3. Periodic and Aperiodic signal.

A signal which varies after a specified time Period is called as Periodic signal.

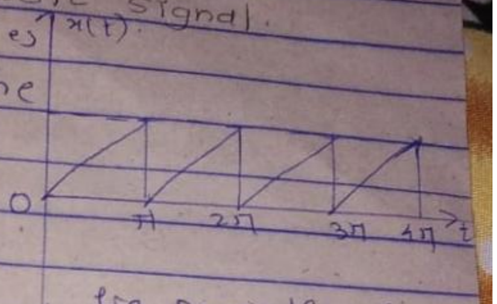


fig. periodic signal

Mathematically if signal satisfies the equation $x(t) = x(t+T)$ then it is Periodic otherwise it is aperiodic.

$x(t) \mid x(t+T)$ + Periodic

$\therefore x(t)$ - amplitude of signal.

4) Unit Impulse signal :-

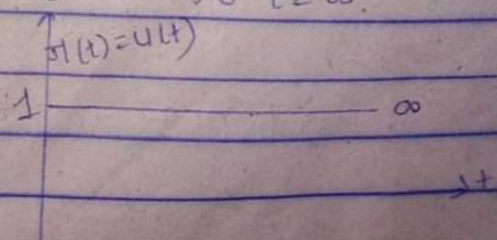
If $x(t) = \begin{cases} 1 & ; \text{ for } t=0 \\ 0 & , \text{ elsewhere} \end{cases}$

then signal is called as unit impulse signal denoted by $\delta(t)$.

5) unit step Signal :-

If $u(t) = \begin{cases} 1 & \text{for } t=0 \text{ to } t=\infty \\ 0 & \text{for } t=-\infty \text{ to } t=\infty \end{cases}$

This signal is called unit step signal.



PAGE NO.
DATE :

$$= \frac{1}{2}\int_{-\infty}^{\infty}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] e^{-j\omega t}\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{j\omega_0 t - j\omega t} + e^{-j\omega_0 t - j\omega t}\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{-jt(\omega_0+\omega)} + e^{-jt(\omega_0+\omega)}\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \frac{1}{2}\left[2\pi\delta(\omega+\omega_0)t + 2\pi\delta(\omega+\omega_0)t\right]$$

$$= \pi\delta t\left[(\omega+\omega_0) + (\omega+\omega_0)\right]$$

$$= 2\pi\delta t(\omega+\omega_0)$$

Find fourier Transform of $\sin\omega_0 t$

$$f(t) = \int_{-\infty}^{\infty}\sin\omega_0 t\; e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty}\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2}\right] e^{-j\omega t}\,dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty}\left[e^{j\omega_0 t - j\omega t} - e^{-j\omega_0 t - j\omega t}\right]dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty}\left[e^{-jt(\omega_0+\omega)} - e^{-jt(\omega_0+\omega)}\right]$$

proof :-

$$f(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

Differentiate

$$\frac{d}{dt} f(\omega) = \int_{-\infty}^{\infty} f(t)\, (-j\omega)\, e^{-j\omega t}\, dt$$

$$\boxed{\frac{d}{dt} f(\omega) = f(\omega)\,(-j\omega)} \qquad \because\ f(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

* Convolution (Folding of signals)

The term convolution means folding of signal it is a special kind of multiplication or mathematical operation used for signal analysis. If F_1 and F_2 are the two functions then $f_1(t)$ convolves with $f_2(t)$ is given by

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(k)\, f_2(t-k)\, dk$$

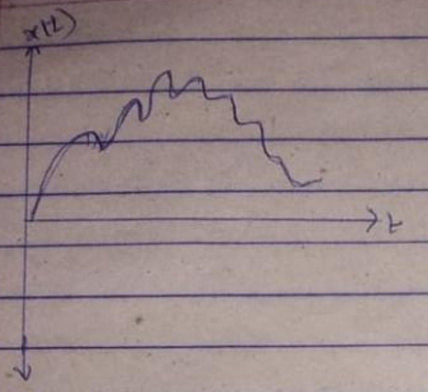


fig. analog signal

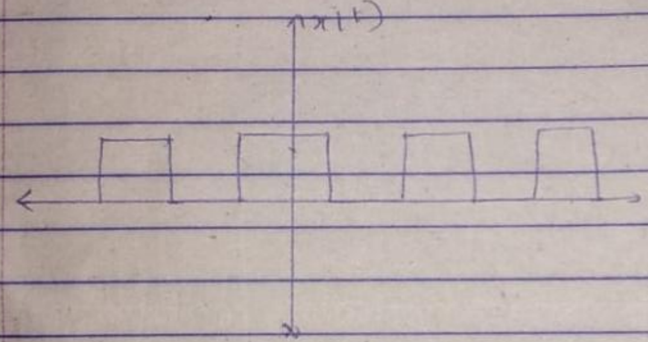


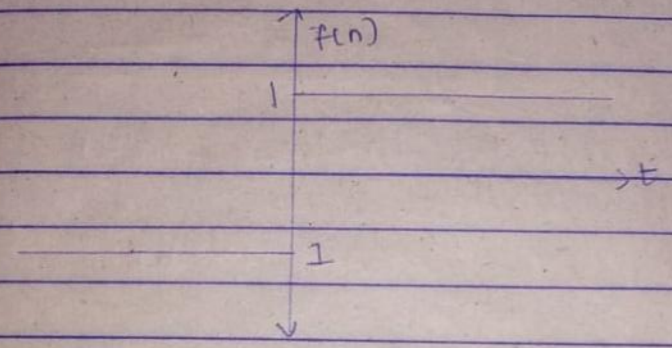
fig. Digital signal

A signal whose amplitude take ~~smi~~ finite number of value and varying in ~~time~~ step in an continuous ~~range~~ is called as digital signal.

3) signam function :-

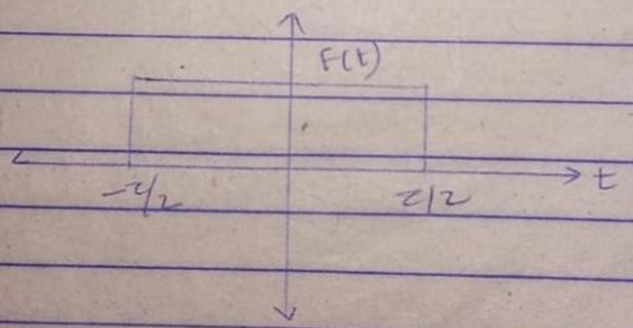
A signam function is given by

$$f(t) = sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



4 Gate function :-

$$F(t) = G_\tau(t) = A \quad \text{for } -\tau/2 \text{ to } \tau/2$$
$$= 0 \quad \text{else where}$$



Time shifting Property :-

$f(t) \longrightarrow f(\omega)$

then $f(t-t_0) \longrightarrow f(\omega)\, e^{-j\omega t_0}$

$f(t+t_0) \longrightarrow f(\omega)\, e^{+j\omega t_0}$

Proof :- $f(t) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt$

$$f(t-t_0) = \int_{-\infty}^{\infty} f(t-t_0)\, e^{-j\omega(t-t_0)}\, dt$$

$x = t - t_0$, $t = x + t_0$

$dx = dt$ $= \int_{-\infty}^{\infty} f(x)$

$$= \int_{-\infty}^{\infty} f(t-t_0)\, e^{-j\omega t_0} \cdot e^{-j\omega t_0}\, dt$$

$$f(t-t_0) = \int_{-\infty}^{\infty} f(x)\, e^{-j\omega t_0} \cdot e^{-j\omega t_0}\, dt$$

$f(t) = f(\omega)$

$\boxed{f(t-t_0) = f(\omega) \cdot e^{+j\omega t_0}}$ Similarly

$\boxed{f(t+t_0) = f(\omega) \cdot e^{-j\omega t_0}}$

6) Energy and Power of signal :-

Total energy dissipiated by a signal or consume by a signal during the interval of $-T/2$ to $T/2$ is given by.

$$E = \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

Avarage Power associated with the signal is given by

$$P = E/T$$

$$P = \lim_{T \to 0} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

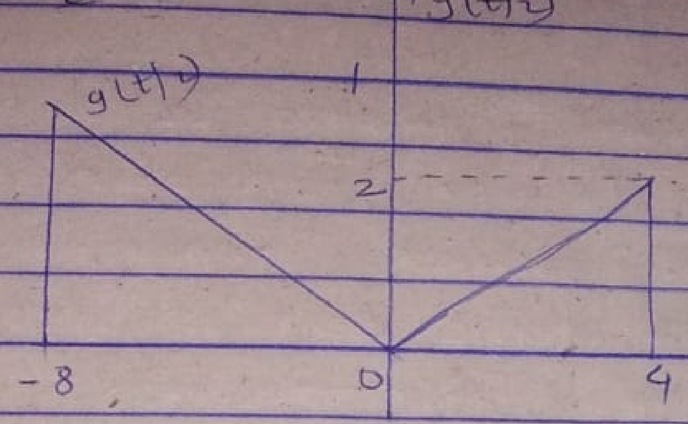
★ System and its Types :-

System :- A system is a set of functional black that are combine together to produce an output in response, to ~~signal~~ input.

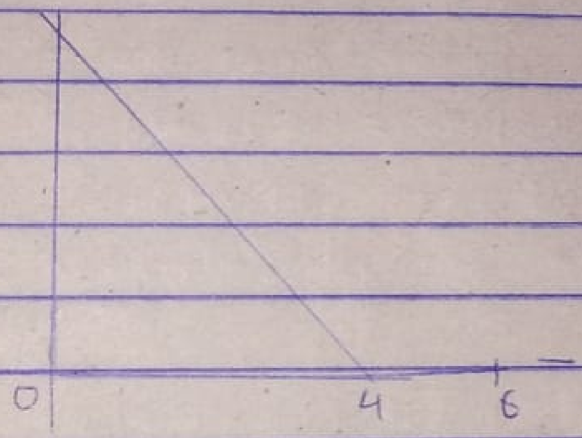
System may be made up of Physical component.

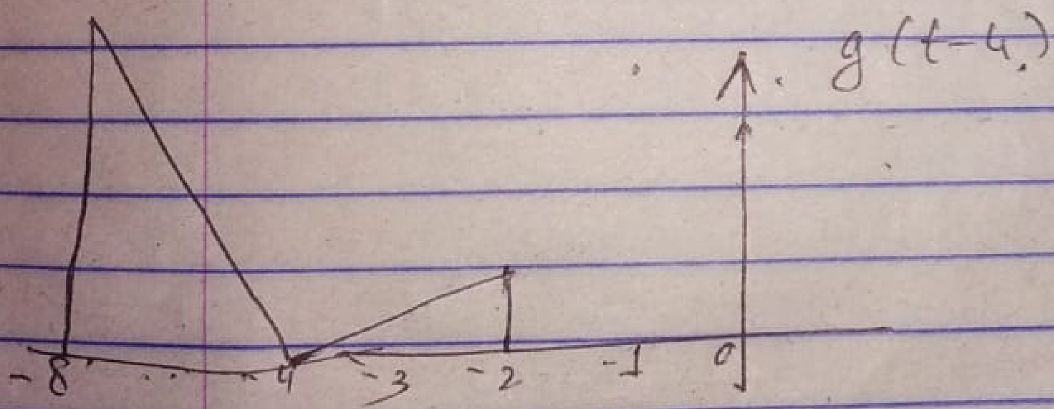
Types of System

(3) g(t/2)



(4) g(t-4)





PAGE NO.
DATE :

* Fourier Transform of Some Important Function.

1) Find fourier transform of exponential function $e^{-at}u(t)$

$F(t) = e^{-at}u(t)$

$F(t) = f(\omega)$

$= \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} \cdot dt$

$= \int_{-\infty}^{\infty} e^{-at} e^{-j\omega t} \cdot dt$

$= \int_{0}^{\infty} e^{-at - j\omega t} dt$

$= \int_{0}^{\infty} e^{-t(a + j\omega)} dt$

$= \left[\frac{e^{-t(a+j\omega)}}{(a+j\omega)} \right]_0^{\infty}$

$= \frac{-1}{a+j\omega} \left[e^{-\infty} - e^{0} \right]$

$= \frac{-1}{a+j\omega} (-1)$

$F(t) = \frac{1}{a+j\omega}$

$e^{-at} u(t) = 1/a+j\omega$

$$E_{g_3} = \int_0^5 [-1]^2 dt = +[t]_0^5 = +[5-0]$$

$$E_{g_3} = +5$$

$$E_{g_4} = \int_0^5 \left| e^{-t/5} \right|^2 dt = \int_0^5 \left| e^{-2t/5} \right| dt$$

$$= \left[\frac{e^{-2t/5}}{-2/5} \right]_0^5 = \frac{e^{-2} - e^{0}}{-2/5}$$

$$= \frac{e^{-2} - 1}{-2/5}$$

$$= \frac{-5}{2} \left[e^{-2} - 1 \right]$$

$$= -5/2 \, (0.1353 - 1)$$

$$E_{g_4} = 2.1616$$

$$C_n = \frac{1}{\sqrt{E_n E_g}} \int_{-\infty}^{\infty} x(t)\, g(t)\, dt$$

$$C_{n_1} = \frac{1}{\sqrt{5 \times 5}} \int_0^5 (1)(1)\, dt$$

$$= 1/5 \, [t]_0^5$$

$$= \frac{1}{5} [5-0]$$

$$\boxed{C_n = 1}$$

time invarient system, otherwise System is Varient.

✱ Correlation Coefficient [C_n] :-

let us consider $x(t)$ and $g(t)$ are the two functions. If these two functions are related to each other then they are said to be correlated.

A coefficient used to correlate these two function is known as correlation coeffient, and it is given by

$$\therefore C_n = \frac{1}{\sqrt{E_x E_g}} \int_{-\infty}^{\infty} x(t)\, g(t)\, dt$$

where, E_g and E_n are the energy at function $g(t)$ and $x(t)$.

$C_n \rightarrow$ correlation coefficient

C_n will always in between -1 to 1.

If $C_n = 1$ then two functions are similar and if $C_n = -1$ then two function are exactly opposite.

(22) $\boxed{e^{-j\omega} = 2\pi\delta\,(\omega+\omega_0)}$

4) Find the fourier Transform of gate function $G_\tau(t) = A$

$$F(t) = F(\omega)$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} G_\tau(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} A \cdot e^{-j\omega t}\, dt$$

$$= A \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-\tau/2}^{\tau/2}$$

$$= -\frac{A}{j\omega}\left[e^{-j\omega(\tau/2)} - e^{-j\omega(-\tau/2)}\right]$$

$$= \frac{A}{-j\omega}\left[e^{-j\omega\tau/2} - e^{j\omega\tau/2}\right]$$

$$F(t) = \frac{A\sin(\omega\tau/2)}{-2j\omega}$$

Find fourier Transform of $\cos\omega_0 t$

$$F(t) = \int_{-\infty}^{\infty} \cos\omega_0 t\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\frac{e^{+j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] e^{-j\omega t}\, dt$$

frequency domain by an amount of $\omega_0 t$ or viceversa.

Time differentiation Property :

$f(t) \longrightarrow f(\omega)$ then

$\frac{d}{dt} f(t) \longrightarrow (j\omega)\, F(\omega)$

statement:- Time differentiation property of fourier transform is given by if

$f(t) \longrightarrow f(\omega)$ then $\frac{d}{dt} f(t) \longrightarrow (j\omega)\, f(\omega)$.

proof:-

$$F^{-1}\{f(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\omega) \cdot e^{j\omega t}\, dt$$

Differentiate

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(\omega)\, e^{j\omega t} (j\omega)\, dt$$

$$\boxed{f(t) = f(\omega)(j\omega)}$$

frequency differentiation property :

Statement : Frequency differentiation property of fourier transform is given by if $f(t) \longrightarrow f(\omega)$ then

$$\frac{d}{dt} f(\omega) \longrightarrow (-j\omega)\, f(t)$$

(15)

PAGE NO.
DATE:

Fourier Transform

Fourier Transform is named after Josuef fourier Transform it is an integral fourier Transform it converts time domain into frequency domain i.e. $f(t) \rightarrow f(\omega)$;

Similarly inverse fourier transform converts the signal $f(\omega) \rightarrow f(t)$

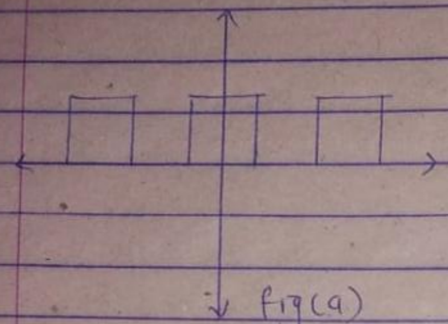
Here $F(\omega)$ is a complex soln i.e

i.e. $F(\omega) = |f(\omega)| e^{i\theta\omega}$

Where $|f(\omega)|$ is magnitude spectrum and $\theta\omega$ phase angle spectrum

$$\therefore F[f(t)] = f(\omega)$$
$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$\therefore F^{-1}(\{f(\omega)\} = f(t)$$
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) e^{j\omega t} dt$$



fig(a)

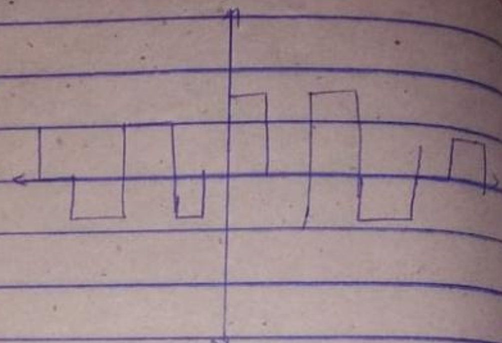


fig (b)

A function f(t) is odd if it is plot asymmetric about vertical axis

$$f(t) = -f(-t)$$

eg:- + $-\sin x$

2) Sampling and Sinc function

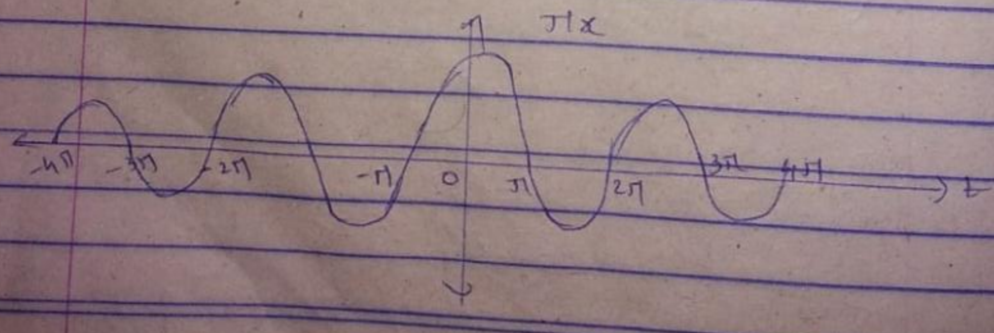
Sampling ⟶ decrement in amplitude

sampling function is given by $Sa(x)$

$$Sa(x) = \sin x / x$$

Sinc function is given by $\operatorname{sinc}(x)$

$$\operatorname{Sinc}(x) = \frac{\sin \pi x}{\pi x}$$



$$C_{n_2} = \frac{1}{\sqrt{5 \times 1.25}} \int_0^5 1\,(0.5)\,dt$$

$$= \frac{2}{5} \int_0^5 0.5\,dt$$

$$= \frac{2}{5} \times 0.5\,[t]_0^5$$

$$= \frac{2}{5} \times 0.5 \times 5$$

$$\boxed{C_{n_2} = 1}$$

$$C_{n_3} = \frac{1}{\sqrt{5 \times 5}} \int_0^5 (1)(-1)\,dt$$

$$= \frac{1}{5}\,-[t]_0^5$$

$$= -\frac{1}{5} \times 5$$

$$C_{n_3} = -1$$

$$C_{n_4} = \frac{1}{\sqrt{5 \times 2.1616}} \int_0^5 1 \times e^{-t/5}\,dt$$

$$= 0.3041 \int_0^5 e^{-t/5}\,dt$$

$$= 0.3041 \left[\frac{e^{-t/5}}{-1/5}\right]_0^5$$

$$= 0.3041 \times (-5)\,[0.3678 - 1]$$

$$C_{n_4} = 0.9612$$

PAGE NO

DATE

Condition for existence of fourier Transform :-

For existance of fourier Transform $F\{F(t)\}$ must satisfy the following condition.

i) $F\{f(t)\}$ must be absolutely integral i.e. $\int_{-\infty}^{\infty} F(t)\, dt < 0$

ii) $F\{f(t)\}$ must be single value function.

iii) $F\{f(t)\}$ must have finite no. of discontinous in an finite interval.

iv) $F\{f(t)\}$ must have finite number of maxima and minima in an finite interval.

* Important function used in signal Analysis :-

1) Even and odd function

Even → Symmetry

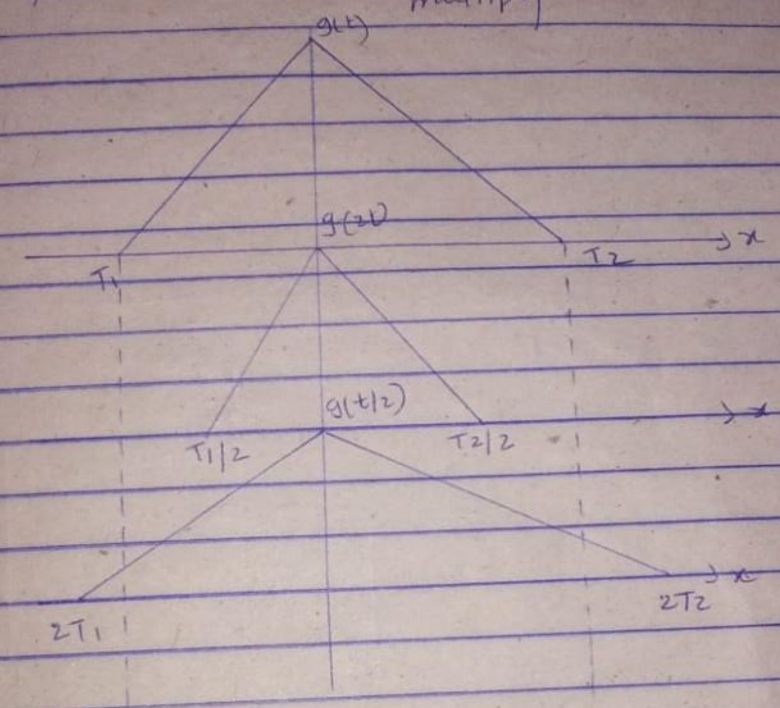
Odd → Asymmetry

A function $F(t)$ is even if the plotting is symmetric about vertical axis $F(t) = F(-t)$

eg → $\cos x$

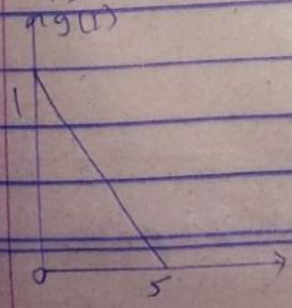
$g(t/a)$ is expanded signal by value of "a".

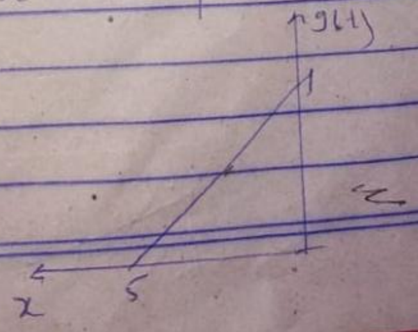
compression → value ~~multiply~~ divide by 2.
expansion → value ~~divide~~ multiply by 2



3. Time Inversion Operation :-

If $g(t)$ is original signal then $g(t)$ is called time inversion operation of $g(t)$



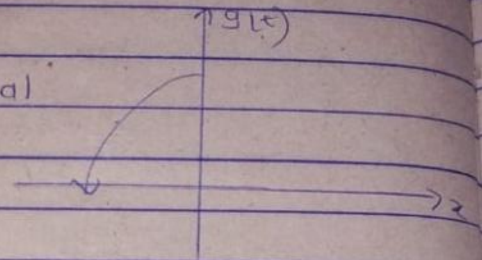


PAGE NO.
DATE:

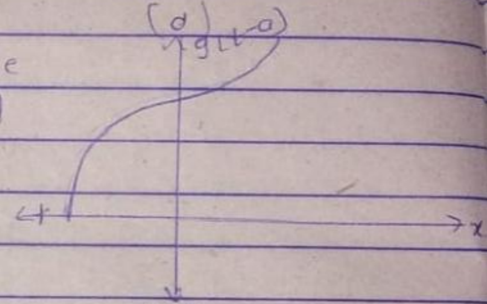
★ Signal Operation :-

1) Time Shifting

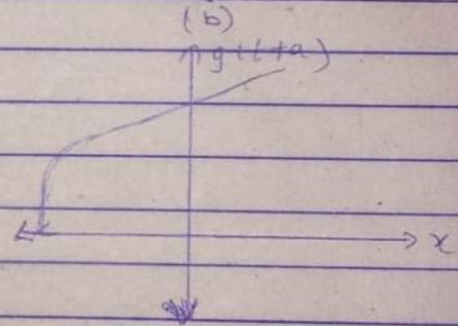
If $g(t)$ is a original signal N given by.



Then $g(t-a)$ will be the signal shifted toward right by a sample is given by.



$g(t+a)$ will be the signal shifted toward left by



2. Time Scaling operation :-

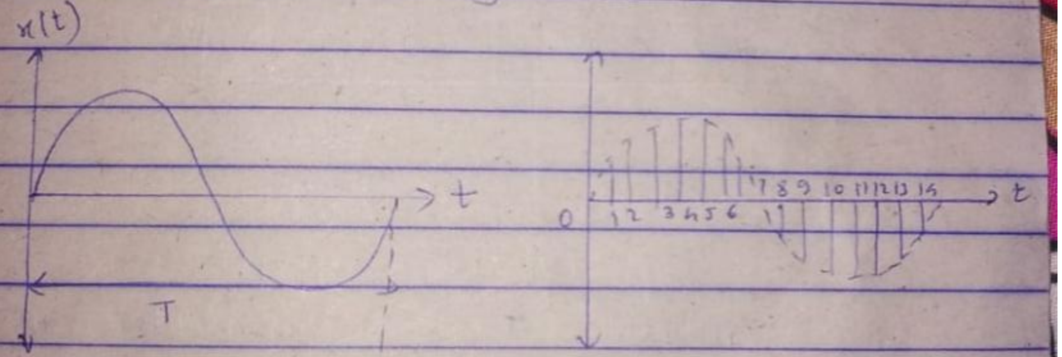
Compression and expansion of signal is known as Scaling. If $g(t)$ is the original signal then $g(at)$ is the compressive Signal by value at "a" independant Parameter is called as Signal.

These signal are used in communica-tion system, medical dignosis like ECG etc.

* Types of signals:—

1) continuous and discrete time signal

A signal which is specified for every value of time is called as continuous time signal.



A signal which is specified for discrete value of time is called as discrete time signal.

2) Analog and Digital time signal:—

A signal whose amplitude can take any number of value in an continuous range is called as analog signal.

unit-1 :-

Signal and System 12 Marks

(1) It is subject deal with system and intersection of System with signal.

(2) Most General eg. of study signal and System is communication through mobile.

(3) Voice act as Signal Here voice signal interacted with mobile system

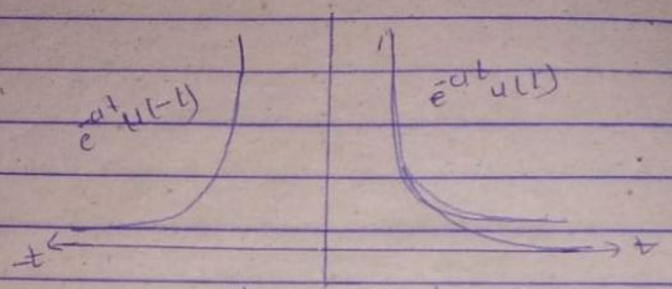
Signal and Types of signal

(1) signal :- Signal is function that carry or onway information one place to another. it can be define as any quantity w.r.t time, space and any other independ Parameter called as Signal.

(21)

DATE :

3) Find the fourier Transform of double sided exponential function (e^{-at}).



$F(t) = e^{-at}u(t) + e^{-at}u(-t)$

$f(\omega) = ?$

$$F(t) = \int_{-\infty}^{\infty} F(t) \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \left[e^{-at}u(t) + e^{-at}u(-t) \right] e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} \left[e^{-at}u(t)\, e^{-j\omega t} + e^{-at}u(-t)\, e^{-j\omega t} \right] dt$$

$$= \int_{0}^{\infty} e^{-at-j\omega t} dt + \int_{0}^{-\infty} e^{-at-j\omega t} dt$$

$$= \left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)} \right]_0^{\infty} + \left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)} \right]_0^{-\infty}$$

$$= \frac{e^{\infty} - e^{0}}{-(a+j\omega)} + \frac{e^{-\infty} - e^{0}}{-(a+j\omega)} = \frac{0-1}{-(a+j\omega)} + \frac{-1}{-(a}$$

$$F(t) = \frac{1}{a+j\omega} + \frac{1}{a+j\omega} \quad \frac{1}{a+j\omega} + \frac{1}{a+j\omega}$$

$$F(t) = \frac{2}{a+j\omega} \quad \frac{2}{a+j\omega}$$

PAGE NO

DATE

(1) find four Transform of unit impulse signal.

Ans:—

(1) unit impulse

$$f(t) = \begin{cases} \delta(t) = 1 & \text{at } t=0 \\ & \text{elsewhere} \end{cases}$$

$$f(t) = f(\omega)$$

$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$= \int e^{-j\omega t}\, dt \Big]_{t=0}$$

$$= \frac{e^{-j\omega t}}{-j\omega} = \frac{e^{0}}{-j\omega} = \frac{-1}{j\omega}$$

$$I(t) = f(\omega) = f(t) = \frac{-1}{j\omega}$$

(2) Find fourier Transform of constant function.

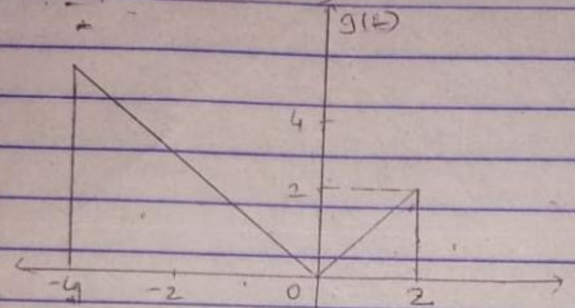
$$\Rightarrow \quad f(t) = F(\omega) \qquad G_\tau = A \qquad -\tau/2 \text{ to } \tau/2$$

$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt$$

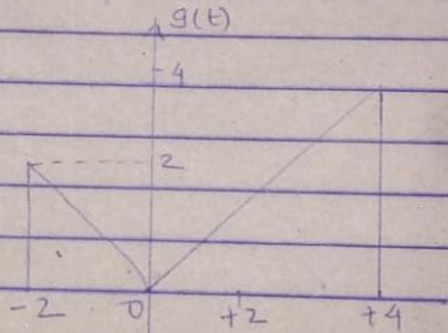
(13)

PAGE NO.
DATE :

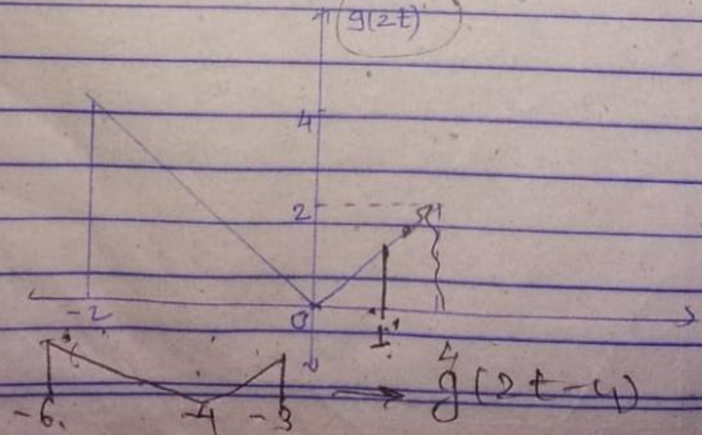
For the signal g(t) shown drawn the waveform for (1) g(-t) (2) g(2t)
(3) g(t/2) (4) g(t-4) (5) g(t+4)
(6) g(2t-4) (7) g(2t+4)



Ans ⟹ (1) g(-t)



(2) g(2t)



⟹ g(2t-4)

$$= \int_{-\tau/2}^{\tau/2} A \cdot e^{-j\omega t} \, dt$$

$$= A \int_{-\tau/2}^{\tau/2} e^{-j\omega t} \cdot dt$$

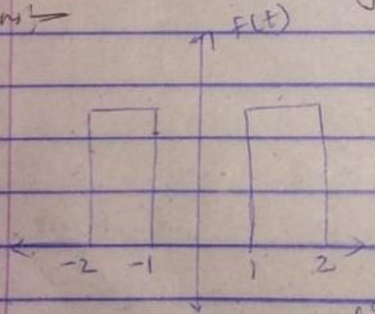
$$= A \left[\frac{e^{-j\omega t}}{-j\omega} \right]_{-\tau/2}^{\tau/2}$$

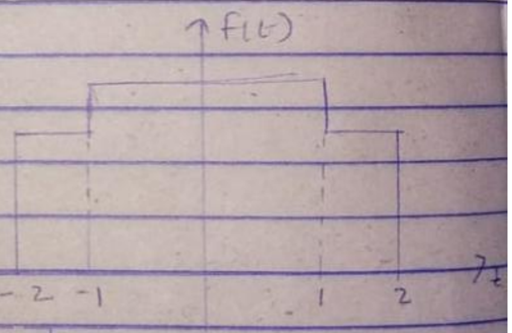
$$= A \left[\frac{e^{-j\omega\tau/2} - e^{j\omega\tau/2}}{-j\omega} \right]$$

$$= \frac{A \sin(\tau/2)}{-2j\omega}$$

(3) find fourier Transform of the Pulse show in fig.

Ans →





$$F(t) = \int_{-\infty}^{\infty} f(t) \, e^{-j\omega t} dt$$

$$f(t) = \int_{-1}^{-2} e^{-j\omega t} dt + \int_{1}^{2} 1 \cdot e^{j\omega t} dt$$

Frequency shifting Property :-

statement :- frequency shifting property of a fourier transform is given by

if $f(t) \longrightarrow f(\omega)$ then

$$f(t)\, e^{+j\omega_0 t} \longrightarrow f(\omega-\omega_0)$$

$$f(t)\, e^{-j\omega_0 t} \longrightarrow f(\omega+\omega_0)$$

proof :-

$$f(t) = f(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt$$

$$f(t)\, e^{j\omega_0 t} = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} \cdot e^{j\omega_0 t}\, dt$$

$$f(t) \cdot e^{j\omega_0 t} = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} \cdot e^{-j\omega_0 t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{jt(-\omega+\omega_0)}\, dt$$

$$f(t) \cdot e^{j\omega_0 t} = \int_{-\infty}^{\infty} f(t) \cdot e^{-jt(-\omega_0+\omega)}\, dt \qquad f(\omega) = f(\omega-\omega_0)$$

$$\boxed{f(t) \cdot e^{j\omega_0 t} = f(\omega-\omega_0)}$$ Similarly

$$\boxed{f(t) \cdot e^{-j\omega_0 t} = f(\omega+\omega_0)}$$

Physical significance

frequency ~~free~~ shifting property state that multiplication of signal by a factor of $e^{-j\omega_0 t}$ in the time domain shifts these spectrum of the signal in

PAGE NO.
DATE :

Types of System:-

1) Casual and Non-casual system:-

If present output = F(post + present) then the system is called as casual system. otherwise it is non-casual system.

Ex: calculator

2) Linear and Non-linear system:-

A System is said to be linear if it is satisfied the superposition theorem i.e. output of system is

$y_1(t) = F\{x_1(t)\}$

$y_2(t) = F\{x_2(t)\}$

$y_3(t) = F\{x_3(t)\}$

then total output must be

$y_1(t) + y_2(t) + y_3(t) = F\{x_1(t) + x_2(t) + x_3(t)\}$

This is linear system, otherwise system is non-linear.

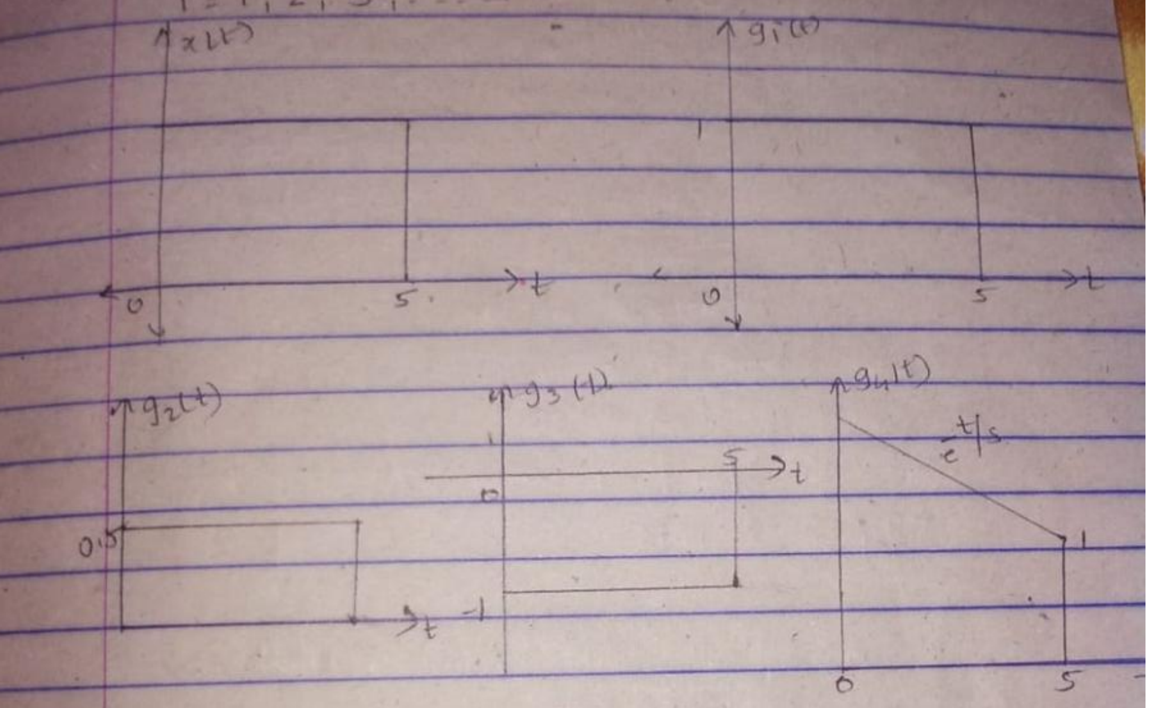
3) Time Varient and Invarient system:-

When input $x(n)$ is delayed by samp (k) then output $y(n)$ is also delayed by (k) Sample.

Such a system is called as

Imp.

Q.1 Define correlation coefficient c_n. Find c_n between pulse $x(t)$ and $g_i(t)$ where $i = 1, 2, 3, \ldots$



$\Rightarrow$ $$c_n = \frac{1}{\sqrt{E_n E_g}} \int_{-\infty}^{\infty} x(t) \cdot g(t)\, dt$$

$E_x \rightarrow$ upper limit i.e $E_n = 5$

$$E_{g_1} = \int_0^5 |1|^2 dt = [t]_0^5 = 5 - 0 = 5$$

$$E_{g_1} = 5$$

$$E_{g_2} = \int_0^5 [0.5]^2 dt = 0.25 [t]_0^5$$

~~$E_{g_1} = 5$~~

$$= 0.25 [5 - 0] = 1.25$$

2) Find the fourier transform of function $e^{at}u(t)$

$$F(t) = e^{at} u(t)$$

$$f(t) = F(\omega)$$

$$= \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{at} u(t) e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{at - j\omega t} dt$$

$$= \int_{0}^{\infty} e^{t(0 - j\omega)} dt$$

$$= \left[\frac{e^{0}(0 - j\omega)}{0 - j\omega} \right]_{-\infty}^{\infty}$$

$$= \frac{1}{0 - j\omega} \left[e^{\infty} - e^{0} \right]$$

$$F(t) = \frac{-1}{0 - j\omega}$$

$$e^{at} u(t) = \frac{-1}{0 - j\omega}$$

$$= \left[\frac{e^{2j\omega} - e^{j\omega}}{-j\omega}\right] + \left[\frac{e^{-2j\omega} - e^{-j\omega}}{-j\omega}\right]$$

$$= \frac{-1}{j\omega}\left[e^{2j\omega} - e^{j\omega} + e^{-2j\omega} - e^{-j\omega}\right]$$

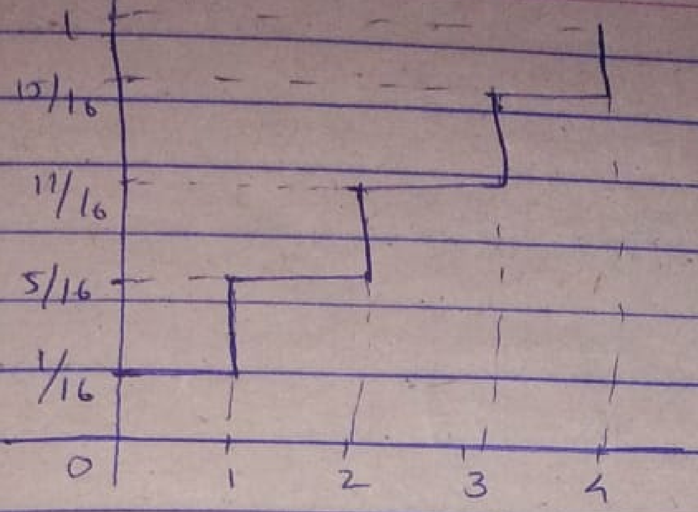
$$f(t) = \int_{-2}^{-1} e^{-j\omega t}\,dt + \int_{-1}^{1} 2e^{-j\omega t}\,dt + \int_{1}^{2} e^{-j\omega t}\,dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-2}^{-1} + 2\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^{1} + \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{1}^{2}$$

$$= \left[\frac{e^{2j\omega} - e^{j\omega}}{-j\omega}\right] + 2\left[\frac{e^{j\omega} - e^{-j\omega}}{-j\omega}\right] + \left[\frac{e^{-j\omega} - e^{-2j\omega}}{-j\omega}\right]$$

$$= \frac{-1}{j\omega}\left[e^{2j\omega} - e^{2j\omega} + 2e^{-j\omega} - 2e^{j\omega} + e^{-2j\omega} - e^{-j\omega}\right]$$

(29)



04/02/2019
Monday

A PDF is given by $f_X(x) = a e^{-b|x|}$ where X is a random variable whose allowable values ranges from $-\infty$ to $+\infty$. find

1) Relation betn A and B.
2) CDF
3) $P(1 \le X \le 2)$

$\Rightarrow$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} a \cdot e^{-b|x|}\, dx = 1$$

$$\int_{-\infty}^{0} a e^{-b|x|}\, dx + \int_{0}^{\infty} a e^{-b|x|}\, dx = 1$$

$$a \left\{ \left[\frac{e^{-bx}}{-b} \right]_{-\infty}^{0} + \left[\frac{e^{-bx}}{-b} \right]_{0}^{\infty} \right\} = 1$$

$$a\left[\frac{1}{-b}+\frac{1}{-b}\right]=1$$

$$a\left[\frac{-2}{b}\right]=1$$

$$\boxed{-2a=b}$$

$$\boxed{a=-b/2}$$

(3) $\int_1^2 f_X(x)\,dx = P(1 \leq x \leq 2)$

$$\int_1^2 a e^{-bx}\,dx = P(1 \leq x \leq 2)$$

$$a\left[\frac{e^{-ax}}{b}\right]_1^2 = P(1 \leq x \leq 2)$$

$$= \frac{a}{b}\left[e^{-2b}-e^{-b}\right] = P(1 \leq x \leq 2)$$

(2) $CDF = \int_{-\infty}^{8} f_X(x)\,dx$

of all the events are always in betn 0 and 1.

hence 0 is ~~greater~~ less than equal to $F_x(x)$

$$0 \le F_X(x) \le (1)$$

2) $F_X(-\infty) = 0$ and $F_X(\infty) = 1$

Here $x = -\infty$ means there is no Possible event has occured. and $x = \infty$ means all the possible event has occured.

Therefore Probability of all the Possible event is equal to 1. hence,

$$\underbrace{F_X(-\infty) = 0}_{\text{Min Prob.}} \text{ and } \underbrace{F_X(\infty) = 1}_{\text{Max Probability}}$$

3) If $x_1 \le x_2$ then $F_X(x_1) \le F_X(x_2)$.

2) Probability distributive function :- (PDF)

let X be a continuous Random Variable the derivative of CDF w.r.t. variable is known as probability distributive function It is denoted by $p(x)$ or $f_X(x)$.

$$\therefore P(x) = \frac{d}{dx} F_X(x)$$

PAGE NO.
DATE :

V.Imp

Random Process :- (Not-fixed)

Q 1) defined Gaussian Random Process.
Give Properties of same with necessary
Two.
OR. differentiate betn Random Process and
Variable with example. OR
Differentiate between ergotic and
stationary Process. OR
Differentiate betn stationary widesense
stationary and erogatic random process.
explain the meaning of stationary, wide
sense stationary and ergotic process.
write short note on stationary and ergotic
Process.

The concept of random process is an extension of random variable consider example of temperature of certainsity in the afternoon. The temp. X is a random variable which will take different values every day to get the complete statistic of X. We need to record value of X in afternoon over many days. from these data we can find $F_X(x)$ i.e CDF and PDF of random variable X.

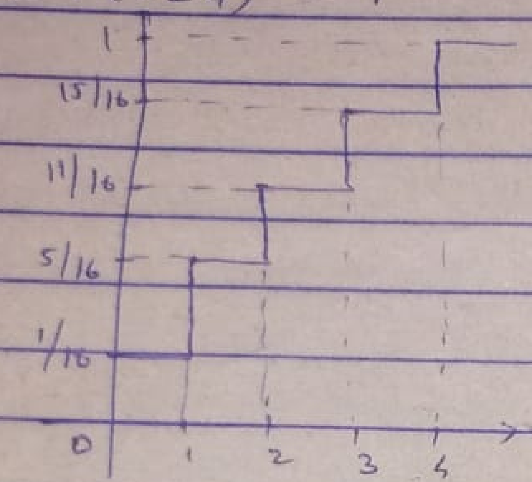
PAGE NO
DATE

$\therefore P(X \leq 4) = P(X=0) + P(X=1) + P(X=2)$
$+ P(X=4)$

$= 1/16 + 4/16 + 6/16 + 4/16 + 1/16$

$= 16/16$

$\therefore P(X \leq 4) = 1$



V.imp (marks)

2) find k such that $f(x) = k\cos(x)$ $-\pi/2 < x < \pi/2$

Qualifies as PDF. find k.

Given:

$f(x) = k\cos(x)$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$$

$$1 = \int_{-\pi/2}^{\pi/2} k\cos x\,dx$$

$$1 = k\int_{-\pi/2}^{\pi/2} \cos x\,dx$$

① $P(X=0) = 1/16$

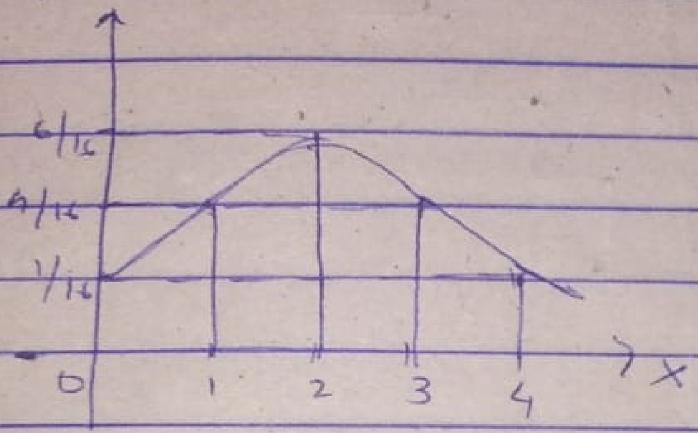
② $P(X=1) = 4/16$

③ $P(X=2) = 6/16$

④ $P(X=3) = 4/16$

⑤ $P(X=4) = 1/16$

curve for PDF :-



② CDF = ?

$F_X(x) = P(X \leq x)$

$P(X \leq 0) = 1/16$

$P(X \leq 1) = P(X=0) + P(X=1) = 1/16 + 4/16$

$P(X \leq 1) = 5/16$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16}$

$P(X \leq 2) = 11/16$

$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=$

$= 1/16 + 4/16 + 6/16 + 4/16$

$P(X \leq 3) = 15/16$

01/02/.. Friday

PAGE NO. (25)
DATE

In a random experiment in a trial consist of four successive toss of a coin if we defined random variable X. as a number of heads appearing in the trial determine PDF and CDF also draw PDF and CDF curve

Ans :-

$CDF = F_X(x) = P(X \le x)$

$PDF = f_X(x) = \frac{d}{dx} CDF$

$n(s) = x^n$

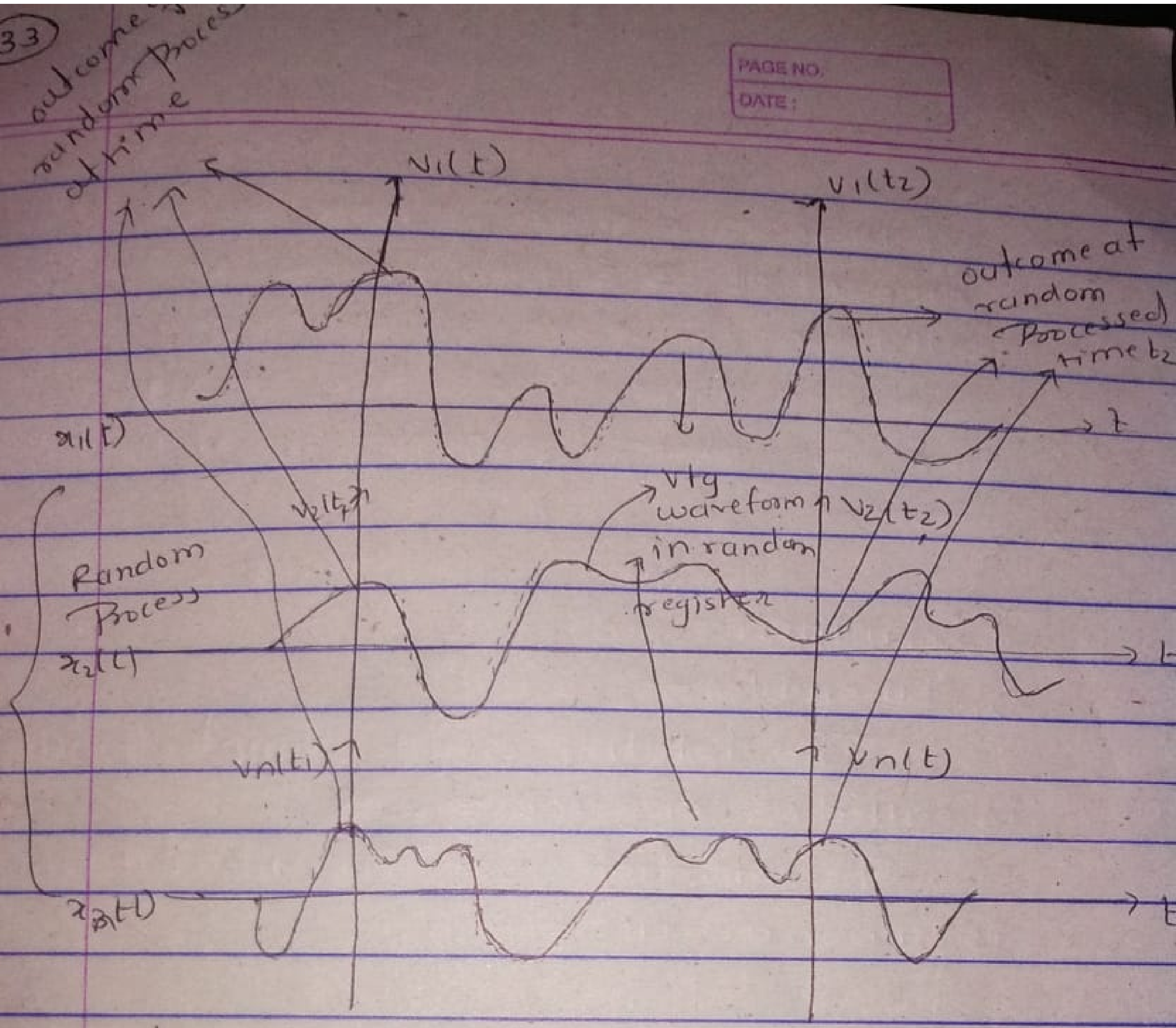
$n(s) = 2^4$ $\quad 0 \to H$

$= 16$ $\quad 1 \to T$

$$S = \left\{ \begin{array}{cc} 0000 & 1000 \\ 0001 & 1001 \\ 0010 & 1010 \\ 0011 & 1011 \\ 0100 & 1100 \\ 0101 & 1101 \\ 0110 & 1110 \\ 0111 & 1111 \end{array} \right\}$$

$X = \{0, 1, 2, 3, 4\}$

(1) PDF = ?



Stationary Process :-

The Process which appears stationary for certain Period of time or specified Period of time is called as stationary.

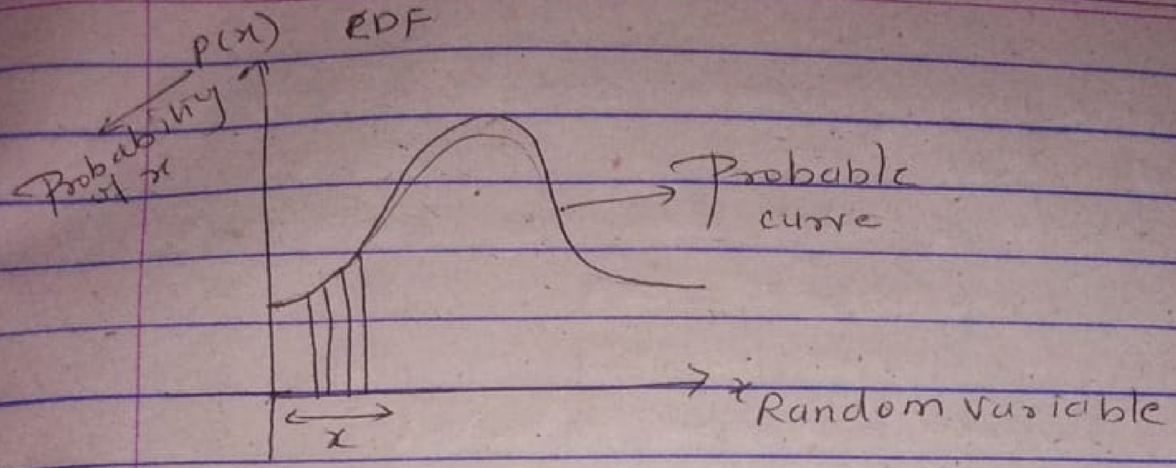
A random Process $x(t)$ is said to be stationary if the statistics or complete observation are not affected by change in time origin

But the Temperature is also a function of time. at 11 am The temperature may have an entirely different distribution that of temperature at afternoon. thus random Variable X. would be a function of time. and can be represented as X(t). "A random variable i.e. function of time. is called as random process. OR if random Variable associated with time i.e. different outcome at different value / time. then it is called as random process or Stochastic Process"

Hence in short Random is the collection of infinite number of random Variable at different time.

Consider a set of voltage waveform. generated by thermal electrons in large number of identical registers whose waveform is given by.

PAGE NO.
DATE :



CDF

$$\boxed{F(x) = CDF = \cancel{P(X)}\ P(X \leq x)}$$

X - probable random variable
x - Random variable

The CDF of random variable X is a Probability that random variable X. takes a value less than or equals to variable x.

CDF is denoted by F(x).

The CDF can be defined for discrete random variable which is given by,

$$F(x) = CDF = P(X \leq x)$$

Properties of CDF :-

1.) $0 \leq f_X(x) \leq 1$

As CDF is the probability that random variable X takes a value less than or equals to x. and we know that probab

(27)

$$1 = K \left[\sin x\right]_{-\pi/2}^{\pi/2}$$

$$1 = K\left[\sin(\pi/2) - \sin(-\pi/2)\right]$$

$$1 = 2K$$

$$\therefore \boxed{K = 1/2}$$

3) A digital ckt produces 4 bit binary Number at random all the numbers are produces with equal probability. A random variable x is defined as no. of ones in the circuit. find CDF. and sketch it.

$\Rightarrow$

$$n(S) = x^n = (2)^4$$

$$n(S) = 16$$

$$S = \begin{cases} 0000 & 1000 \\ 0001 & 1001 \\ 0010 & 1010 \\ 0011 & 1011 \\ 0100 & 1100 \\ 0101 & 1101 \\ 0110 & 1110 \\ 0111 & 1111 \end{cases}$$

$$X = \{0, 1, 2, 3, 4, 5\}$$

$$f_X(x) = PDF = P(X) = \frac{d}{dx} F_X(x)$$

Properties of PDF ÷

1) $f_X(x) \geq 0$

For the PDF to be exist function $f(x)$ must be Positive. i.e. $f_X(x) \geq 0$.

2) $\int_{-\infty}^{\infty} f_X(x)\, dx = 1$

Area under PDF curve always equals to 1.

3) $f_X(x) = \frac{d}{dx} F_X(x)$

PDF can be obtained by differentiating CDF w.r.t. variable x.

$\therefore PDF = \frac{d}{dx} CDF$

4) ~~PDF~~ CDF $= F_X(x) = \int_{-\infty}^{\infty} f_X(x)\, dx$

CDF can be obtained by integrating PDF w.r.t. variable x.

i.e. $CDF = \int_{-\infty}^{\infty} PDF$

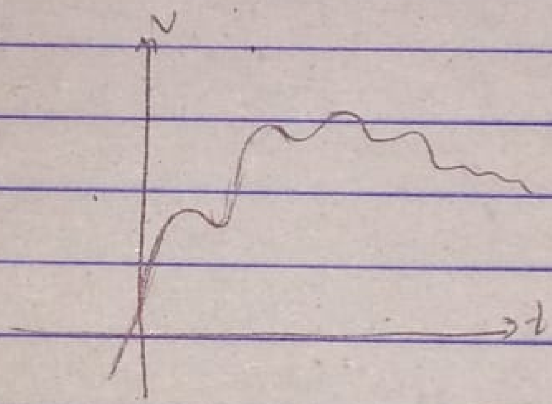
5) $P(x_1)$ $P(x_1 \leq X \leq x_2) = \int_{x_1}^{x_2} f_X(x)\, dx$

then random variable x will take 6 values which are finite and countable.

2) Continuous Random Variable :-

A Random variable x is said to be continuous random variable if it takes infinite uncountable value from a sample space.

ex. Noise of voltage generated by random one.



Imp

Distribution Function :-

① What do you mean by CDF and PDF. What are properties. state and prove it.

⇒ CDF → cumulative

PDF → Probability

there are two types of distribution function

1) cumulative distribution function

2) Probability distribution function

PDF = ?

1. $P(X=0) = 1/16$
2. $P(X=1) = 4/16$
3. $P(X=2) = 6/16$
4. $P(X=3) = 4/16$
5. $P(X=4) = 1/16$

CDF = ?

$P(X \le 0) = 1/16$

$P(X \le 1) = P(X=0) + P(X=1)$

$= 1/16 + 4/16 = 5/16$

$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 1/16 + 4/16 + 6/16$

$= 11/16$

$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= 1/16 + 4/16 + 6/16 + 4/16$

$= 15/16$

$P(X \le 4) = P(X=0) + P(X=1) + P(X=2)$

$+ P(X=3) + P(X=4)$

$= 1/16 + 4/16 + 6/16 + 4/16 + 1/16$

$= 1$

Autocorelation:-

It is denoted by $R(\tau)(t_1 t_2)$ and it can also be represents in the term of ensembler mean as

$$\therefore R_\tau(t_1 t_2) = E[x(vt_1) x(vt_2)]$$

$$= E[x(t_1) x(t_2)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f(x_1, x_2)\, dx_1\, dx_2$$

Ergodic Process:-

A Random Process is called as ergodic Process.

Que. consider a random process $x(t) = A\cos(\omega_0 + \theta)$. where A and ω_0 are constant and θ is a random variable distributed on $-\pi$ to π. check whether $x(t)$ is the widesense stationary Process. OR check whether $x(t)$ is the ergodic Process.

$\Rightarrow$ $x(t) = A\cos(\omega_0 t + \theta)$

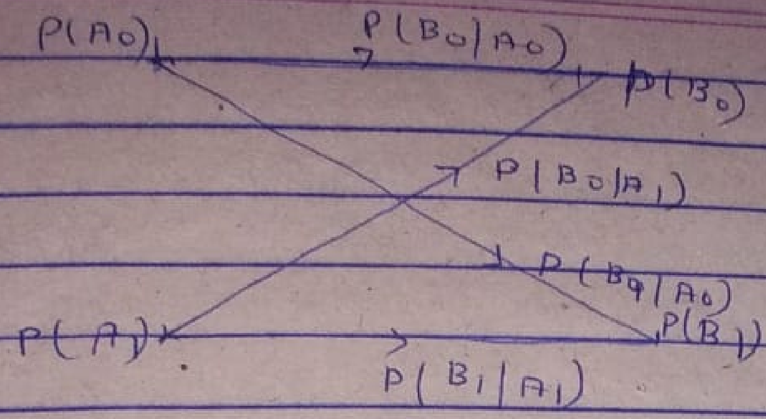
$-\pi$ to π

θ - upperlimit - lowerlimit.

$= \pi - (-\pi)$

$\theta = 2\pi$

(15)



$\therefore P(B_0) = P(B_0/A_0)\cdot P(A_0) + P(B_0/A_1)\cdot P(A_1)$

$= 0.9 \times 0.7 + 0.4 \times 0.3$

$= 0.63 + 0.12$

$P(B_0) = 0.75$

$P(B_1) = P(B_1/A_1)\, P(A_1) + P(B_1/A_0)\cdot P(A_0)$

$= 0.6 \times 0.3 + 0.1 \times 0.7$

$P(B_1) = 0.25$

$P(C) = P(B_0/A_0)\, P(A_0) + P(B_1/A_1)\cdot P(A_1)$

$= 0.9 \times 0.7 + 0.6 \times 0.3$

$= 0.63 + 0.18$

$P(C) = 0.81$

$P_e = 1 - P_c = 1 - 0.81 = 0.19$

ORA = ?

Condn: $P(B_0/A_0)\, P(A_0) > P(B_0/A_1)\, P(A_1)$

$0.9 \times 0.7 > 0.4 \times 0.3$

$0.63 > 0.12$

Hence Ao was transmitted and Bo was Received.

PAGE NO.
DATE :

Conditional probability:-

Defined conditional joined and independant Probability find relation between them.

Consider and experiment involves to event A and B. probability of A given that B has occured. it represent by P(A B).

Probability B given that of A has occured is given by P(B/A).

P(B|A) and P(A|B) are called as conditional Probability which is given by

$$P(A/B) = \frac{P(AB)}{P(B)} \quad ; \quad P(B/A) = \frac{P(AB)}{P(A)}$$

Probability of Independent event:-

If A and B are two Possible event of an experiment and Possiblity of occurs A does not dependent of occur of B.

or vice versa, Then event A and B are called as Independent event and Possibility this event are called as Possibility of Independent event

PAGE NO.
DATE :

Probability of Events

It is given by $P(A) = \frac{n(A)}{n(S)}$ which is

equal to
$$P(A) = \frac{\text{no. of Possible event}}{\text{Total no. of desired o/P}}$$

* If event A and B cannot occur simultaneously then event A and B set to be mutual exculsive event. and if A and B are mutually exclusive event then probability of event A and B it is given by

$$P(A+B) = P(A) + P(B)$$
$$= \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$$

If A and B are not mutually exclusive then $P(A) + P(B) - P(AB)$

5

A = Exactly two tails

$n(A) = 10$

$$P(A) = \frac{n(A)}{n(\Omega)} = \frac{10}{32} = \frac{5}{16}$$

S = {
00000
00001
00010
00011
00100
00101
00110
00111
01000
01001
01010
01011
01100
01101
01110
01111
10000
10001
10010
10011
10100
10101
10110
10111
11000
11001
11010
11011
11100
11101
11110
11111

36

$f(\theta) = 2\pi$

$$M_X(t) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{-\pi}^{\pi} A\cos(\omega_0 t + \theta)\, 2\pi\, d\theta$$

$$= A 2\pi \int_{-\pi}^{\pi} \cos(\omega_0 t + \theta)\, d\theta$$

$$= A 2\pi \left[\sin\theta\right]_{-\pi}^{\pi}$$

$$= 0$$

$$\boxed{M_X(t) = 0}$$

Autocorelation = ?

$$R_Z(t_1 t_2) = E[X(v_1 t_1)\, X(v t_2)]$$

$$= E[X(t_1)\, X(t_2)]$$

$$= E[A\cos(\omega_0 t + \theta)\;\; A\cos(\omega_0 t_2 + \theta)]$$

$$= E\left[A\cos(\omega_0 t_1 + \theta) \cdot A\cos(\omega_0 t_2 + \theta)\right]$$

$\therefore \cos A \cdot \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$$= \frac{A^2 E}{2}\left[\cos(\omega_0 t_1 + \theta - \omega_0 t_2 - \theta) + \cos(\omega_0 t_1 + \theta + \omega_0 t_2 + \theta)\right]$$

$$= \frac{A^2 E}{2}\left[\cos(\omega_0 t_1 - \omega_0 t_2) + \cos(\omega_0 t_1 + \omega_0 t_2 + 2\theta)\right]$$

PAGE NO

DATE

$$R_\tau(t_1 t_2) = \frac{A^2}{2} E\left[\cos(\omega_0 t_1 - \omega_0 t_2) + \cos(\omega_0 t_1 + \omega_0 t_2 + 2\theta)\right]$$

$$R_\tau = \frac{A^2 \cos(\omega_0 t_1 - \omega_0 t_2)}{2} + \frac{A^2}{2}\int_{-\pi}^{\pi} \cos(\omega_0 t_1 + \omega_0 t_2 + 2\theta)\, d\theta$$

$$R_\tau(t_1 t_2) = \frac{A^2 \cos(\omega_0 t_1 - \omega_0 t_2)}{2}$$

$$R_\tau(t_1 t_2) = \frac{A^2 \cos[\omega_0(t_1 - t_2)]}{2}$$

Ass que 2) b) What is the Purpose of fourier

(1) A town has a Population of 10,000 People of this 60,000 are male and 4000 female also 300 male and 400 female of this Population are the unemployed and unemployed Person is choose that random what is the Probability that he is male.

→

$T = 10000$ n

$M - 6000$ ⟷

$F - 4,000$

$n(A) = 300$ UEM $TUE = 700$

$n(B) = 400$ WEM

12 Marks

Monday
28/01/2019 ①

2. Probability

What is the importance of Probability in communication system.

⇒ In the chapter of fourier Transform we have study different signals and for this signal's we can write mathematical equation by using Transform and fourier Series we can easily analyzed. Then in time and frequency domain.

But their are other class of signal for which we cannot right mathematical eqn and hence fourier Transform Series and analysis of this Signal are not Possible.

exam:-

Noise Signal which can random in nature we can not write mathematic eqn of these Signal and hence fourier Transform and series analysis of this Signal is not Possible in commn system using Probability therby we can find out Probability of current respectrum i.e PC and Probability of error

Tuesday 29/01/19

PAGE NO. 6

DATE:

1) Two dies are Thrown determine the Probability the sum of dies are even.

$\Rightarrow$ x^n

n-experiment

$(6)^2 = 36$

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(S) = 36$

Event A :- Sum of dies are even

$n(A) = 18$

Probability $P(A) = \frac{n(A)}{n(S)}$

$= \frac{18}{36}$

$\therefore \boxed{P(A) = \frac{1}{2}}$

2) A coin is tosed 4 times in succession determine the probability of obtaining exactly two heads.

①

Definitions:-

Experiment:- The process in which outcomes or not known is called as experiment.
ex:- Tossing a coin Rolling a dices.

Sample space:- The total Number of outcomes in at experiment is known as sample space.
ex:- If we tossed a coin two time then sample space will be

$S = \{TT, HH, HT, TH\}$

$\therefore n(S) = 4$

Event:- It is desired outcome from an experiment.
ex. If we roll 6 sided dies then A is the event occuring even no. then event is
$A = \{2, 4, 6\}$
$n(A) = 3$

PAGE NO.

DATE :

P(UEM)

$$P(A) = \frac{n(A)}{n(S)} = \frac{300}{6000} = \frac{1}{20}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{400}{4000} = \frac{1}{10}$$

Conditional Probability

$$P(A|B) = P(AB) / P(B)$$

$$P(AB) = P(A) \cdot P(B)$$

$$= \frac{1}{20} \times \frac{1}{10}$$

$$= \frac{1}{200}$$

$$P(A|B) = \frac{1/200}{1/10}$$

$$= \frac{10}{200}$$

$$= \frac{1}{20}$$

$$\boxed{P(A|B) = 1/20}$$

Q. If there is a shifft in time by an amount of ϵ then Process at $X(t+\epsilon)$ will have the same statistics or a statistical Property as it had at $X(t)$ then this process is called as stationary Process.

Definition :-

1) Ensembler :-

The totality of all sample function is called as ensembler. OR

The collection of all Possible outcomes is known as ensembler.

2) Ensembler Mean :-

If $V_1(t_1)$, $V_2(t_1)$, $V_3(t_1)$ .... $V_n(t_1)$ are Possible outcome at time t_1. then ensembler mean is given by.

Ensembler mean denoted by $m_X(t)$

$$m_X(t) = V_1(t_1) + V_2(t_1) + V_3(t_1) \ldots V_n(t_1)$$

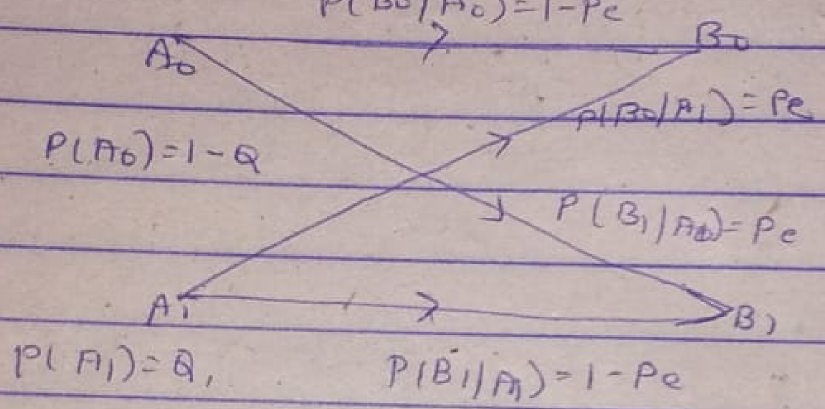
$$m_X(t) = E[X(V, t_1)]$$

$$= \int_{-\infty}^{\infty} x\, f(x)\, dx$$

= CDF

Q.1) A Binary symmetric channel has error Probability P_e. The probability of Transmitting 1 is Q. and that of Transmitting is zero is $1-Q$. determine the probability of receiving 1 and 0. at the Receiver.

Ans:-



$$P(B_0) = P(B_0|A_0)P(A_0) + P(B_0|A_1)\cdot P(A_1)$$
$$= (1-P_e)\cdot(1-Q) + P_e \cdot Q$$
$$= 1 - Q - P_e + P_eQ + P_eQ$$
$$P(B_0) = 1 - Q - P_e + 2P_eQ$$

$$\therefore P(B_1) = P(B_1|A_1)\,P(A_1) + P(B_1|A_0)\cdot P(A_0)$$
$$= (1-P_e)(\cancel{1-}Q) + P_e(1-Q)$$
$$= \cancel{Q+Q}\; Q - P_eQ + P_e - P_eQ$$
$$P(B_1) = Q + P_e - 2P_eQ.$$

A = Sum of dies are seven

$n(A) = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$n(A) = 6$

$P(A) = \frac{n(A)}{n(S)}$

$= 6/36$

$P(A) = 1/6$

4) A coin is tossed five times in succession determine the Probability of obtaining exactly two tails.

⇒

$(2)^5 = 32$

$n(S) = 32$

$0 \rightarrow H$

$1 \rightarrow T$

$\therefore S = \{$

00000	~~1010~~	11001	01010
11111	~~0101~~	11101	01110
01111	~~0110~~	01000	00110
00111	~~0100~~	00100	01001
00011	~~1101~~	00010	11010
10000	~~0010~~	10111	01000
11000	1011	10011	
10001		10100	
00001		10010	
11100		11010	
11110		~~10110~~	
10001		10101	

$\}$

6) Two dies are thrown determine the Probability that sum of the dies are odd Number.

$\Rightarrow$

$x^n = (6)^2 = 36$

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

$n(s) = 36$

A = Sum of dies are odd Number

$$n(A) = \begin{Bmatrix} (1,2) & (1,4) & (1,6), & (2,1) & (2,3) & (2,5) \\ (3,2) & (3,4) & (3,6) & (4,1) & (4,3) & (4,5) \\ (5,2) & (5,4) & (5,6) & (6,1) & (6,3) & (6,5) \end{Bmatrix}$$

$n(A) = 18$

$$P(A) = \frac{n(A)}{n(s)} = \frac{18}{36} = \frac{1}{2}$$

(16)

PAGE NO.

DATE :

$cond^n 2 :-$ $P(B_1/A_1)\,P(A_1) > P(B_1/A_0)\cdot P(A_0)$

$0.6 \times 0.3 > 0.1 \times 0.7$

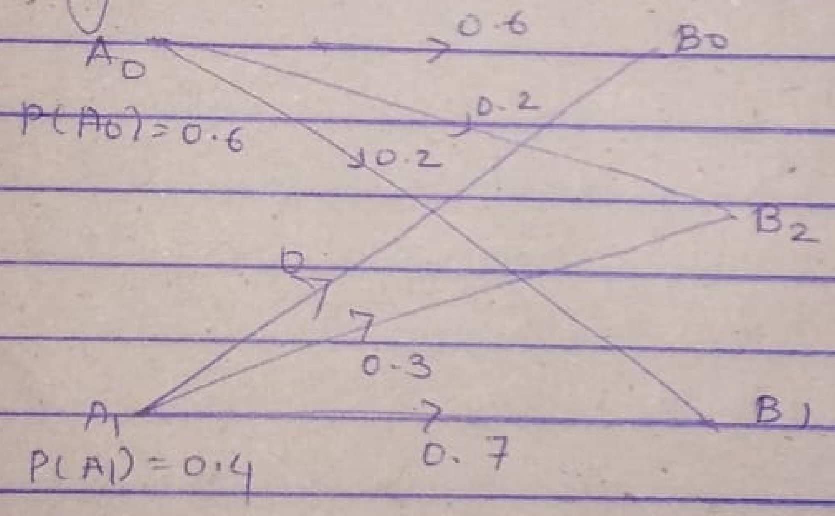
$0.18 > 0.07$

Hence A_1 was Transmitted and B_1 was Receiver.

8 marks

2) For the system shown A_0 will generate B_0. A_1 generate B_1. with certainity if there is no noise. B_0 will never occur, calculate the probability of B_0 i.e $P(B_0)$. B_1 i.e $P(B_1)$, B_2 i.e $P(B_2)$ also find probability of error and apply ORA.

$\Rightarrow$



$P(B_0) = P(A_0)\cdot P(B_0/A_0) + P(B_0/A_1)\,P(A_1)$

$= 0.6 \times 0.6 + 0 \times 0.4$

$P(B_0) = 0.36$

(10)

PAGE NO.
DATE :

5) A two contains Three socket ~~from~~ for 4 bulb from the collection of bulb ~~of~~ out of veg. four are detective 3 bulbs are selected at random and put into the socket. find the Probability that ~~The~~ room is brighter.

⇒ Number of bulb
↳ 4 defective - 3 socket

A	B	C
0	0	0
0	0	1
0	1	0
1	0	0
1	1	0
1	1	1
1	0	1
0	1	1

0 → detective
1 → Brighter

$n(s) = 8$

A = Room is brighter

$n(A) = 7$

$\therefore P(A) = \dfrac{n(A)}{n(s)} = \dfrac{7}{8}$

V. Imp

* Random Variable

The function which takes any value from the sample space and its range is some set of real Number then it is called as random variable.

ex (1)

If we tossed a coin, the Possible outcome are Head and tail i.e. sample space
$S = \{H, T\}$

ex. (2)

A dies is roll, and we get six diff. event at ~~six~~ 6 different instant. The sample space.

$$S = \{1, 2, 3, 4, 5, 6\}$$

* Two types of Random Variable :-
1) Discrete Random Variable :- (countab
2) continuous Random Variable :-

(1) Discrete Random Variable :-

A Random Variable x is set to be discrete If it takes finite countable numbers of values from a sample space.

ex. Tossing a coin in which Random Variable x will take only two values Head or Tail

Similarly, if we thrown a dies

S - source
D - destination

$P(B_1/A_0)$ - Probability that B_1 is Receiver and A_0 is Transmitter.

Binary data can be 0 and 1. These data are Transmitted through the channel from S to D. The channel can be wire or wireless. and satelite etc.

The data been Transmitted can be corrected by noise. If (1) is Transmitted 0 is Receive.

Similarly, if 0 is Transmitted 1 is received by creating error in the Transmission.

* Probability of Receiving 0 is given by $P(B_0)$

$$P(B_0) = P(A_0) \cdot P(B_0/A_0) + P(A_1) \cdot P(B_0/A_1)$$

* Probability of Receiving 1 is given by

$$P(B_1) = P(A_0) \; P(B_1/A_0) + P(A_1) \cdot P(B_1/A_1)$$

* Probability of correct Reception is given by

$$\therefore P_c = P(B_0/A_0) \, P(A_0) + P(B_1/A_1) \cdot P(A$$

(17)

PAGE NO

DATE

$$P(B_1) = P(B_1/A_0)P(A_0) + P(B_1/A_1)P(A_1)$$
$$= 0.2 \times 0.6 + 0.7 \times 0.4$$
$$= 0.12 + 0.28$$
$$P(B_1) = 0.4$$

$$P(B_2) = P(B_2/A_0) \cdot P(A_0) + P(B_2/A_1)P(A_1)$$
$$= 0.2 \times 0.6 + 0.3 \times 0.4$$
$$= 0.12 + 0.12$$
$$P(B_2) = 0.24$$

$$P(C) = P(B_0/A_0)P(A_0) + P(B_1/A_1)P(A_1) + P(B_2/A_0)P(A_0)$$
$$= 0.6 \times 0.6 + 0.7 \times 0.4 + 0.2 \times 0.6$$
$$P(C) = 0.76$$

$$P_e = 1 - P_C = 1 - 0.76 = 0.24$$

ORA = ?

cond^n 1 :- $P(B_0|A_0)P(A_0) > P(B_0|A_1)P(A_1)$
$$= 0.6 \times 0.6 > 0 \times 0.4$$
$$= \cancel{0.36}\ 0.36 > 0$$

Hence, A_0 was Transmitted and B_0 was Received.

cond^n 2 :- $P(B_1|A_1)P(A_1) > P(B_1|A_0)P(A_0)$
$$= 0.7 \times 0.4 > 0.2 \times 0.6$$
$$0.28 > 0.12$$

Hence, A_1 was Transmitted and B_1 was Received.

PAGE NO.
DATE :

Probability of error is given by
$P_e = 1 - P_c$

Optimum Receiver algorithm :-
Optimum Receiver algorithm deals with determination of Transmitted bit w.r.t Received bit.

1) If B_0 is Received then select A_0 if and only if.

$$P(B_0/A_0)P(A_0) > P(B_0/A_1)P(A_1)$$

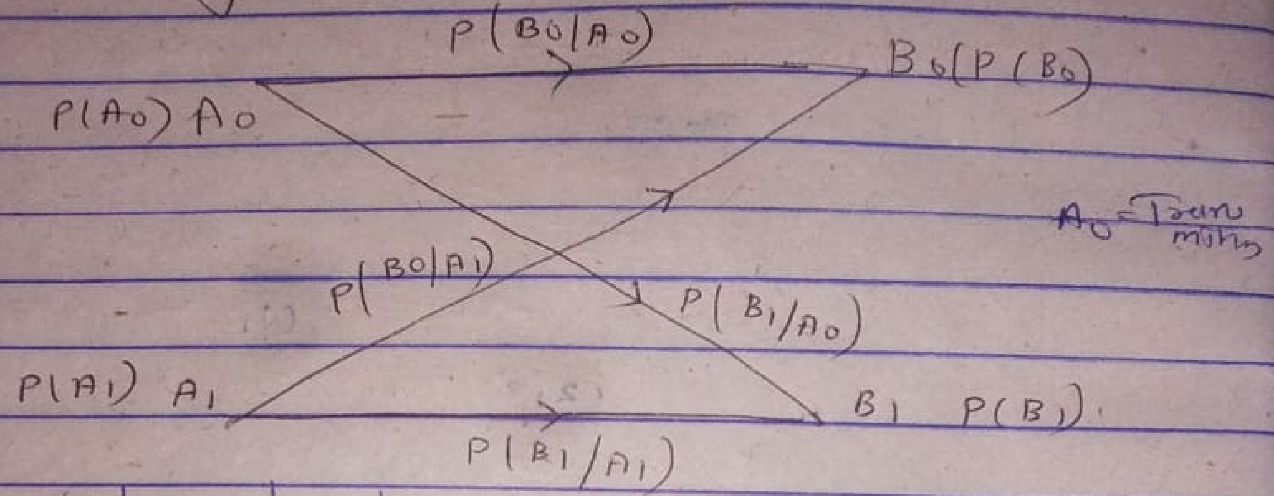
Note 2) If B_1 is Received, then select A_1 if and only if

$$P(B_1/A_1)P(A_1) > P(B_1/A_0)P(A_0)$$

else, select A_0

Q:1) The probability of given binary symmetric channel are. $P(A_0) = 0.7$ $P(A_1) = 0.3$
$P(B_0|A_0) = 0.9$, $P(B_1|A_1) = 0.6$
$P(B_0/A_1) = 0.4$, $P(B_1|A_0) = 0.1$ find
$P(B_0)$, $P(B_1)$, $P(C)$ and ORA.

Binary Communication:-



Q.1) Write short Note on binary symmetric channel. what is optimum Receiver algorithm

⟹ A_0, A_1 – event of Transmitting binary 0 and 1 respectively.

B_0, B_1 – event of Receiving binary Number 0 and 1 Respectively.

$P(A_0)$ and $P(A_1)$ – Probability of Transmitting binary 0 and 1 respectively.

$P(B_0)$ and $P(B_1)$ – Probability of Receiving binary 0 and 1 respectively.

$P(B_0/A_0)$ – Probability that A_0 is Transmitted and B_0 is receiving

$P(B_1/A_1)$ – Probability that B_1 is receiver and A_1 is Transmitter.

$P(B_0/A_1)$ – Probability that B_0 is receiver and A_1 is Transmitter

7

PAGE NO:
DATE:

Ans :- $n(s) = (x)^n$ Assume $0 \rightarrow T$

$= (2)^4$ $1 \rightarrow H$

$n(s) = 16$

$S = \begin{cases} 0000 & 1001 \\ 1111 & 1101 \\ 0111 & 1011 \\ 0011 & 0100 \\ 0001 & 1010 \\ 1000 & 0101 \\ 1100 & 0110 \\ 1110 & 0100 \end{cases}$

A = Exactly Two head

$n(A) = 6$

$P(A) = \frac{n(A)}{n(s)}$

$\therefore P(A) = 6/16$

Two dies are thrown determine probabi-lity that sum of dies are seven.

→

$n(s) = (6)^2 = 36$

$S = \begin{cases} (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) \\ (2,1) (2,2) (2,3) (2,4) (2,5) (2,6) \\ (3,1) (3,2) (3,3) (3,4) (3,5) (3,6) \\ (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) \\ (5,1) (5,2) (5,3) (5,4) (5,5) (5,6) \\ (6,1) (6,2) (6,3) (6,4) (6,5) (6,6) \end{cases}$

$$P(A)\,P(B) = P(AB)$$

Joined Probability:

Joined probabilty of xy is represented by $P(x,y)$ let x,y with two random variable Then there are two times of joined probability.

① Joined probability of desired random variable. It is given by

$$P(x,y) = \sum_{x_i}\sum_{y_j} f(x_i)\,f(y_j)$$

$P(x,y)$ = will occur only $f(x_i y_j) \geq 0$

② Joined Probability for continous random Variable is given by

$$\therefore P(x,y) = \int_{-\infty}^{a}\int_{\infty}^{b_1} f(x)\,f(y)\,dx\,dy$$

1) $f(x_i y_j) \geq 0$

2) $\int_{-\infty}^{a}\int_{-\infty}^{b} f(x_i y_j)\,dx\,dy = 1$