -216 & 1320 \end{bmatrix}$$

$$|Q_c| = 24 [0 - (24 \times 24)] = -13824 \neq 0$$

∴ System is controllable.

* Observable matrix,

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$CA^2 = CA(A) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ -24 & -26 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|Q_0| = 1[1 \times 1 - 0] = 1 \neq 0$$

$\therefore$ System is observable.

S-12
W-11

Q- check the observability & controllability.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

* Controllable matrix

$$Q_c = [B \quad AB \quad A^2B]$$

$$AB = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \\ 4 & 0 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 0 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 0 \\ 10 & 0 \end{bmatrix}$$

$$|Q_c| = \begin{vmatrix} 0 & 1 & 0 & 2 & 0 & 4 \\ 1 & 0 & 2 & 0 & 4 & 0 \\ 1 & 0 & 4 & 0 & 10 & 0 \end{vmatrix}$$

$$= |-1(4-2) : 0| = |-2 \ 0| \neq 0$$

$\therefore$ System is controllable

* observable matrix, $Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

$$CA^2 = (CA)A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 0 & 0 \\ 0 & 4 & 0 \end{bmatrix}$$

$$|Q_0| = \begin{vmatrix} 0 \\ 0 \end{vmatrix} = 0$$

∴ System is unobservable.

Type-III Transfer function

$$TF = C \times (SI - A)^{-1} B$$

$$\text{i.e., } TF = C \times \frac{1}{|SI - A|} \times \text{Adj}(SI - A) B$$

Q- A Single input single output system is given as.

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = [1 \ 0 \ 2] x(t)$$

Test for controllability and observability.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad C = [1 \ 0 \ 2]$$

* Controllable matrix

$$Q_c = [B \ AB \ A^2B]$$

$$AB = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$$

$$|Q_c| = \begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \\ 0 & 0 & 0 \end{vmatrix}$$

$$|Q_c| = 0$$

$\therefore$ system is uncontrollable

* observable matrix, $Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & -6 \end{bmatrix}$$

$$CA^2 = (CA)A = \begin{bmatrix} -1 & 0 & -6 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 18 \end{bmatrix}$$

$$|Q_o| = \begin{vmatrix} 1 & 0 & 2 \\ -1 & 0 & -6 \\ 1 & 0 & 18 \end{vmatrix}$$

$$|Q_o| = 0$$

System is unobservable

S-17

Q - Determine whether the system given below is completely controllable and observable or not.

$$\dot{x} = \begin{bmatrix} -6 & -18 & -6 \\ 2 & 3 & 1 \\ -4 & -8 & -3 \end{bmatrix} x + \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 3 & 1 \end{bmatrix} x$$

$$A = \begin{bmatrix} -6 & -18 & -6 \\ 2 & 3 & 1 \\ -4 & -8 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 3 & 1 \end{bmatrix}$$

* Controllable matrix

$$Q_c = [B \quad AB \quad A^2B]$$

$$AB = \begin{bmatrix} -6 & -18 & -6 \\ 2 & 3 & 1 \\ -4 & -8 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ -5 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} -6 & -18 & -6 \\ 2 & 3 & 1 \\ -4 & -8 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -5 \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \\ -1 \end{bmatrix}$$

$$|Q_c| = \begin{vmatrix} 2 & 0 & -6 \\ -3 & 2 & 1 \\ 7 & -5 & -1 \end{vmatrix}$$

$$|Q_c| = 0$$

$\therefore$ system is uncontrollable

* observable matrix, $Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} -6 & -18 & -6 \\ 2 & 3 & 1 \\ -4 & -8 & -3 \end{bmatrix} = \begin{bmatrix} -4 & -17 & -6 \end{bmatrix}$$

$$CA^2 = CA(A) = \begin{bmatrix} -4 & -17 & -6 \end{bmatrix} \begin{bmatrix} -6 & -18 & -6 \\ 2 & 3 & 1 \\ -4 & -8 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 14 & -69 & 25 \end{bmatrix}$$

$$|Q_0| = \begin{vmatrix} 1 & 3 & 1 \\ -4 & -17 & -6 \\ 14 & 69 & 25 \end{vmatrix}$$

$$|Q_0| = 1(-425 + 414) - 3(-100 + 84) + 1(-276 + 238)$$

$$|Q_0| = -1 \neq 0$$

∴ System is observable.

S-14

Q - Find observability and controllability

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -9 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} y(t)$$

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -9 & 11 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad C = [1 \ 0 \ 0]$$

* Controllable matrix.

$$Q_c = [B \ AB \ A^2B]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -9 & 11 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -9 & 11 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}$$

$$|Q_c| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 7 \end{vmatrix}$$

$$|Q_c| = 0 - 1(7-2) + 1(2-1) = -4 \neq 0$$

$\therefore$ System is controllable

* observable matrix, $Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$

$$CA = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -9 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$CA^2 = CA(A) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -9 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

$$|Q_o| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$|Q_o| = 1(1-0) = 1 \neq 0$$

$\therefore$ system is observable.

Q- Check controllability and observability

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad C^T = \begin{bmatrix} 10 \\ 5 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix}$$

* Controllable matrix

$$Q_c = [B \quad AB \quad A^2B]$$

$$AB = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ -6 & -11 & -6 & 1 & -12 \end{bmatrix}$$

$$A^2B = A(AB) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -12 \\ -6 & -11 & -6 & -12 & 61 \end{bmatrix}$$

$$|Q_c| = \begin{vmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -12 & 1 & -12 \\ 1 & -12 & 61 & -12 & 61 \end{vmatrix}$$

$$|Q_c| = 1(61 - 144) - 0 + 1(0 - 1)$$

$$|Q_c| = -84 \neq 0$$

$\therefore$ system is controllable

* Observable matrix

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

* Observable matrix,

$$Q_o = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}$$

$$CA = \begin{bmatrix} 10 & 5 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = \begin{bmatrix} -6 & -1 & -1 \end{bmatrix}$$

$$CA^2 = CA(A) = \begin{bmatrix} -6 & -1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} = \begin{bmatrix} 6 & -9 & 5 \end{bmatrix}$$

$$|Q_o| = \begin{vmatrix} 10 & 5 & 1 \\ -6 & -1 & -1 \\ 6 & -9 & 5 \end{vmatrix}$$

$$|Q_o| = 10(-30+6) - 5(-30+6) + 1(30+6)$$

$$|Q_o| = 9 \neq 0$$

∴ System is controllable.

TYPE-3

Date 14/4/18
Page

Q-1) Consider the following no. of state space representation. Examine controllability & observability also convert state space ~~represent~~ equation in to T.F.

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, B = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, C = \begin{bmatrix} -2 & -2 \end{bmatrix}$$

Controllable matrix, $Q_c = [B \ AB]$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -2 & -10 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} -2 & -2 \\ -2 & -10 \end{bmatrix}$$

$$|Q_c| = (-2 \times -10) - (-2 \times -2) = -24 \neq 0$$

$\therefore$ System is controllable.

* Observable matrix, $Q_o = \begin{bmatrix} C \\ CA \end{bmatrix}$

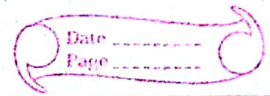
$$CA = \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} -2 & -2 \\ 4 & 4 \end{bmatrix}$$

$$|Q_o| = (-2 \times 4) - (-2 \times 4) = 0$$

$\therefore$ System is unobservable.

Transfer function:-



$$TF = C \times \frac{1}{|SI-A|} \times \text{Adj}(SI-A) B \quad \text{--- (1)}$$

$$SI-A = S \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix}$$

$$|SI-A| = S(S+3) - (-1 \times 2) = S^2 + 3S + 2$$

$$\text{Adj}(SI-A) = \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix}$$

put in eqn (1)

$$T.F = \begin{bmatrix} -2 & -2 \end{bmatrix} \cdot \frac{1}{S^2 + 3S + 2} \begin{bmatrix} S+3 & 1 \\ -2 & S \end{bmatrix} \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$= \frac{1}{S^2 + 3S + 2} \begin{bmatrix} -2 & -2 \end{bmatrix} \begin{bmatrix} -2S - 8 \\ 4 - 2S \end{bmatrix}$$

$$= \frac{1}{S^2 + 3S + 2} \begin{bmatrix} -2(-2S - 8) - 2(4 - 2S) \end{bmatrix}$$

$$= \frac{1}{S^2 + 3S + 2} \begin{bmatrix} 4S + 16 - 8 + 4S \end{bmatrix}$$

$$T.F = \frac{8S + 8}{S^2 + 3S + 2}$$

Q-2 Find T.F. of system given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$T.F. = C \times \frac{1}{|sI - A|} \times \text{Adj}(sI - A) B \quad \text{--- (1)}$$

$$|sI - A| = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 3 & s+2 \end{bmatrix}$$

$$|sI - A| = s(s+2) - (-1 \times 3) = s^2 + 2s + 3$$

$$\text{Adj}(sI - A) = \begin{bmatrix} s+2 & 1 \\ -3 & s \end{bmatrix}$$

put in eqn (1)

$$T.F. = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{s^2 + 2s + 3} \times \begin{bmatrix} s+2 & 1 \\ -3 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2 + 2s + 3} \times \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$T.F. = \frac{1}{s^2 + 2s + 3}$$

Note:- For Adjoint

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \quad \text{• calculate for row & write in column.}$$

S-14
7M

Imp

Q-3 Find the T.F. for equation of given state model when,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -7 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\text{T.F.} = C \times \frac{1}{|SI-A|} \times \text{Adj}(SI-A) \times B \quad \text{--- (1)}$$

$$|SI-A| = \begin{vmatrix} s & 1 & 0 & 0 \\ 0 & s & 1 & 0 \\ 0 & 0 & s & 1 \\ 0 & 0 & 1 & s+7 \end{vmatrix} = \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 3 & s+7 \end{vmatrix}$$

$$\begin{aligned} |SI-A| &= s[s(s+7)+3] + 1(0+1) \\ &= s(s^2+7s+3) + 1 \\ &= s^3 + 7s^2 + 3s + 1 \end{aligned}$$

$$\text{Adj}(SI-A) = \begin{bmatrix} +(s^2+7s+3) & -(-s-7) & +(1) \\ -(1) & +(s^2+7s) & -(-s) \\ +(-s) & -(3s+1) & +(s^2) \end{bmatrix}$$

$$= \begin{bmatrix} s^2+7s+3 & s+7 & 1 \\ -1 & s^2+7s & s \\ -s & -3s-1 & s^2 \end{bmatrix}$$

put in eqn ①

$$T.F. = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \frac{1}{s^3 + 7s + 3s + 1} \begin{bmatrix} s^2 + 7s + 3 & s + 7 & 1 \\ -1 & s^2 + 7s & s \\ -s & -3s - 1 & s^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^3 + 7s + 3s + 1} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{s} \\ s \\ s^2 \end{bmatrix}$$

$$T.F. = \frac{1}{s^3 + 7s + 3s + 1}$$

W-16
gm

Q- Find T.F. of system check whether the system is observable & controllable or not.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$(1-s) \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} + (s+6) \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = |A - sI|$$

$$1 + (s+6)(s+1) = 0$$

$$1 + 2s + s^2 + 6s + 6 = 0$$

$$\begin{aligned} (1) &= (1-s) - (s+6)(s+1) = (1-s) - (s^2+7s+6) = -s^2-8s-5 \\ (2) &= (s+6) - (1-s) = 2s+5 \\ (3) &= (s+6) - (s+6) = 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -s^2-8s-5 \\ 2s+5 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$